NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF MECHATRONICS ENGINEERING

COURSE MATERIALS

MR404 POWER ELECTRONICS AND DRIVES

VISION OF THE INSTITUTION
To mould true citizens who are millennium leaders and catalysts of change through excellence in education.
MISSION OF THE INSTITUTION
NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values. We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- Established in: 2013
- Course offered: B.Tech Mechatronics Engineering
- Approved by AICTE New Delhi and Accredited by NAAC
- Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

To develop professionally ethical and socially responsible Mechatronics engineers to serve the humanity through quality professional education.

DEPARTMENT MISSION

1) The department is committed to impart the right blend of knowledge and quality education to create professionally ethical and socially responsible graduates.

2) The department is committed to impart the awareness to meet the current challenges in technology.

3) Establish state-of-the-art laboratories to promote practical knowledge of mechatronics to meet the needs of the society

PROGRAMME EDUCATIONAL OBJECTIVES

I. Graduates shall have the ability to work in multidisciplinary environment with good professional and commitment.

II. Graduates shall have the ability to solve the complex engineering problems by applying electrical, mechanical, electronics and computer knowledge and engage in lifelong learning in their profession.

III. Graduates shall have the ability to lead and contribute in a team with entrepreneur skills, professional, social and ethical responsibilities.

IV. Graduates shall have ability to acquire scientific and engineering fundamentals necessary for higher studies and research.

PROGRAM OUTCOME (PO'S)

Engineering Graduates will be able to:

PO 1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO 2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO 3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO 4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO 5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO 6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO 7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO 8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO 9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO 10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO 11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO 12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOME(PSO’S)

PSO 1: Design and develop Mechatronics systems to solve the complex engineering problem by integrating electronics, mechanical and control systems.

PSO 2: Apply the engineering knowledge to conduct investigations of complex engineering problem related to instrumentation, control, automation, robotics and provide solutions.

COURSE OUTCOME

After the completion of the course the student will be able to

C411.1	Understand the concepts of power semi conductor devices
C411.2	Describe about phase controlled converters
C411.3	Acquire knowledge about the design of choppers nad switching regulators
C411.4	Understand the woring of fixed DC to variable AC converters an to learn the modulation techniques employed in inverters
C411.5	Determine the performance parameters of controlled rectifiers and AC voltage controllers
C411.6	Acquire knowledge about the concepts of electric drive

CO VS PO'S AND PSO'S MAPPING

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PS01	PSO2
C411.1	3	-	-	-	-	-	-	-	-	-	-	**2**	**2**	**2**
C411.2	**3**	-	-	-	-	-	-	-	-	-	-	**2**	**2**	**2**
C411.3	**3**	-	**2**	-	-	-	-	-	-	-	-	**2**	**2**	**2**
C411.4	**3**	-	**2**	-	-	-	-	-	-	-	-	**2**	**2**	**2**
C411.5	**3**	-	**2**	-	-	-	-	-	-	-	-	**2**	**2**	**2**
C411.6	**3**	-	**2**	-	-	-	-	-	-	-	-	**2**	**2**	**2**

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

Course code	Course Name	L-T-P - Credits	Year of Introduction
MR 404	**Power Electronics and Drives**	**3-0-0:3**	**2016**

Prerequisite : Nil

Course Objectives

- To give an overview of different types of power semiconductor devices and their switching characteristics.
- To understand the operation, characteristics and performance parameters of controlled rectifiers.
- To study the operation, switching techniques and basic topologies of switching regulators

Syllabus

Power semi conductor devices- characteristics of power diodes- SCR- TRIAC- GTO- power BJT-power MOSFET and IGBT –- phase controlled converters-single phase full converters- 3 phase half converter and 3 phase full converter – input power factor – thyristor triggering circuits- dc to dc choppers-dc chopper – step up and step down chopper – forced commutation – different techniques – voltage- current and load – commutated choppers – inverters-voltage source inverters – series- parallel and bridge inverters – PWM inverters – current source inverters- ac voltage controllers and cyclo converters-single phase ac voltage controller – multistage sequence control – step up and step down cyclo converters –introduction to electric drives– advantages- parts of electrical drives – fundamental torque equation – four quadrant operation – components of load torque

Expected outcome .

The students will be able to

- analyse the dynamic and switching characteristics of power semiconductor devices.
- determine the performance parameters of controlled rectifiers and AC voltage controllers.
- design Choppers and Switching Regulators.
- understand the working of Fixed DC to Variable AC converters and learn the Modulation Techniques employed in Inverters

Text Books:

1. Bhimbra P S, *Power Electronics,* Khanna Publishers, 2001
2. Reshid M.H., Power Electronics – Circuits Devices and Application, Prentice Hall International, New Delhi, 3rd Edition, 2004

References:

1. Dubey, G.K., Doradia, S.R., Joshi, A. and Singh, R.M., Thyristorised Power Controllers, Wiley Eastern Limited, 1986.
2. Joseph Vithayathil, Power Electronics – Principle and Applications, and Robbins, McGraw-Hill Inc, New York, 1995.
3. Lander, W., Power Electronics, McGraw-Hill and Company, 3rd Edition, 1993.
4. Mohan Undeland and Robbins, Power Electronics, John Wiley and Sons, New York, 1995
5. Singh, M.D., Khanchandani, K.B., Power Electronics, Tata McGraw-Hill, 1998.

Course Plan

Module	Contents	Hours	Sem. Exam Marks
I	**POWER SEMI CONDUCTOR DEVICES** Principle of operation – Characteristics of power diodes- SCR-TRIAC- GTO- Power BJT- Power MOSFET and IGBT – Thyristor protection circuits.	7	15%

II	**PHASE CONTROLLED CONVERTERS** Single phase full converters- 3 phase half converter and 3 phase full converter – inverter operation – input power factor – effect of source inductance – Thyristor triggering circuits.	7	15%
FIRST INTERNAL EXAMINATION			
III	**DC TO DC CHOPPERS** DC Chopper – Principle of operation – step up and step down chopper – Forced commutation – different techniques – voltage- current and load – commutated choppers – step up and step down chopper.	7	15%
IV	**INVERTERS** Voltage source inverters – series- parallel and bridge inverters – PWM inverters – current source inverters.	7	15%
SECOND INTERNAL EXAMINATION			
V	**AC VOLTAGE CONTROLLERS AND CYCLOCONVERTERS** Single phase AC voltage controller – multistage sequence control – step up and step down cyclo converters – three phase to single phase and three phase cyclo converters.	7	20%
VI	**INTRODUCTION TO ELECTRIC DRIVES** Electrical Drives – advantages of electric drives - parts of electrical drives – fundamental torque equation – four quadrant operation – components of load torque - friction- windage & load torques – steady state stability	7	20%
END SEMESTER EXAM			

QUESTION PAPER PATTERN

Maximum Marks : 100 Exam Duration:3 hours

PART A: FIVE MARK QUESTIONS

8 compulsory questions –1 question each from first four modules and 2 questions each from last two modules (8 x 5= 40 marks)

PART B: 10 MARK QUESTIONS

5 questions uniformly covering the first four modules. Each question can have maximum of three sub questions, if needed. Student has to answer any 3 questions (3 x10 = 30 marks)

PART C: 15 MARK QUESTIONS

4 questions uniformly covering the last two modules. Each question can have maximum of four sub questions, if needed. Student has to answer any two questions

(2 x15 = 30 marks)

QUESTION BANK

MODULE I

Q:NO:	QUESTIONS	CO	KL	
1	Explain the principle of operation of power diodes.	CO1	K1	
2	Illustrate the characteristics of power diodes.	CO1	K2	
3	Discuss briefly about SCR	CO1	K2	
4	Write a short note on TRIAC	CO1	K2	
5	Elucidate about power BJT	CO1	K1	
6	Explain about power MOSFET	CO1	K1	
7	Describe about IGBT	CO1	K2	
8	Interpret about Thyristor protection circuit	CO1	K2	

MODULE II

Q:NO:	QUESTIONS	CO	KL	
1	Explain the principle of operation of Single phase full converters.	CO2	K2	
2	Illustrate the characteristics of 3 phase half converter.	CO2	K1	
3	Discuss briefly about 3 phase full converter	CO2	K1	
4	Write a short note on inverter operation	CO2	K2	
5	Elucidate about input power factor	CO2	K2	
6	Explain about effect of source inductance	CO2	K3	
7	Describe about Thyristor triggering circuits	CO2	K2	
8	Interpret about phase controlled converters	CO2	K2	

MODULE III				
1	Explain the principle of operation of DC Chopper	CO3	K1	
2	Illustrate the characteristics of DC TO DC CHOPPERS	CO3	K2	
3	Discuss briefly about step up chopper	CO3	K2	
4	Write a short note on step down chopper	CO3	K2	
5	Elucidate about Forced commutation	CO3	K1	
6	Explain about different techniques of commutation.	CO3	K1	
7	Describe about DC Chopper using R-L load	CO3	K2	
8	Interpret about different classes of DC choppers.	CO3	K2	
MODULE IV				
1	Explain the principle of operation of INVERTERS	CO4	K1	
2	Illustrate the characteristics of Voltage source inverters	CO4	K2	
3	Discuss briefly about series inverters	CO4	K2	
4	Write a short note on parallel inverters	CO4	K2	
5	Elucidate about bridge inverters	CO4	K1	
6	Explain about different techniques of commutation.	CO4	K1	
7	Describe about PWM inverters	CO4	K2	
8	Interpret about current source inverters.	CO4	K2	

MODULE V				
1	Explain the principle of operation of AC VOLTAGE CONTROLLERS	CO5	K1	
2	Illustrate the characteristics of CYCLOCONVERTERS	CO5	K2	
3	Discuss briefly about Single phase AC voltage controller	CO5	K2	
4	Write a short note on multistage sequence control	CO5	K2	
5	Elucidate about step up cyclo converters	CO5	K1	
6	Explain about three phase to single phase cyclo converters.	CO5	K1	
7	Describe about three phase cyclo converters	CO5	K2	
8	Interpret about single phase to three phase cyclo converters..	CO5	K2	

MODULE VI				
1	Explain the principle of operation of Electrical Drives	CO6	K1	
2	Illustrate the characteristics of advantages of electric drives	CO6	K2	
3	Discuss briefly about parts of electrical drives	CO3	K2	
4	Write a short note on fundamental torque equation	CO6	K2	
5	Elucidate about four quadrant operation	CO6	K2	
6	Explain about components of load torque	CO6	K2	

7	Describe about friction	CO6	K2	
8	Interpret about windage & load torques.	CO6	K2	
9	Explain the principle of operation of steady state stability	CO6	K2	

APPENDIX 1

CONTENT BEYOND THE SYLLABUS

S:NO	**TOPIC**
1	DIAC
2	INDUSTRIAL APPLICATION OF POWER ELECTRONICS

MODULE 1

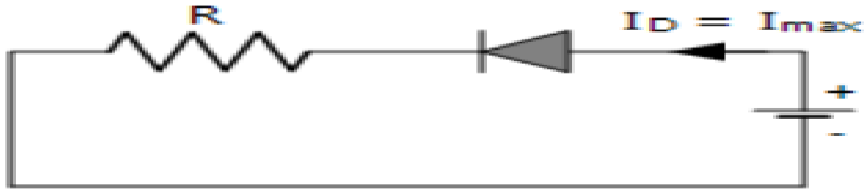
MODULE 1 POWER SEMICONDUCTOR DEVICES

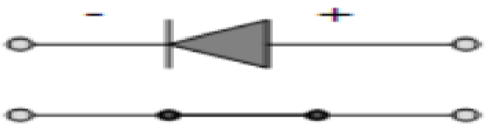
Principle of operation – Characteristics of power diodes- SCR -TRIAC- GTO- Power BJT- Power MOSFET and IGBT – Thyristor protection circuits.

Power Semiconductor Devices The first SCR was developed in late 1957. Power semiconductor devices are broadly categorized into 3 types:

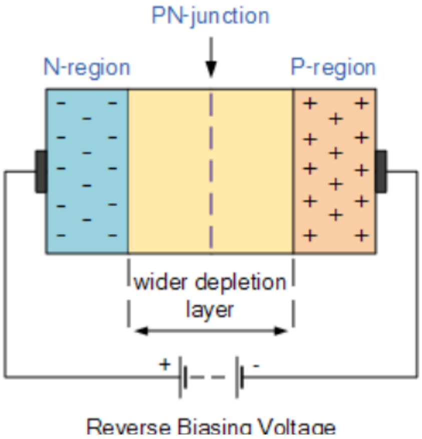
1. Power diodes 2. Transistors 3. Thyristors Power diodes
   Power diode principle: Diode is a two terminal P-N junction semiconductor device, with terminals anode (A) and cathode (C). Symbol:
   A semiconductor may be doped with acceptor impurities such as Boron (P-type doping), so that it contains mobile charges which are mainly holes.
   • The junction region itself has no charge carriers and is known as the depletion region.
   • The junction (depletion) region has a physical thickness that varies with the applied voltage.
   • When a diode is Zero Biased no external energy source is applied and a natural Potential Barrier is developed across a depletion layer which is approximately 0.5 to 0.7v for silicon diodes and approximately 0.3 of a volt for germanium diodes.
   • When a junction diode is Forward Biased the thickness of the depletion region reduces and the diode acts like a short circuit allowing full current to flow.
   • When a junction diode is Reverse Biased the thickness of the depletion region increases and the diode acts like an open circuit blocking any current flow, (only a very small leakage current).

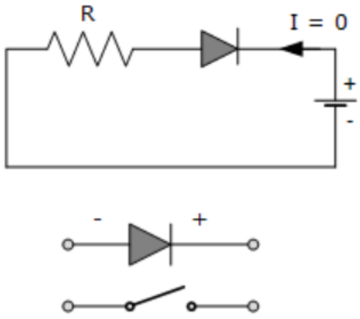
PN JUNCTION DIODE FORWARD BIAS



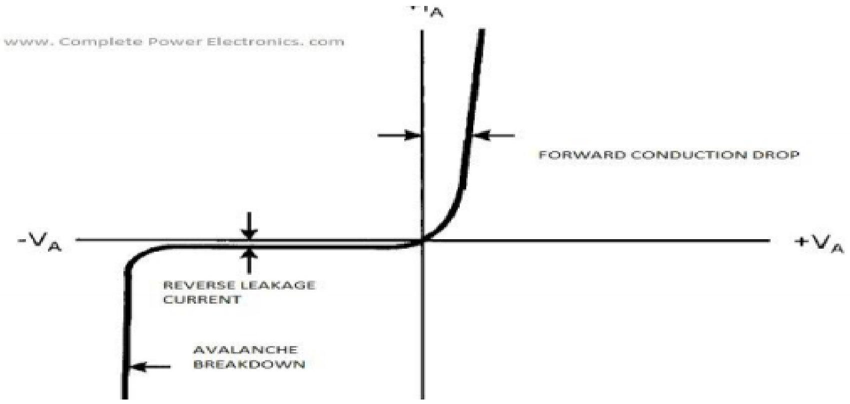


PN JUNCTION DIODE REVERSE BIAS





Power Diode Characteristics:



The reverse recovery characteristics of the Power diode is shown in the following figure. From the figure, we can understand the turn off characteristic of the diode. The Reverse recovery time trr is the time interval between the application of reverse voltage and the reverse current dropped to 0.25 of IRR. Parameter ta is the interval between the zero crossing of the diode current to it reaches IRR. Parameter tb is the time interval from the maximum reverse recovery current to 0:25 of IRR . The lower trr means fast diode switching. The ratio of the two parameters ta and tb is known as the softness factor SF.

Diode Protection: Snubber circuits are essential for diodes used in switching circuits. It can save a diode from overvoltage spikes, which may arise during the reverse recovery process. A very common snubber circuit for a power diode consists of a capacitor and a resistor connected in parallel with the diode.

Power Diode Applications:

As a rectifier Diode, For Voltage Clamping As a Voltage Multiplier, As a freewheeling Diode.

Types of Power Diode:

Schottky diodes: These diodes are used where a low forward voltage drop (usually 0.3V) is needed in low output voltage circuits. These diodes are limited in their blocking voltage capabilities to 50 – 100V.

Fast Recovery diodes: These are used in high frequency circuits in combination with controllable switches where a small reverse recovery time is needed. At power levels of several hundred volts and several hundred amperes, these diodes have trr ratings of less than a few microsecond.

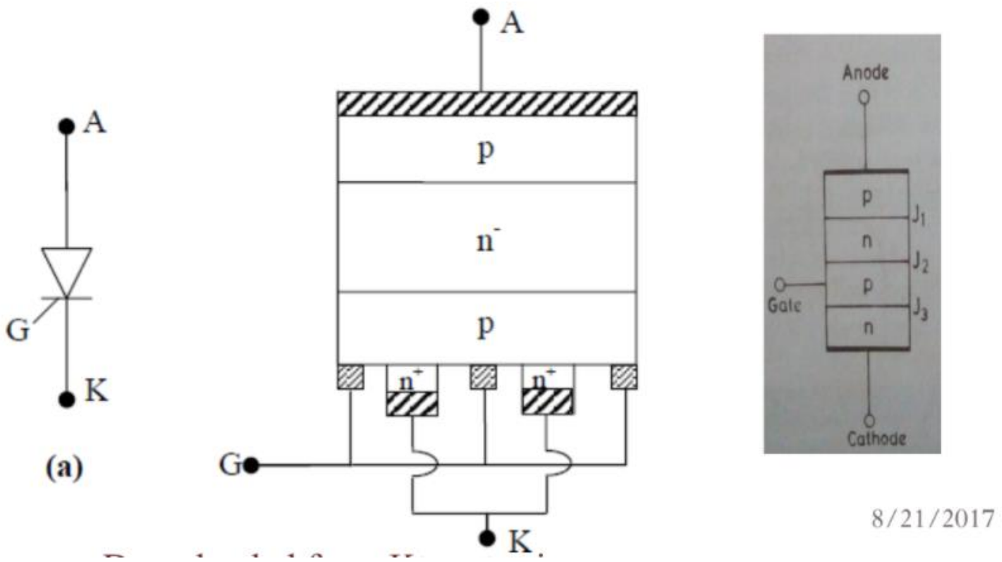
Line – frequency diodes: The on state voltage of these diodes is designed to be as low as possible and as a consequence have larger trr, which are acceptable for line frequency applications.

These diodes are available with blocking voltage ratings of several kilovolts and current ratings of several kilo amperes. Moreover, they can be connected in series and parallel to satisfy any voltage and current requirement.

SILICON CONTROL RECTIFIER [construction]

- Four layer, three junction, p-n-p-n semiconductor switching
- device
- Basically a thyristor consist of 4 layers of alternate p-type and n-type silicon semiconductors forming three junctions J1, J2 and J3. Gate terminal is usually kept near the cathode terminal. T
- The terminal connected to outer p region is called the anode. The terminal connected to outer n region is called the cathode.
- And that connected to inner p region is called gate. An SCR is so called because silicon is used for its construction and its operation as a rectifier can be controlled.
- Like the diode SCR is a unidirectional device, that blocks the current flow from cathode to anode.
- Unlike the diode, a thyristor also blocks the current flow
- from anode to cathode until it is triggered into conduction by a proper gate signal between gate and cathode terminal.

SCR (Silicon Controlled Rectifier)



SCR – Static V-I Characteristics

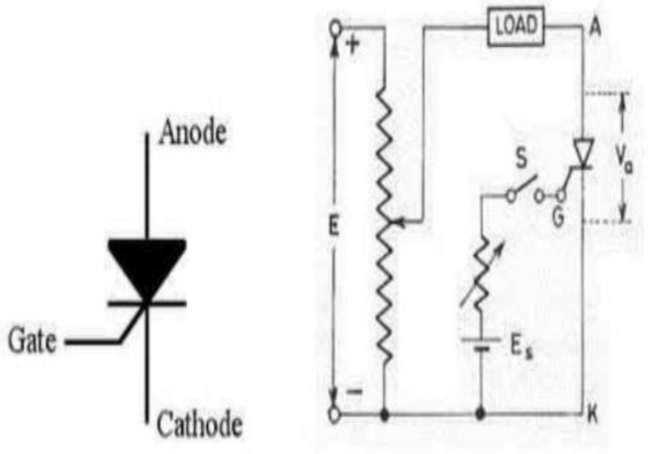
- An elementary circuit diagram for obtaining static V-I characteristics of a thyristor is shown below
- The anode and cathode are connected to main source through load.
- The gate and cathode are fed from a source of Es which provides +ve gate current from gate to cathode.
- The characteristics reveals that a thyristor has three basic modes of operation.

Reverse blocking mode:

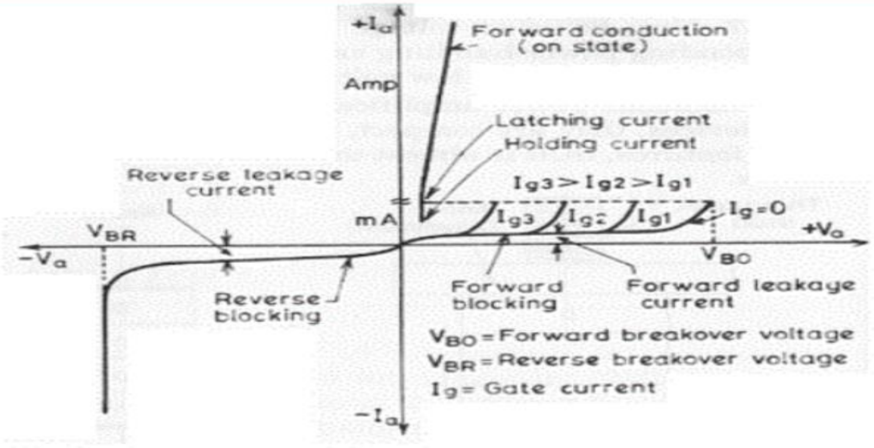
- When cathode is made +ve w.r.t anode with switch S open thyristor is reverse biased.
- Junction J1 and J3 are seen to be reverse biased.
- A small leakage current of the order of a few milli-amperes flows. This is reverse blocking mode,called the off state of a thyristor.
- If the reverse voltage is increased, then at a critical breakdown level, called reverse breakdown voltage VBR, an avalanche occur at J1 and J3, Reverse current increases rapidly.

Forward blocking mode:

- When anode is +ve w.r.t cathode, with gate circuit open, thyristor is said to be forward biased
- J1 and J3 are forward biased, J2 is reverse biased.
- In this mode, a small current called forward leakage current flows.
- As the forward leakage current is small, SCR offer a high impedance. Therefore, a thyristor can be treated as an open switch even in the forward blocking mode.



SILICON CONTROL RECTIFIER CIRUIT AND SYMBOL.



V-I characteristics of thyristor.

Forward conduction mode:

- When anode to cathode forward voltage is increased with gate, circuit open, reverse biased junction J2 will have an avalanche breakdown at a voltage called forward break-over voltage VBO,
- After this breakdown thyristor gets turned on with point M at once shifting to N and then to a point where between N and K. Here NK represents the forward conduction mode.
- A thyristor can be brought from forward blocking mode to forward conduction mode by turning on by applying

1)A positive gate pulse between gate and cathode
2)A forward break-over voltage across anode and cathode.

Latching current : The latching current may be defined as the minimum value of anode current which at must attain during turn ON process to maintain conduction even if gate signal is removed.

Holding current: It is the minimum value of anode current below which if it falls, the SCR will turn OFF.

Switching characteristics of thyristors:

The time variation of voltage across the thyristor and current through it during turn onand turn off process gives the dynamic or switching characteristic of SCR.

Switching characteristic during turn on Turn on time, It is the time during which it changes from forward blocking state to ON state.
Total turn on time is divided into 3 intervals:

1. Delay time 2. Rise time 3. Spread time

Delay time: If Ig and Ia represent the final value of gate current and anode current. Then the delay time can be explained as time during which the gate current attains 0.9 to the instant anode current reaches 0.1 or the anode current rises from forward leakage current to 0.1 .

1. Gate current 0.9 to 0.1 .

2. Anode voltage falls from V to 0.9a.

3. Anode current rises from forward leakage current to 0.1 .

Rise time (t) Time during which

1. Anode current rises from 0.1 to 0.9

2. Forward blocking voltage falls from 0.9 v to 0.1v . is the initial forward blocking anode voltage.

Spread time (t)

1. Time taken by the anode current to rise from 0.9 to I

2. Time for the forward voltage to fall from 0.1 to on state voltage drop of 1 to 1.5V. During turn on, SCR is considered to be a charge controlled device. A certain amount of charge is injected in the gate region to begin conduction. So higher the magnitude of gate current it requires less time to inject the charges. Thus, turn on time is reduced by using large magnitude of gate current.

Switching Characteristics During Turn Off Thyristor turn off means it changed from ON to OFF state.

Once thyristor is ON there is no role of gate. As we know thyristor can be made turn OFF by reducing the anode current below the latching current. Here we assume the latching current to be zero ampere. If a forward voltage is applied across the SCR at the moment it reaches zero then SCR will not be able to block this forward voltage.

Because the charges trapped in the 4-layer are still favourable for conduction and it may turn on the device.

So to avoid such a case, SCR is reverse biased for some time even if the anode current has reached to zero. So now the turn off time can be different as the instant anode current becomes zero to theinstant when SCR regains its forward blocking capability. tq= trr+tqr, Where, tq is the turn off time, trr is the reverse recovery time, tqr is the gate recovery time

At 1 anode current is zero. Now anode current builds up in reverse direction with same /dt slope.

This is due to the presence of charge carriers in the four layers.

The reverse recovery current removes the excess carriers from j1 and j3 between the instants t1 and t3.

At instant t3 the end junction j1 and j3 is recovered. But j2 still has trapped charges which decay due to recombination only so the reverse voltage has to be maintained for some more time.

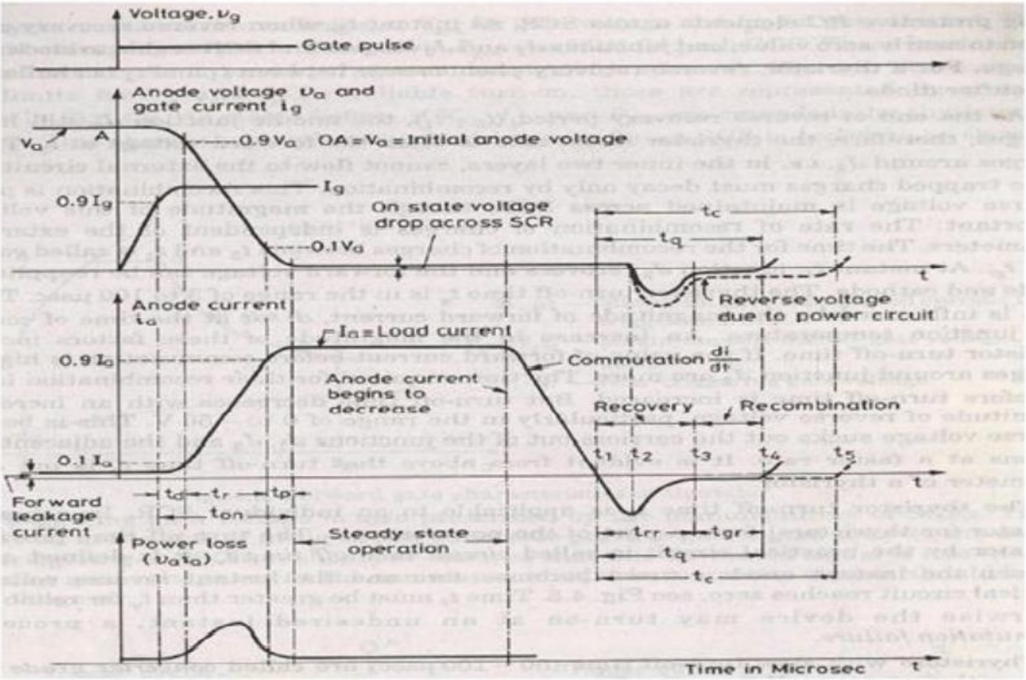
The time taken for the recombination of charges between j3 and j4 is called **gate recovery time** .

Junction 2 recovered and now a forward voltage can be applied across SCR.

The turn off time is affected by:

1. Junction temperature

2. Magnitude of forward current Turn off time decreases with the increase of magnitude of reverse applied voltage.



GTO: GATE TURN OFF THYISTOR

A gate turn off thyristor is a pnpn device.
In which it can be turned ON like an ordinary SCR by a positive gate current.
However, it can be easily turned off by a negative gate pulse of appropriate magnitude existence.

The salient features of GTO are:

1. GTO turned on like conventional SCR and is turned off by a negative gate signal of sufficient magnitude.
2. It is a non latching device.
3. GTO reduces acoustic and electromagnetic noise.

It has high switching frequency and efficiency.
A gate turn off thyristor can turn on like an ordinary thyristor but it is turn off by negative gate pulse of appropriate magnitude.

Advantage:
It is compact and cost less Switching performance
1. For turning ON a GTO first TR1is turned on.
2. This in turn switches on TR2 so that a positive gate current pulse is applied to turn on theGTO.
3. Thyristor 1 is used to apply a high peak negative gate current pulse.

Disadvantage The negative gate current required to turn off a GTO is quite large that is 20% to 30 %of anode current

Gate turn-on and turn off characteristic:

Gate turn-on

1. The gate turn on characteristics is similar to a thyristor. Total turn on time consists of delay time, rise time, spread time.
2. The turn on time can be reduced by increasing its forward gate current.

Gate turn off

Turn off time is different for SCR.
Turn off characteristics is divied into 3 period
1. Storage time
2. Fall time
3. Tail time

Tq=ts+tf+tt
At normal operating condition gto carries a steady state current.
The turn off process starts as soon as negative current is applied after t=0.

STORAGE TIME

During the storage pd the anode voltage and current remains constant.

The gate current rises depending upon the gate circuit impedance and gate applied voltage.

The beginning of pd is as soon as negative gate current is applied.

The end of storage pd is marked by fall in anode current and rise in voltage, what we have to do is remove the excess carriers. The excess carriers are removed by negative carriers.

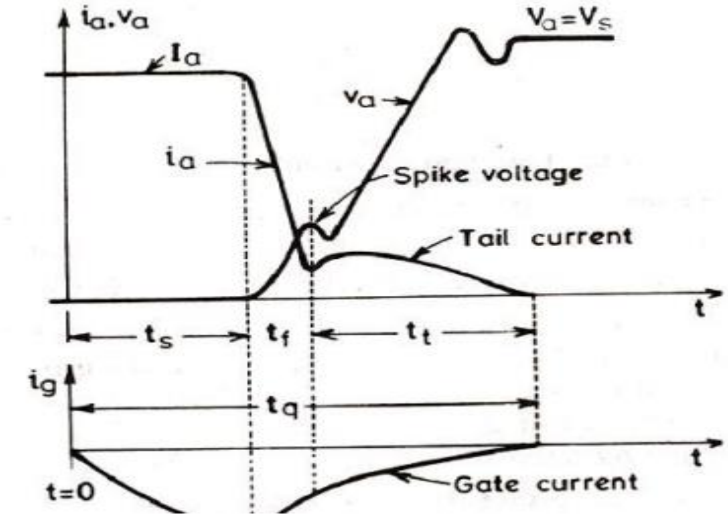
FALL TIME

After ts, anode current begins to fall rapidly and anode voltage starts rising. After falling to a certain value, then anode current changes its rate to fall. This time is called fall time SPIKE IN VOLTAGE During the time of storage and fall time there is a change in voltage due to abrupt current change.

TAIL TIME

During this time ,the anode current and voltage continues towards the turn off values.

The transient overshoot is due to the snubber parameter and voltage stabilizes to steady state value.

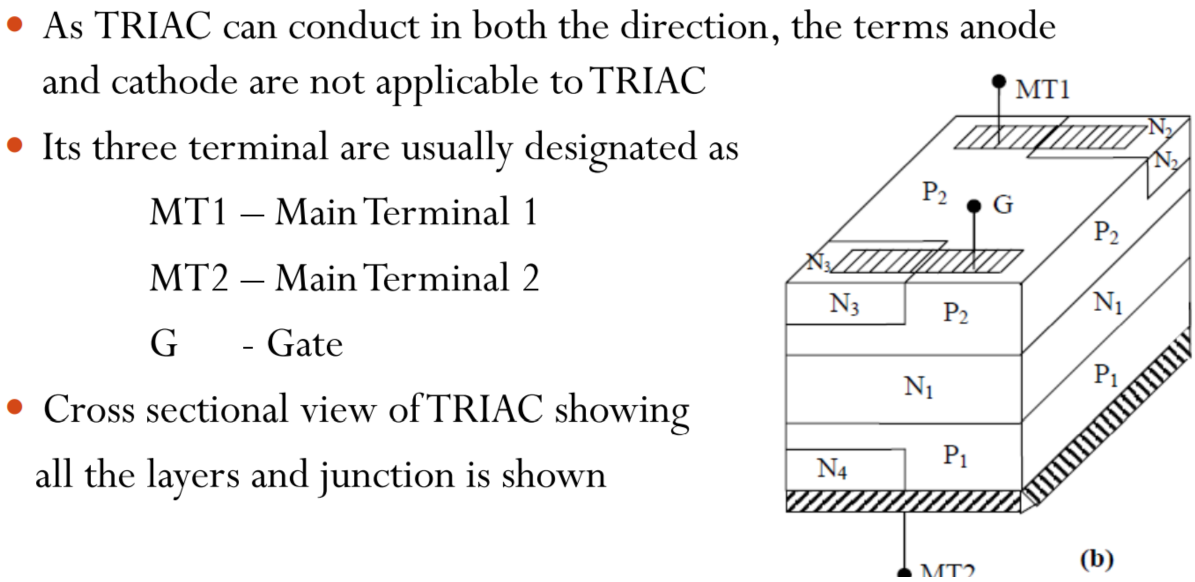


TRIAC:

An SCR is a unidirectional device an it can conduct from anode to cathode only and not from cathode to anode

- A TRIAC can conduct in both the directions
- TRIAC – bidirectional thyristor with three terminals

- Extensively used for the control of power in ac circuits (residential lamp dimmers, heater control, speed control of small single phase series and IM
- The word derived form combing the capital letters from the word TRIode and AC
- When in operation, a TRIAC is equivalent to two SCRs connected in anti-parallel
- The circuit symbol and characteristics are shown



- As TRIAC can conduct in both the direction, the terms anode and cathode are not applicable to TRIAC
- Its three terminal are usually designated as

  MT1 – Main Terminal 1

  MT2 – Main Terminal 2

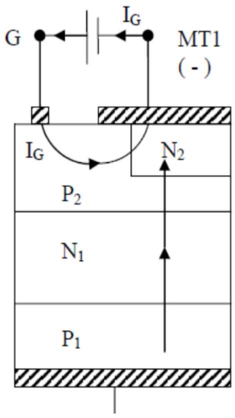
  G - Gate
- Cross sectional view of TRIAC showing all the layers and junction is shown

- The gate G is near terminal MT1
- The cross hatched strip shows that G is connected to N3 as well as P2
- MT1 is connected to P2 and N2
- MT2 is connected to P1 and N4
- Since TRIAC is a bidirectional device and can have its terminals at various combinations of +ve and –ve voltages, there are 4 possible electrode potential combinations as given below
- MT2 is +ve w.r.t MT1, G +ve w.r.t MT1
- MT2 is +ve w.r.t MT1,G -ve w.r.t MT1
- MT2 is -ve w.r.t MT1, G -ve w.r.t MT1
- MT2 is -ve w.r.t MT1, G +ve w.r.t MT1

- Triggering sensitivity is highest – for combination 1 &3
- For bidirectional control and uniform gate trigger – 2 & 3 and 4 usually avoided

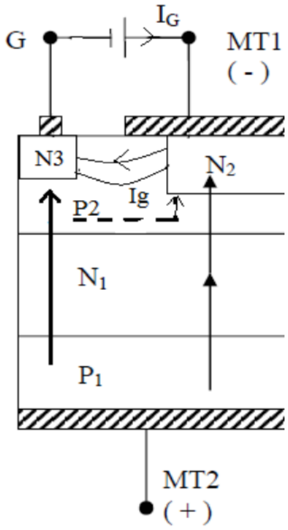
Mode – 1(MT2 is +ve w.r.t MT1, G +ve w.r.t MT1)

- When MT2 is +ve w.r.t MT1 – junction P1N1, P2N2 are forward biased
- Junction N1P2 reverse biased
- When G is +ve w.r.t MT1-
  gate current flows mainly through P2N2
- When gate current has injected sufficient charges into P2 layer, reverse biased junction N1P2 breaks down
- As a result TRIAC starts conducting through - P1N1P2N2



Mode – 2 (MT2 is +ve w.r.t MT1, G -ve w.r.t MT1)

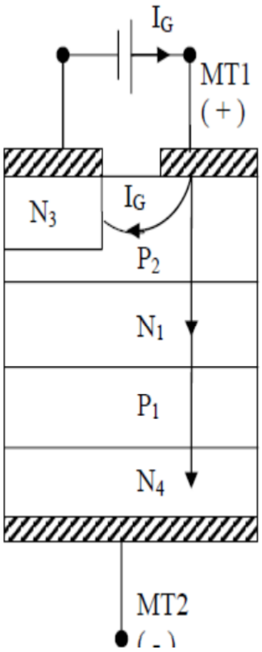
- When G is –ve w.r.t MT1, gate current flows through P2N3
- N1P2 – forward biased
- Conduction through – P1N1P2N3
- With the above conduction,
  voltage drop across this path falls
  but P2N3 rises towards MT2
- A potential gradient exist across P2
- Left hand region become higher potential than right hand region
- Current (shown in dotted line) established



- As a result right hand portion
  P1N1P2N2 begins to conduct
- The device structure P1N1P2N3 regarded as – pilot SCR
  P1N1P2N2- main SCR
- Device with MT2 +ve and G –ve is less sensitive
- Therefore more gate current is required

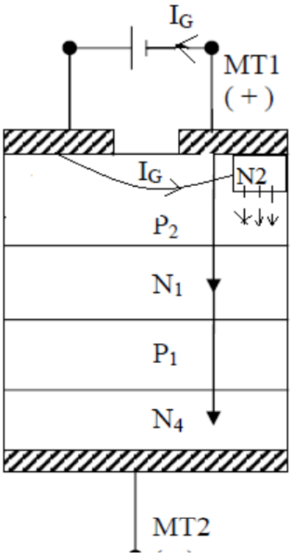
Mode 3 (MT2 is -ve w.r.t MT1, G -ve w.r.t MT1)

- N3 act as a remote gate
- Gate current flows from – P2 to N3
- Reverse biased junction – N1P1 is broken
- Finally P2N1P1N4 is turned on completely
- Operation is in 3rd quadrant
- Device is more sensitive under this condition



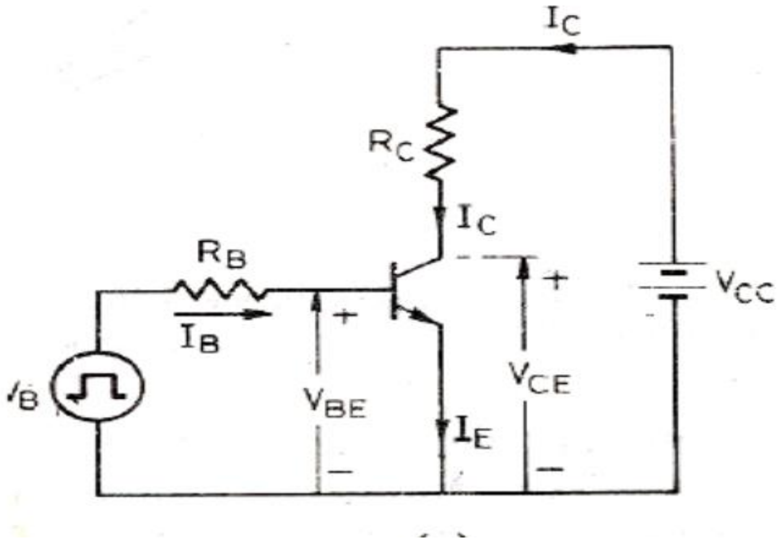
Mode 4 (MT2 is -ve w.r.t MT1, G +ve w.r.t MT1)

- Gate current forward biases junction – P2N2
- N2 inject electrons into P2 layer (shown by dotted arrows)
- As a result reverse biased junction N1P1 breaks down
- The structure P2N1P1N4 is completely turned on
- Current after turn on is limited by external load
- Device is less sensitive
- Operation is in 3rd quadrant



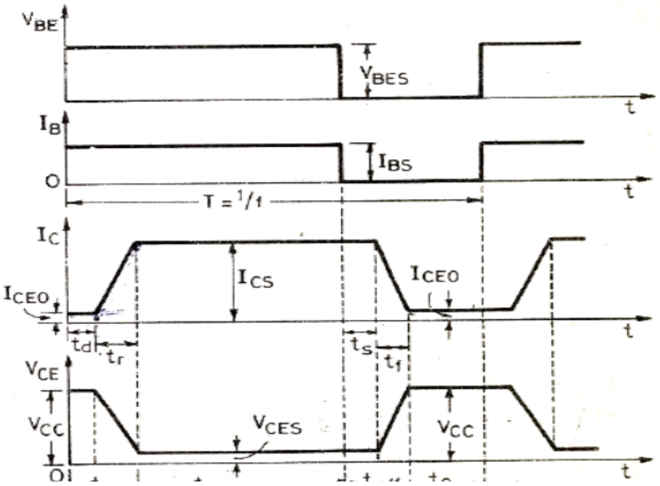
POWER BJT

- Power BJT means a large voltage blocking in the OFF state and high current carrying capability in the ON state.
- In most power application, base is the input terminal. Emitter is the common terminal. Collector is the output terminal.



1.7.1 Signal level of BJT

n+ doped emitter layer ,doping of base is more than collector.Depletion layer exists more towards the collector than emitter



Power BJT Construction:

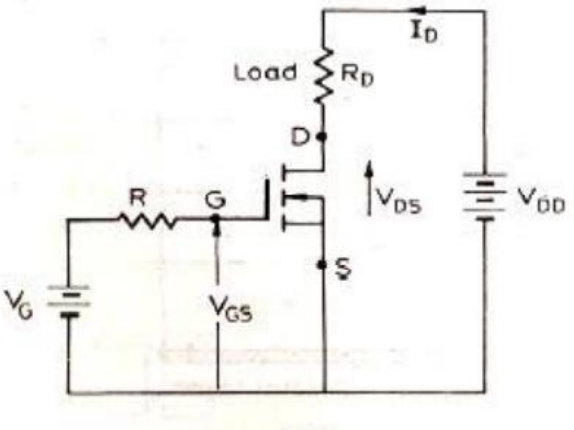
- The maximum collector emitter voltage that can be sustained across the junction, when it is carrying substantial collector current.
- Vceo=maximum collector and emitter voltage that can be sustain by the device. Vcbo=collector base breakdown voltage with emitter open

- Primary Breakdown It is due to convention avalanche breakdown of the C-B junction and its associated large flow of current.
- The thickness of the depletion region determines the breakdown voltage of the transistor.The base thickness is made as small as possible, in order to have good amplification capability.

- The Doping Levels
  1.The doping of the emitter layer is quite large.
  2.The base doping is moderate.
  3.n- region is lightly doped.
  4.n+ region doping level is similar to emitter.

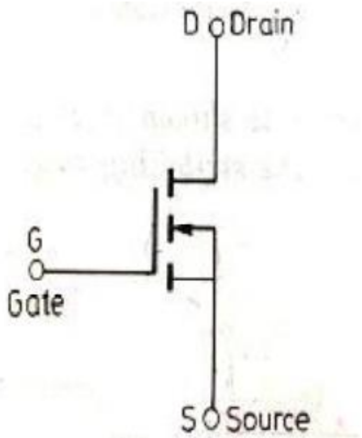
- Thickness of Drift Region-It determines the breakdown length of the transistor. The Base Thickness -Small base thickness- good amplification capability
- Too small base thickness- the breakdown voltage of the transistor has to be compromised. For a relatively thick base, the current gain will be relatively small. so, it is increase the gain.
- Secondary breakdown is due to large power dissipation at localized site within the semiconductor. The transistor is assumed to operate in active region.
- There is no doped collector drift region. It has importance only in switching operation, in active region of operation.
- B-E junction is forward biased and C-B junction is reverse biased.
- Electrons are injected into base from the emitter. Holes are injected from base into the emitter. Quasi Saturation, Initially, we assume that, the transistor is in active region.
- Base current is allowed to increase then, collector rises in response to base current. So, there is a increase voltage drop across the collector load. So, C-E voltage drops. Because of increase in collector current, there is a increase in voltage in drift region.
- This eventually reduces the reverse biased across the C-B junction.
- so, n-p junction get smaller, at some point the junction become forward biased. So now injection of holes from base into collector drift region occurs. Charge neutrality requires the electron to be injected in the drift region of the holes.
- Since a large no of electron is supplied to the C-B junction via injection from emitter and subsequent diffusion across the base.
- As excess carrier build up in the drift region begins to occur quasi saturation region is entered.
- Hard saturation obtained when excess carrier density reaches the n+ side. During quasi saturation,Hard saturation occurs when excess carriers have completely swept across the drift region.

POWER MOSFET



(a)

(Circuit diagram)



(a)

(circuit symbol)

- A power MOSFET has three terminal device.
- Arrow indicates the direction of current flow.
- MOSFET is a voltage-controlled device.
- The operation of MOSFET depends on flow of majority carriers only.

Switching Characteristics:-

- The switching characteristic is influenced by internal capacitance of the device.
- Internal impedance of the gate drive circuit
- Total turn on time is divided into
  1.Turn on delay time
  2.Rise time
- Turn on time is affected by impedance of gate drive source. During turn on delay time, gate to source voltage attains its threshold value .
- After tdn and during rise time gate to source voltage rise to
  Vgsp , a voltage which is sufficient to drive the MOSFET to ON state.

- The turn off process is initiated by removing the gate to source voltage. Turn off time is composed of turn off delay time to fall time.
- Turn off delay time To turn off the MOSFET the input capacitance has to be discharged .
- During tdf the input capacitance discharge from V1to Vgsp. During , fall time ,the input capacitance discharges from VGsp to VGst . During tdf drain current falls from to ID to zero. So when VGs ≤ , MOFSET turn off is complete.

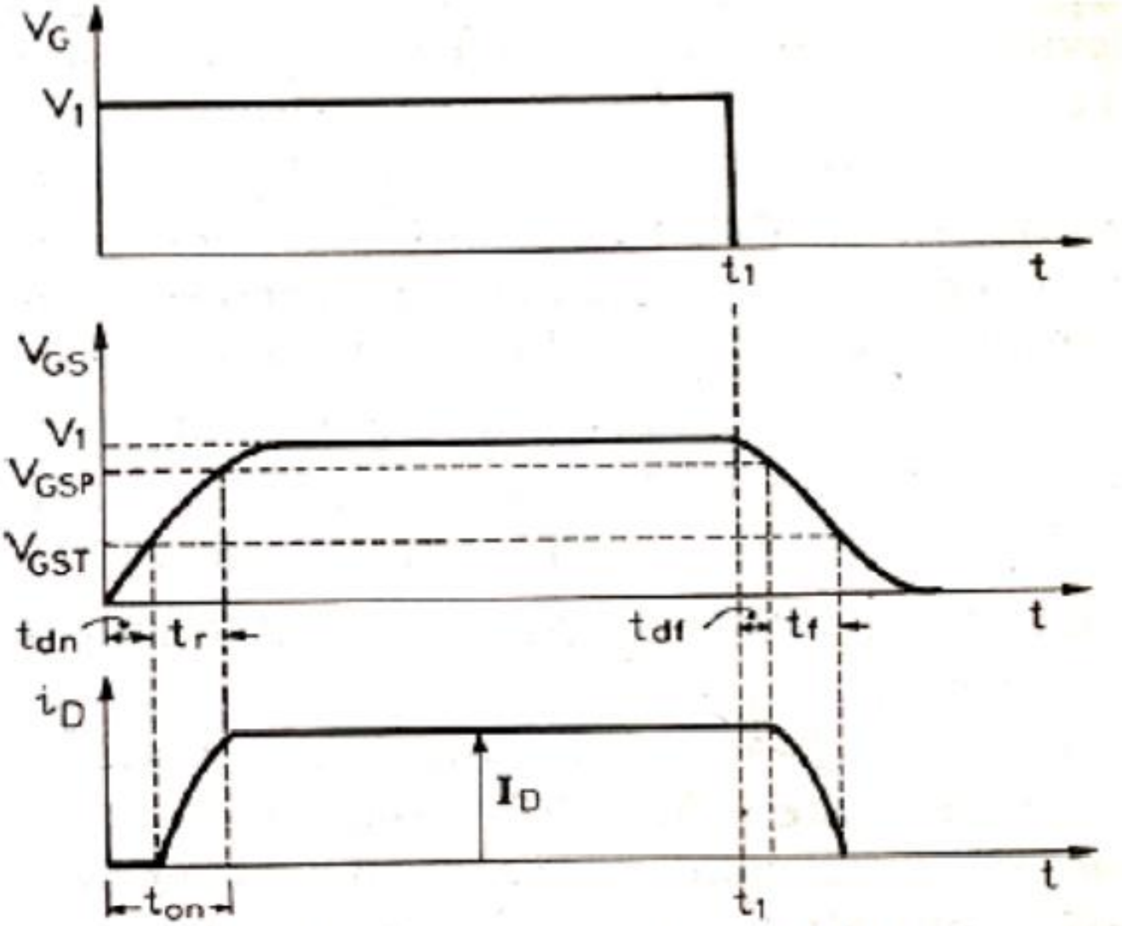


Fig. Switching waveform of power MOSFET

IGBT

- Developed by combining the best qualities of both BJT and MOSFET
- Thus, an IGBT posses high input impedance like MOSFET and has low on state power loss as in BJT
- Free from secondary breakdown problem
- Also known as Metal Oxide Insulated GateTransistor (MOSIGT)
- Conductively Modulated Field EffectTransistor (COMFET)
- Gain Modulated FET (GEMFET) Insulted GateTransistor (IGT)
- Body region and emitter is shorted to minimize possible turn on of the thyristor. n+ buffer layer is between P+ and n- drift layer, is not essential for the operation of the IGBT.
- (IGBT with buffer layer is called punch- through type, PT IGBT‟s and without buffer layer is called non-punch through type, NPT- IGBT‟s)
- Buffer layer improves the operation of IGBT.

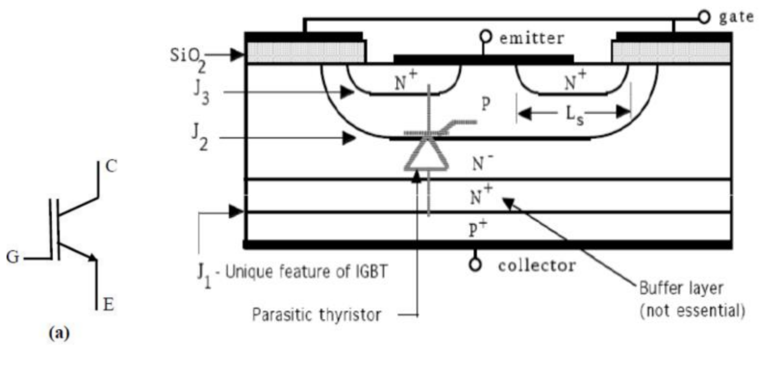
IGBT - Device operation

- Applied Collector emitter voltage is dropped across junction J2 and only very small leakage current flows.
- Depletion region of the J2 junction extends principally into the n- drift region (since P type body region is more doped than n- drift region)
- Thickness of drift region is large enough to accommodate depletion layer so that depletion layer boundary does not touch P+ layer.
- So it can block reverse voltage (magnitude same as forward voltage)
- This type of IGBT is known as symmetrical IGBT or non
- punch through IGBT.
- This reverse voltage blocking capability is useful in some ac circuit applications.
- If thickness of drift region is reduced , depletion layer may touch P+ .
- To avoid that we keep a buffer layer, n+ layer.
- This type of structure is called anti symmetric or punch through IGBT
- Shorter drift region means lower on-state losses.
- Presence of buffer layer reverse voltage capability quite low.

On state operation

- Gate – emitter voltage increases to more than threshold value , an inversion layer is formed beneath the gate of IGBT.
- This inversion layer shorts the n- drift region to the n+ source region exactly as in the MOSFET.
- An electron current flows through this inversion layer which in turn causes substantial hole injection from the P+ drain contact layer in to n- drift region as shown in figure.
- The injected holes move across the drift region by both drift and diffusion, taking a variety of path, and reach the p type body region that surrounds the n+ source region.
- As soon as the holes are in the p type body region, their space charge attracts electrons from the emitter metallization that contacts the body region, and the excess holes are quickly recombined

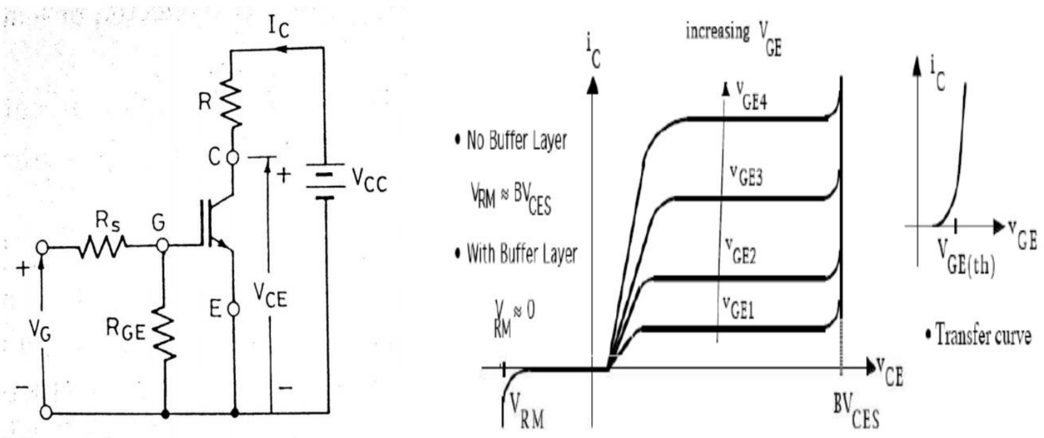
Basic structure of an IGBT

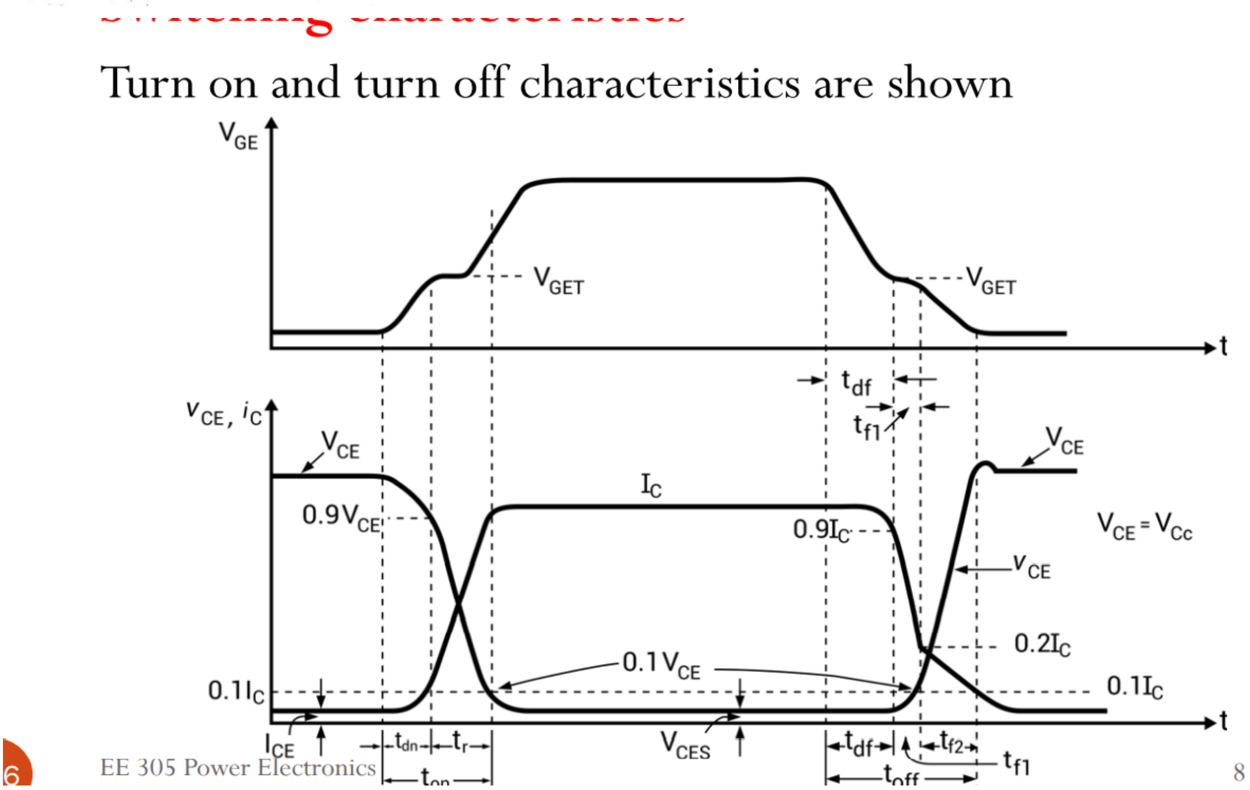


The symbol of IGBT is shown in the above figure.

V-I characteristics

- Output characteristics is the plot of Ic versus VCE
- Transfer characteristics – plot of Ic versus VGE





Turn on and turn off characteristics are shown

EE 305 Power Electronics

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Turn on characteristics

- Turn on time is composed of two time called, delay time tdn and rise time tr

- Delay time – time for the collector emitter voltage to fall from VCE to 0.9VCE
- Also defined as the time for collector current to rise from its initial leakage current ICE to 0.1 IC
- Rise time – time during which collector emitter voltage falls from 0.9VCE to 0.1VCE
- It is also defined as the time during which collector current rises from 0.1Ic to its final value Ic

Turn off characteristics

- It consist of three intervals
- Delay time tdf, initial fall time tf1, final fall time tf2
- Delay time – time during which gate voltage fall from VGE to threshold voltage VGET, the collector current from Ic to 0.9Ic
- First fall time – time during which collector current falls from 90 to 20% of its initial value Ic or collector emitter voltage rises from VCES to 0.1VCE
- Final fall time – time during which collector current falls from 20 to 10% of Ic or collector emitter voltage rises from 0.1VCE to VCE

Comparison of IGBT with MOSFET

IGBT

- Three terminal called, gate, emitter and collector
- High input impedance
- Voltage controlled device
- Can designed for higher voltage rating than PMOSFET

MOSFET

- Three terminal called, gate, source and drain
- High input impedance
- Voltage controlled device
- On state voltage drop and losses rises rapidly than IGBT with rise in temperature.

THYRISTOR PROTECTION CIRCUITS

- Reliable operation of a thyristor demands that its specified ratings are not exceeded
- In practice, a thyristor may be subjected to overvoltage or over-currents
- During SCR turn on, di/dt may be prohibitively large
- There may be false triggering of SCR by high value of dv/dt

- A thyristor must be protected against all such abnormal conditions for satisfactory and reliable operation of SCR circuit and the equipment

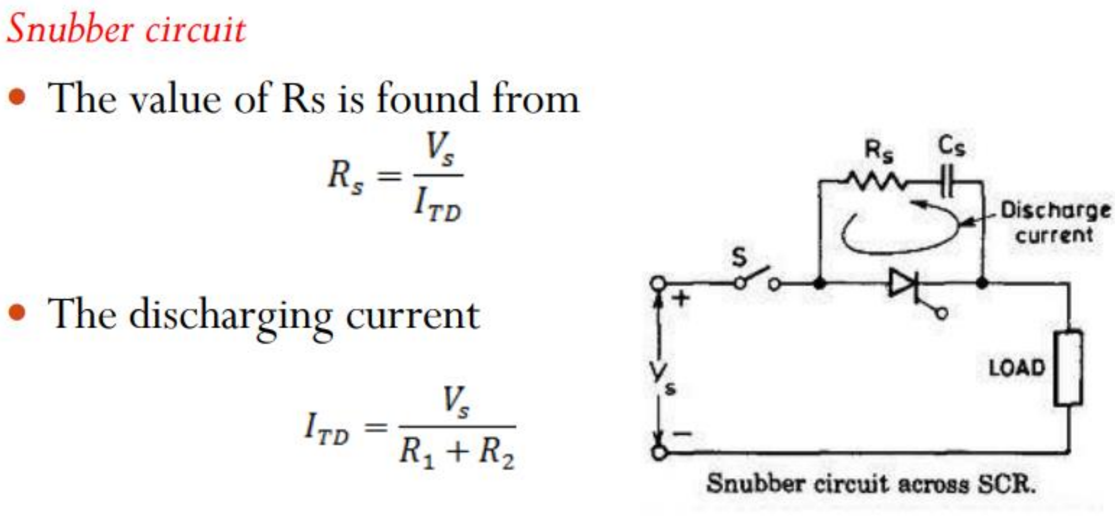
di/dt protection

- When a thyristor is forward biased and is turned on by a gate pulse, conduction of anode current begins in the immediate neighbourhood of the gate cathode junction
- Thereafter the current spread across the whole area of the junction
- The thyristor design permit the spread of conduction to the whole junction area as rapidly as possible
- However, if the rate of rise of anode current i.e., di/dt is large as compared to the spread velocity of carriers, local hot spot will be formed near the gate junction
- This localized heating may destroy the thyristor
- Therefore, the rate of rise of anode current at the time of turn on must be kept below the specified limiting value
- The value of di/dt can be maintained below acceptable limit by using a small inductor called, di/dt inductor in series with the anode circuit
- Typical di/dt limit values of SCR are 20 – 500 A/ μ sec
- Local hot spot heating can also be avoided by ensuring that the conduction spreads to the whole area as early as possible
- This can be achieved by applying agate current nearer to the maximum specified gate current
- di/dt=Vs/Ls
- Ls – series inductance including stray inductance
- dv/dt protection
- If rate of rise of suddenly applied voltage across thyristor is high, the device may get turned on
- Such phenomena of turning on a thyristor is called dv/dt turn on
- And this must be avoided as it leads to false operation of the thyristor circuit
- For controllable operation of the thyristor, the rate of rise of anode to cathode voltage dVa/dt must be kept below the specified limit
- Typical value of dv/dt are 20 – 500V / μ sec
- False turn on of a thyristor by large dv/dt can be prevented by using a snubber circuit in parallel with the device.

Snubber circuit

- Consist of a series combination of resistance Rs and capacitance Cs in parallel with the thyristor
- Cs in parallel with the device is sufficient to prevent unwanted dv/dt triggering of the SCR
- When switch S is closed,a sudden voltage appear across the circuit

- Capacitor Cs behaves like a short circuit, therefore voltage across SCR is zero
- With the passage of time voltage across Cs builds up at a slow rate such that dv/dt across Cs and therefore across SCR is less than the specified maximum dv/dt rating of the device
- When the SCR is turned on capacitor discharges through the SCR and sends a current equal to Vs/ (resistance of the path formed by Cs and SCR)
- As this resistance is quite low, the turn di/dt will tend to be excessive and as a result SCR may be destroyed
- In order to limit the magnitude of discharge current a resistance Rs is inserted in series with Cs
- The value of snubber circuit constant τ=Rs/Cs can be determined from for a known value of dv/dt



Snubber circuit

- The value of Rs is found from

$$R_s = \frac{V_s}{I_{TD}}$$

- The discharging current

$$I_{TD} = \frac{V_s}{R_1 + R_2}$$

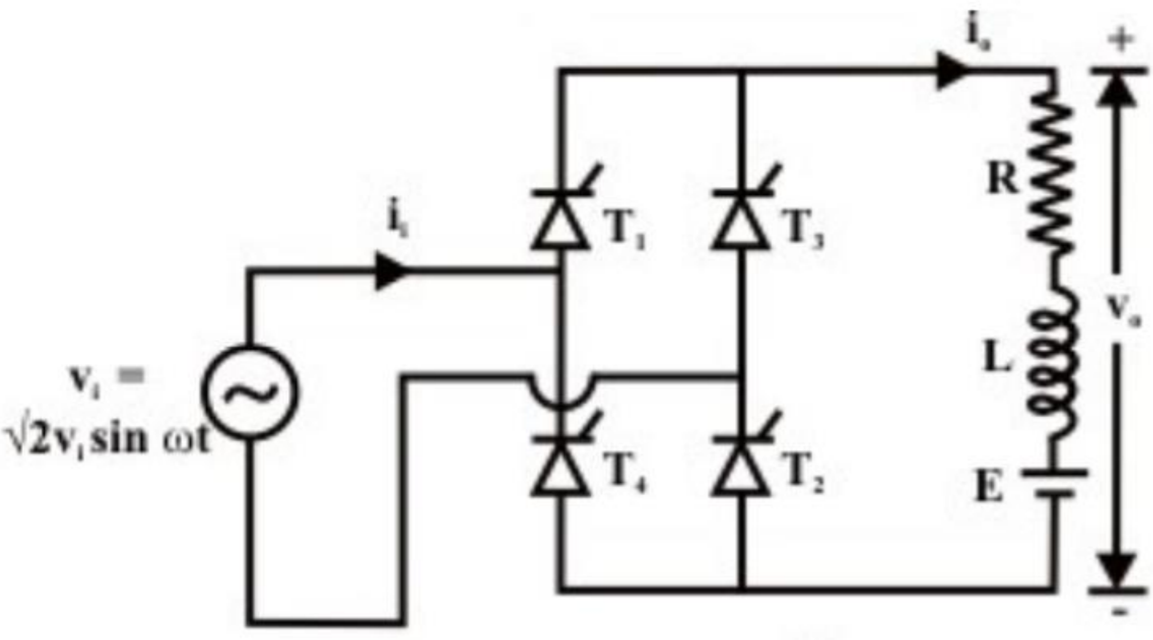
Snubber circuit across SCR.

MODULE 2

MODULE 2 PHASE CONTROLLED CONVERTERS

Single phase full converters- 3 phase half converter and 3 phase full converter – inverter operation – input power factor – effect of source inductance – Thyristor triggering circuits

SINGLE PHASE FULL CONVERTER



(a)

- Fig (a) shows the circuit diagram of a single phase fully controlled bridge converter.
- Indeed, the R–L–E load shown may represent the electrical equivalent circuit of a separately excited dc motor.

- The single phase fully controlled bridge converter is obtained by replacing all the diode of the corresponding uncontrolled converter by thyristors. Thyristors T1 and T2 are fired together while T3 and T4 are fired 180º after T1 and T2.

- From the circuit diagram, it is clear that for any load current to flow at least one thyristor from the top group (T1, T3) and one thyristor from the bottom group (T2, T4) must conduct.
- It can also be argued that neither T1T3 nor T2T4 can conduct simultaneously. For example, whenever T3 and T4 are in the forward blocking state and a gate pulse is applied to them, they turn ON and at the same time a negative voltage is applied across T1 and T2 commutating them immediately.

- Similarly, for T1 and T2. For the same reason T1T4 or T2T3 cannot conduct simultaneously. Therefore, the only possible conduction modes when the output current can flow are T1T2 and T3T4.
- Once the load current becomes zero all thyristors remain off, in this mode the load current remains zero.
- Under normal operating condition of the converter the load current may or may not remain zero over some interval of the input voltage cycle. If load current is always greater than zero then the converter is said to be operating in the **continuous conduction mode.** In this mode of operation, the converter, T1T2 and T3T4 conducts for alternate half cycle of the input supply.
- However, in the **discontinuous conduction mode** none of the thyristors conduct over some portion of the input cycle. The load current remains zero during that period.
- In the continuous conduction mode of operation load current never becomes zero, therefore, either T1T2 or T3T4 conducts.
- The firing angle of the converter is α. As T1T2 are fired at ωt = α they turn on, commutating T3T4 immediately. T3T4 are again fired at ωt = π + α, Till this point T1T2 conducts.
- The period of conduction of different thyristors are pictorially depicted in the second waveform (also called the conduction diagram) of Fig B.
- It is observed that the emf source E is greater than the dc link voltage till ωt = α. Therefore, the load current i0 continues to fall till this point. However, as T1T2 are fired at this point load voltage, becomes greater than E and load current starts increasing through R-L and E.
- At ωt = π – θ , v0 again equals E. Depending upon the load circuit parameters io reaches its maximum at around this point and starts falling afterwards. Continuous conduction mode will be possible only if i0 remains greater than zero till T3T4 are fired at ωt = π + α where upon the same process repeats.
- The input ac current waveform is obtained by noting that whenever T1T2 conducts i1 = i0 and i1 = - i0 whenever T3T4 conducts.
- The last waveform shows the typical voltage waveform across the thyristor T1. It is to be noted that when the thyristor turns off at ωt = π + α a negative voltage is applied across it for a duration of π – α.
- The thyristor must turn off during this interval for successful operation of the converter. It is noted that the dc voltage waveform is periodic over half the input cycle.
- The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

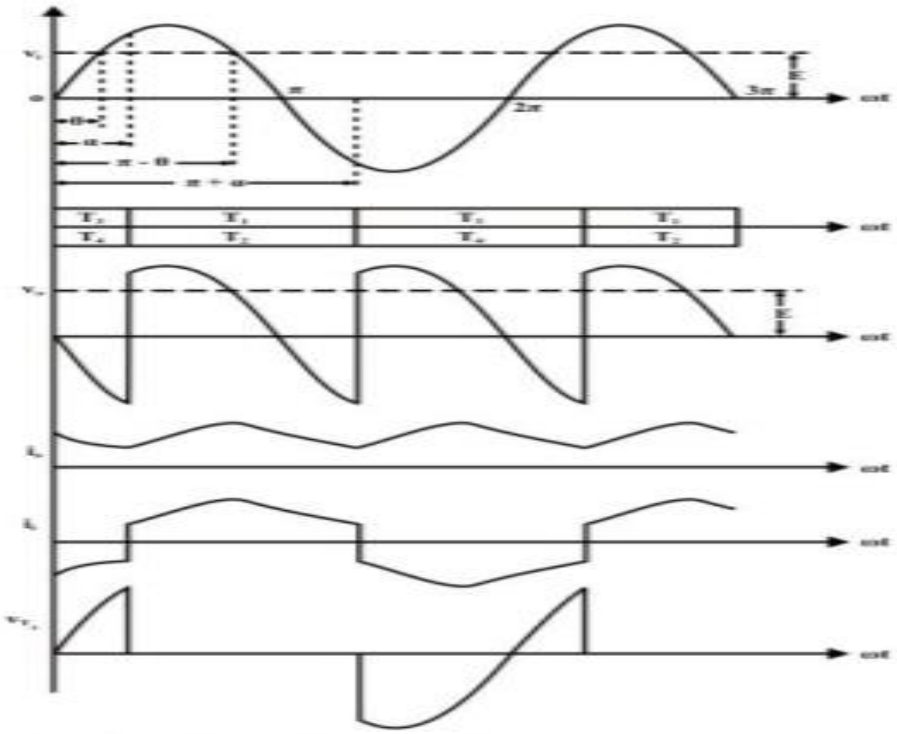


FIG B WAVEFORMS OF SINGLE PHASE FULL CONVERTER
IN CONTINOUS CONDUCTION MODE.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load. Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

Features of 3-phase controlled rectifiers are

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current

3-phase half wave converter

The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load.

The thyristor T_1 in series with one of the supply phase windings *‘a-n’* acts as one half wave controlled rectifier.

The second thyristor T_2 in series with the supply phase winding *'b-n'* acts as the second half wave controlled rectifier.

The third thyristor T_3 in series with the supply phase winding acts as the third half wave controlled rectifier.

The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor T_1 is triggered at $\omega t=(\prod/6 + \alpha)=(30^\circ + \alpha)$, the phase voltage V_{an} appears across the load when T_1 conducts.

The load current flows through the supply phase winding *'a-n'* and through thyristor T_1 as long as T_1 conducts. When thyristor T_2 is triggered at $\omega t=(5\prod/6\alpha)$, T_1 becomes reverse biased and turns-off.

The load current flows through the thyristor and through the supply phase winding *'b-n'* .

When T_2 conducts the phase voltage v_{bn} appears across the load until the thyristor T_3 is triggered .

When the thyristor T_3 is triggered at $\omega t=(3\prod/2 + \alpha)=(270^\circ+\alpha)$, T_2 is reversed biased and hence T_2 turns-off.

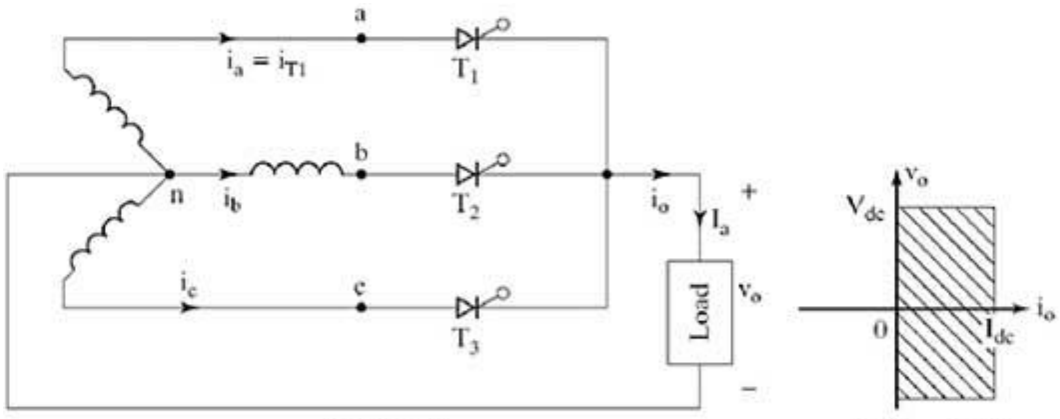
The phase voltage V_{an} appears across the load when T_3 conducts.

When T_1 is triggered again at the beginning of the next input cycle the thyristor T_3 turns off as it is reverse biased naturally as soon as T_1 is triggered.

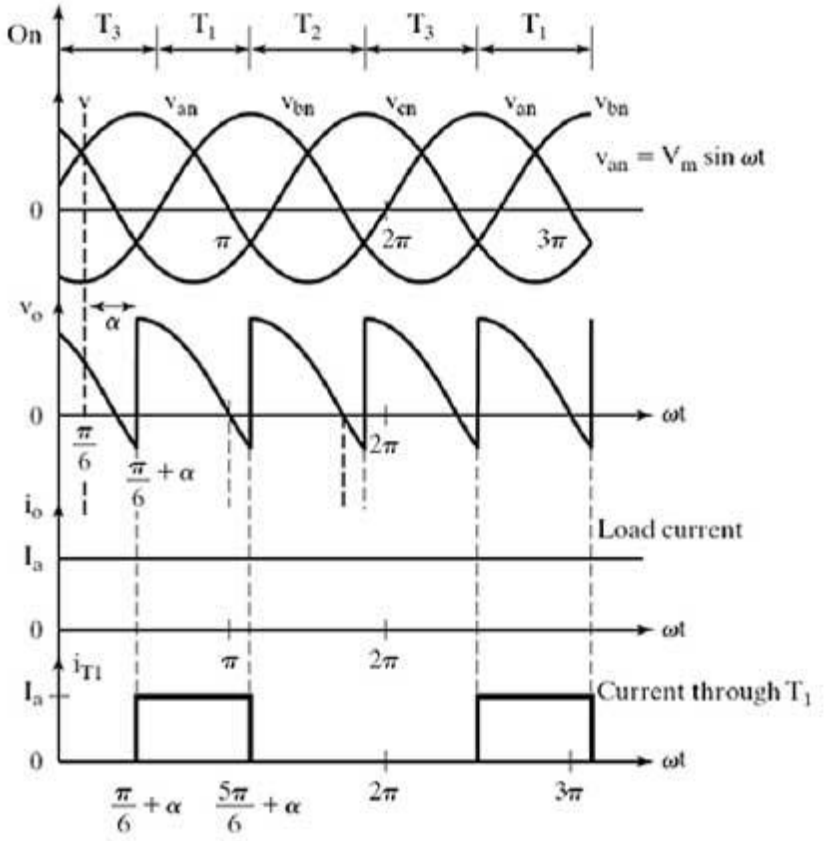
The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor T_1.

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha >(\prod/6)$, the load current appears as discontinuous load current and each thyristor is naturally commutated.

when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is f_s, where f_s is the input supply frequency.



(a) Circuit (b) Quadrant



(c) For inductive load

Fig. Three Phase half converter circuit and its input /output waveforms.

THREE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{ Max. Phase Voltage}$$

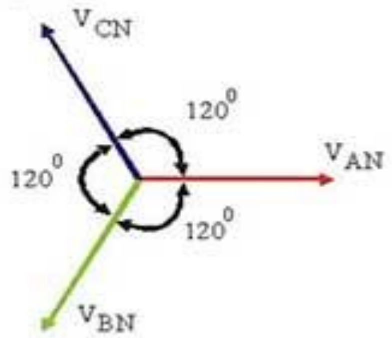
$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$



Vector diagram of 3-phase supply voltages

To derive an expression for the average output voltage of a 3-phase half wave converter for continuous load current

The reference phase voltage is $v_{RN}=v_{an}=V_m sin\omega t$. When the phase supply voltage V_{an} begins its positive half cycle at $\omega t=0$, the first cross over point appears at $\omega t=(\prod/6)$*radians* 30°.

The trigger angle α for the thyristor T_1 is measured from the cross over point at . The thyristor T_1 is forward biased during the period $\omega t=30°$ to 150° ,

when the phase supply voltage v_{an} has higher amplitude than the other phase supply voltages. Hence T_1 can be triggered between 30° to 150°.

When the thyristor T_1 is triggered at a trigger angle α, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} v_O . d(\omega t)\right]$$

Output voltage $v_O = v_{an} = V_m \sin \omega t$ for $\omega t = (30^0 + \alpha)$ to $(150^0 + \alpha)$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t . d(\omega t)\right]$$

As the output load voltage waveform has three output pulses during the input cycle of 2π radians

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin \omega t . d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[(-\cos \omega t)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Three phase full converter

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration.

All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals.

The three phase full converter is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operations is required.

The figure shows a three phase full converter with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter.

The thyristors are triggered at an interval of ($\prod/3$) radians (i.e. at an interval of 30°). The frequency of output ripple voltage is $6f_s$ and the filtering requirement is less than that of three phase semi and half wave converters**.**

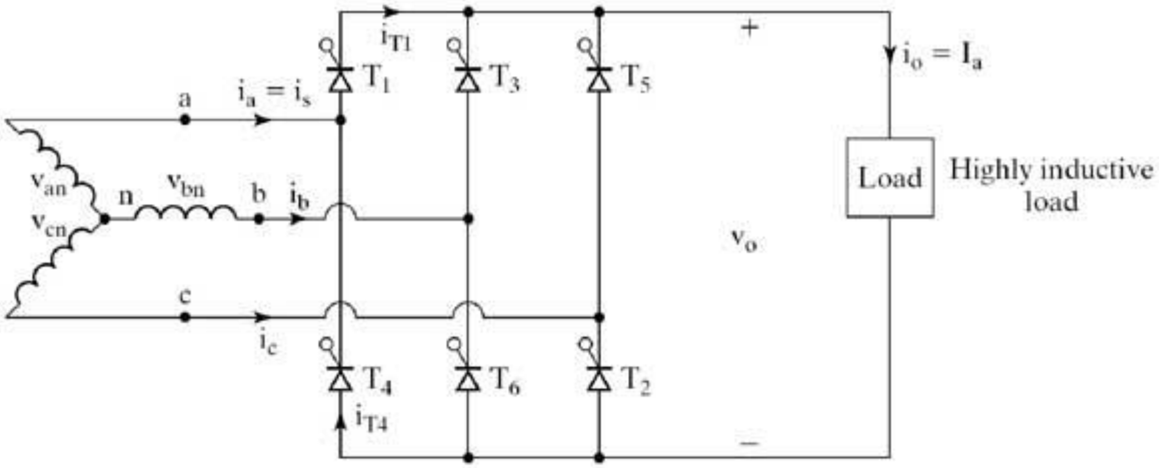


Fig. Three Phase Full converter circuit

At $\omega t=(\prod/6 +\alpha)$, when the thyristor is turned on by applying the gating signal to the gate of T1, T6 .

During the time period $\omega t=(\prod/6 +\alpha)$ to ($\prod/2 +\alpha$), thyristors conduct together and the line to line supply voltage appears across the load.

At $\omega t=(\prod/2 +\alpha)$, the thyristor T_2 is triggered and T_6 is reverse biased immediately and T_6 turns off due to natural commutation.

During the time period $\omega t=(\prod/ +\alpha)$ *to* $(5\prod/6 +\alpha)$, thyristor T_1 and T_2 conduct together and the line to line supply voltage appears across the load.

At $(5\prod/6 +\alpha)$, T1 turns off, T2,T3 starts conducting, line voltage vbn, appears across the load.

The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered.

The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on.

The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through T_1 and T_4, the supply current through the line ‘a’.

We define three line neutral voltages (3 phase voltages) as follows, abc = ryb

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \qquad V_m = \text{ Max Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right) = V_m \sin\left(\omega t - 240^0\right)$$

Where V_m the peak phase voltage of a star (Y) is connected source.

The corresponding line-to-line voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3} V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

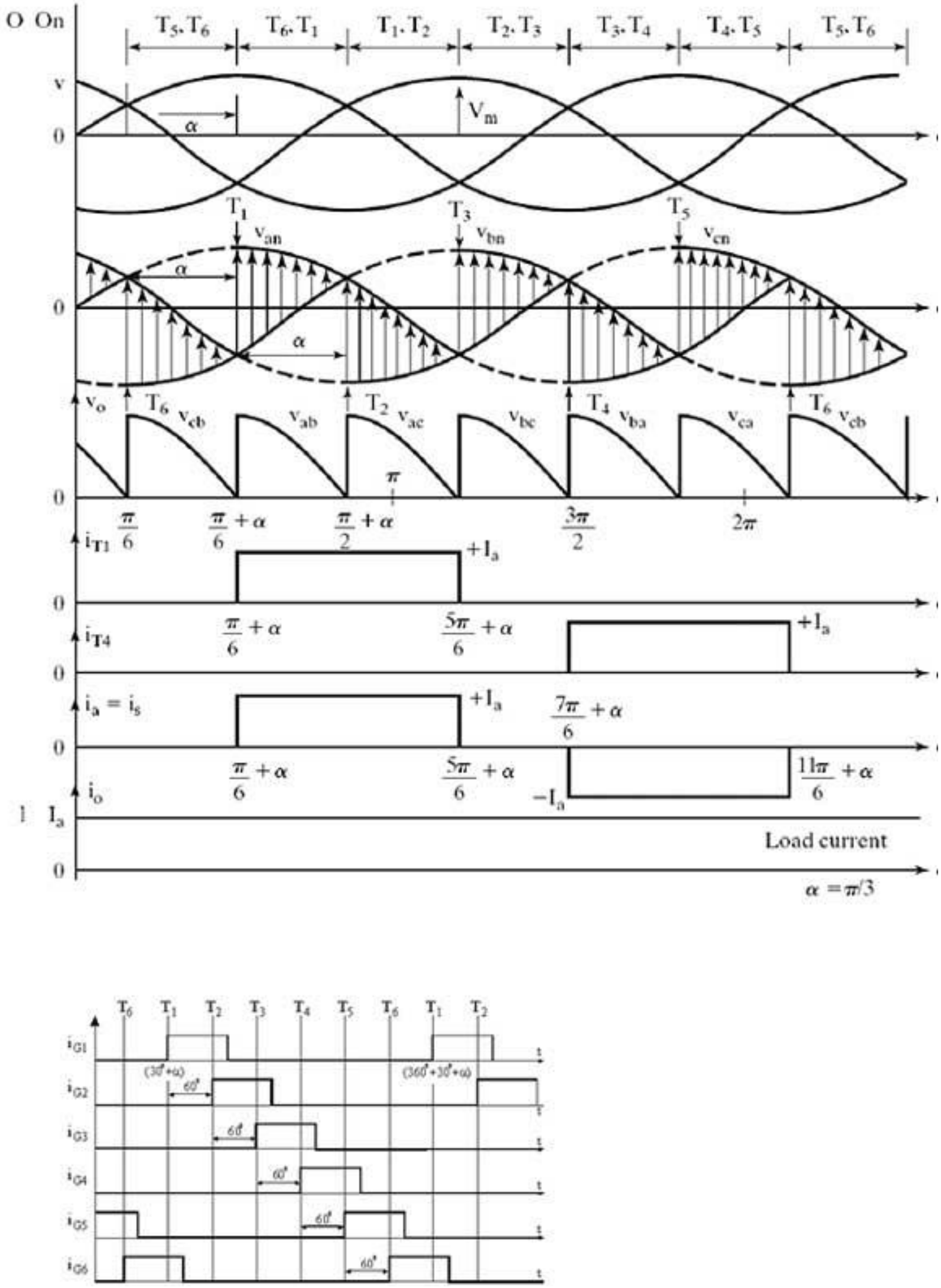


Fig C CURRENT AND VOLTAGE WAVEFORMS OF THREE PHASE FULL CONVERTER

To derive an expression for the average output voltage of three phase full converter with highly inductive load assuming continuous and constant load current

The output load voltage consists of 6 voltage pulses over a period of 2∏ radians, hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O \, d\omega t \quad ;$$

$$v_O = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = \frac{3V_{mL}}{\pi}\cos\alpha$$

Where $V_{mL} = \sqrt{3}V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$

$$V_{dc(max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2 \, d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^2 \, d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right) d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m\left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right)^{\frac{1}{2}}$$

Effect of inductance:

Actually inductance and resistance must be present in the supply source,and the time is required for the current change to take place.

The net result is current commutation is delayed, it takes time for the current to decay to zero on the outgoing thyristor, at the same time current will rise in the incoming thyristor.

Commutation process takes some time, during which both thyristors are conducting is called overlap period.

During this commutation overlap period, the waveforms of the voltage at the outpit terminals of the converter and voltage and current waveforms at the input terminals are different.

At the output terminals, the effect of input source inductance is to cause a loss of mean voltage and modify harmonic distortions.

While at input terminals slight reduction of displacement factor and modification of distortion of current waveform takes place.

Inductive reactance of ac supply is much greater than resistance. The source impedance is resistive. there will be a voltage drop across the resistance, the average output voltage across the converter gets reduced, by an amount equvivalent to average drop.

Since the source resistance is very small, the commutation angle during which the load current is transferred from outgoing to incoming thyristors is neglected.

For converters which are used regularly as dc supplies, an output filter is required to reduce the ripple in dc current and voltage across load.

Therefore inductance is made reasonably large to act as a filter choke. With purely resistive load, inductance zero load current is discontinuous, thyristors turn off by natural commutation.

INPUT POWER FACTOR:

Power Factor Improvement in the Converter

The operation of Phase controlled converter is very simple, reliable and less costly.

The commutation circuit is not necessary in this type of converter.

The power factor becomes low when the output voltage less than the supply voltage (particularly when the firing angle is high).

The displacement angle (The angle between fundamental component of alternating line current and phase to neutral voltage is known as displacement angle.) between the supply voltage and supply current increases as the firing angle increases.

This will result in power factor decreases and lagging reactive power flows from load to supply side and power factor decreases.

There are following methods to improve power factor in the phase controlled converters.

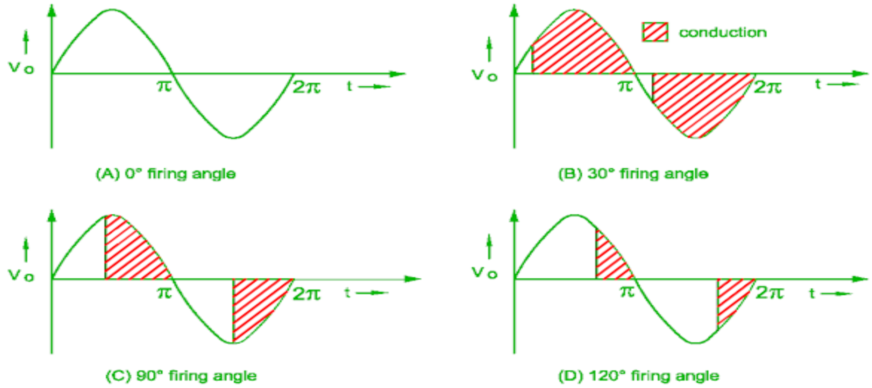
Power factor of phase controlled converters

(a) Phase angle control

The output voltage decreases as the firing angle of the SCR increases.

The input displacement factor and input power factor decreases as the output voltage decreases in the semi converter and full converter.

The effect of the firing angle on the output voltage is shown in the figure A.



(A) 0° firing angle

(B) 30° firing angle

(C) 90° firing angle

(D) 120° firing angle

FIG A : EFFECT OF FIRING ANGLE ON OUTPUT VOLTAGE

Semiconverter operation of full converter

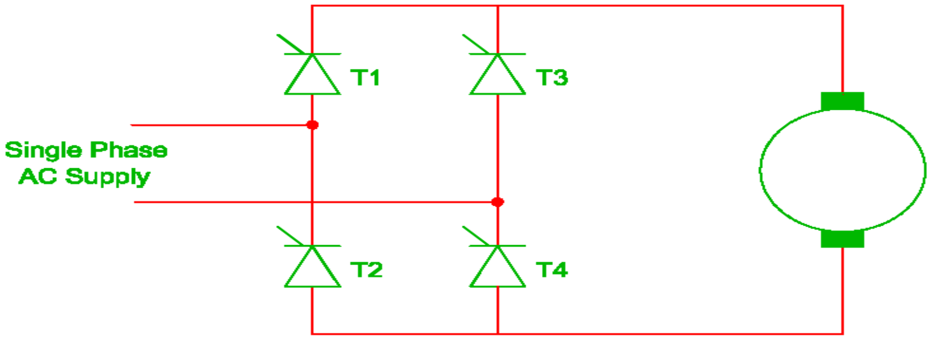


FIG B : POWER CIRCUIT OF FULL CONVERTER

- The full converter is used when regeneration is required.
- The operation of full converter in the semi converter mode is explained below.

- The change in controlled circuit is necessary in order to operate semi converter in to full converter.
- **Rectifier mode**
- The SCR T4 and SCR T2 are turned on during positive and negative half cycle of alternating supply respectively.
- The SCR T4 and SCR T2 acts as switch in this mode.
- The output voltage is adjusted by controlling firing angle of SCR T1 and SCR T3.
- The firing of SCR T4 and SCR T2 is kept at zero in this mode of operation therefore the output voltage is adjusted from maximum to minimum by controlling the firing angle of SCR T1 and SCR T3.
- When the firing angle of positive group SCR T1 and SCR T3 is kept zero, the output voltage becomes maximum positive and the output voltage becomes zero when the firing angle becomes 180^o.

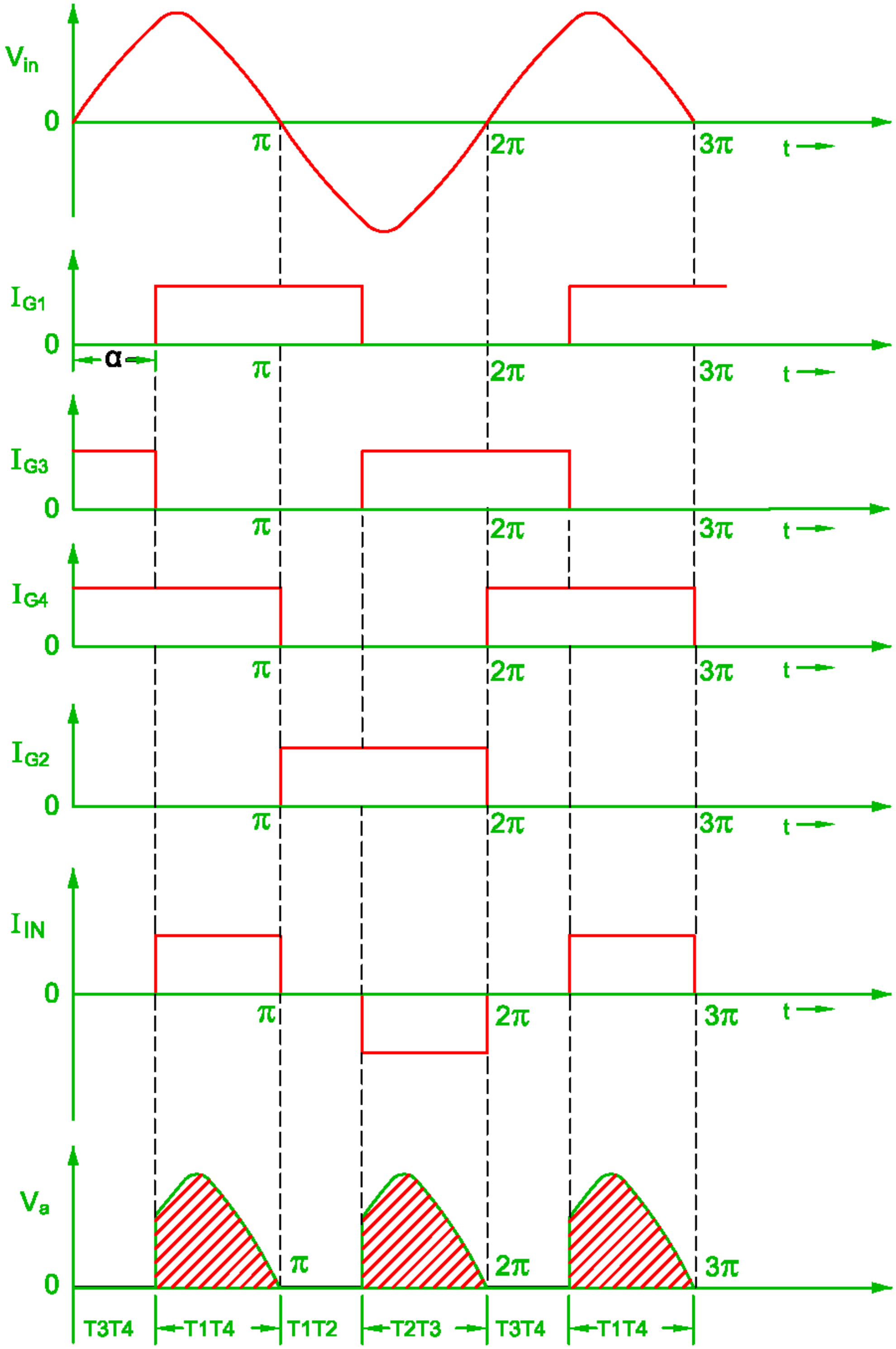


FIG C : WAVEFORMS FOR RECTIFICATION

ASYMMETRICAL FIRING

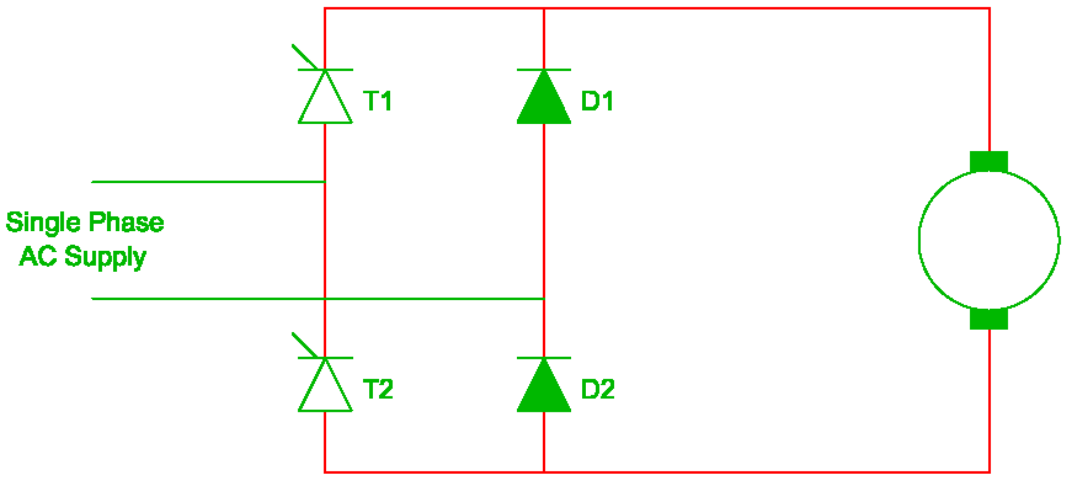


FIG E : BASIC ASYMMETRICAL SEMICONVERTER CIRCUIT

- The firing angle of SCRs is kept same in the symmetrical firing angle control method whereas the firing angle of SCRs is kept different in the asymmetrical firing angle control scheme.
- Let us consider that the firing angle is kept 90° for output voltage of 0.5 pu in the symmetrical firing angle control.
- Let us consider that the firing of the SCR T1 is kept at 60° and SCR T2 is kept at 120° in the asymmetrical firing angle control.
- The power factor improves too some extent when the firing angle of SCR T1 is kept small.
- There are following disadvantages of asymmetrical firing angle control scheme.(1) It generates DC and 2nd, 4th and 6th harmonics. (2) The motor current becomes discontinuous.
- The asymmetrical firing angle control scheme is only for theory point of view due to above mentioned disadvantages.

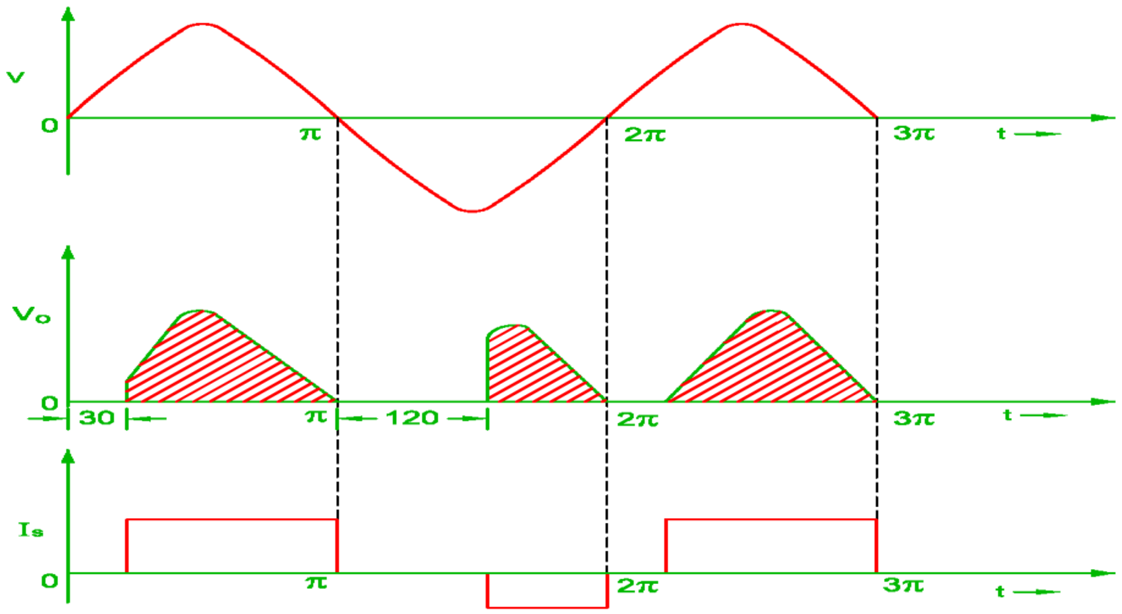


FIG F : WAVEFORMS OF AN ASYMMETRICAL TRIGGERING

Extinction angle control (EAC)

The single phase semi converter is shown in the figure G. The SCR or GTO is used as switch S1 and S2. The square shows commutation circuit of the SCR or GTO.

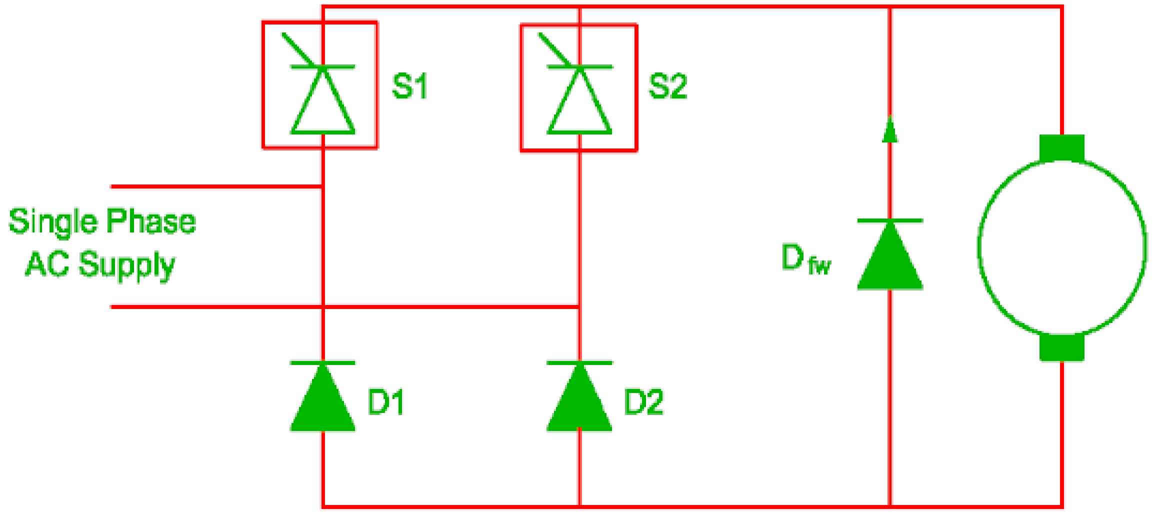


FIG G : SINGLE PHASE SEMICONVERTER CIRCUIT

- The waveform of the extinction angle control is shown in the figure H.The switch S1 is turned on at ωt = α angle and turned off at angle ωt = β. The switch S2 is turned on at ωt = π + α and turned off at angle ωt = π + β.
- The output voltage is controlled by controlling extinction angle of switch S1 and S2.
- The waveform for input voltage, output voltage, input current and output current is shown in the figure H.
- The fundamental component of supply current I_1 leads the supply voltage therefore the displacement angle becomes leading.
- The displacement factor becomes leading for extinction angle control and lagging for firing angle control.

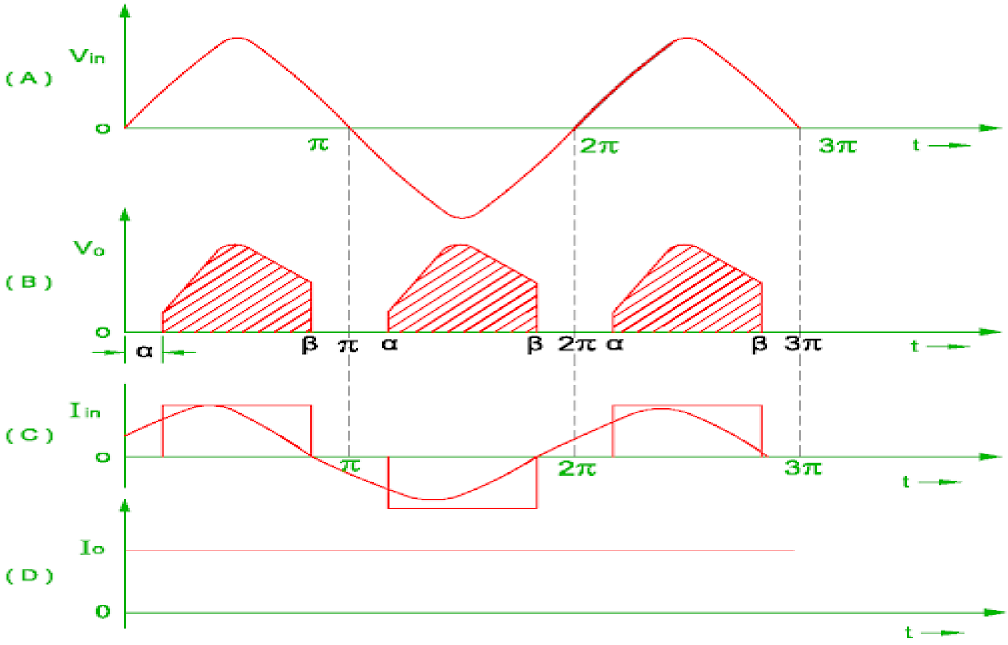


FIG H : VOLTAGE AND CURRENT WAVEFORMS
(A) INPUT VOLTAGE (B) OUTPUT VOLTAGE
(C) INPUT CURRENT (D) OUTPUT CURRENT

INVERTER OPERATION

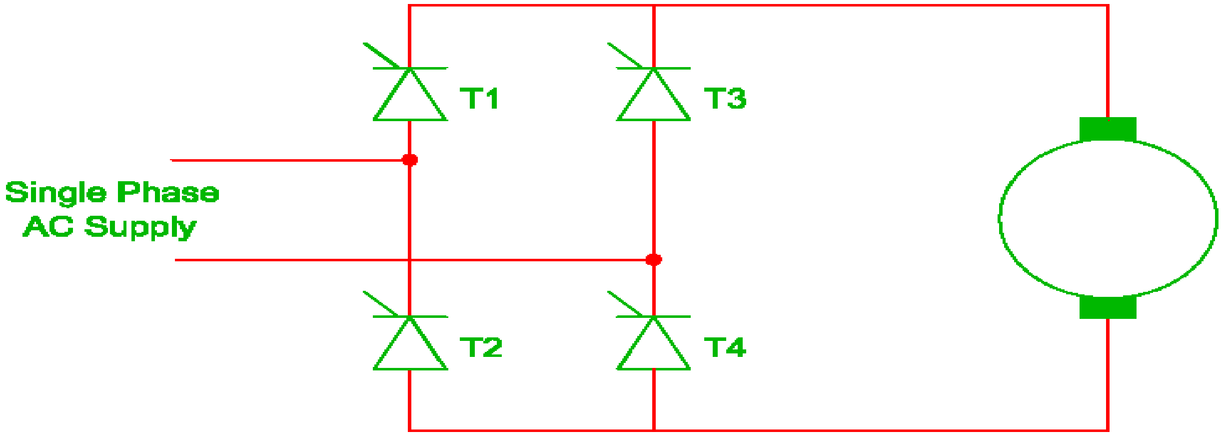


FIG B : POWER CIRCUIT OF FULL CONVERTER

Inverting mode

- The SCR T1 and SCR T3 are turned on during positive half cycle and negative half cycle of alternating supply respectively.
- The SCR T1 and SCR T3 works as switch in this mode and the output voltage is controlled by controlling firing angle of SCR T2 and SCR T4.
- The firing angle of positive group of SCR T1 and SCR T3 is kept 180^o.
- When the firing angle of negative group of SCR T2 and SCR T4 is kept at 180^o, the output voltage becomes maximum negative.
- The output voltage becomes zero when the firing angle is kept 0^o.
- Actually the firing angle is kept less than 180^o in order to keep commutation margin.
- The system firing angle improves when the full converter operates in the semi converter mode.

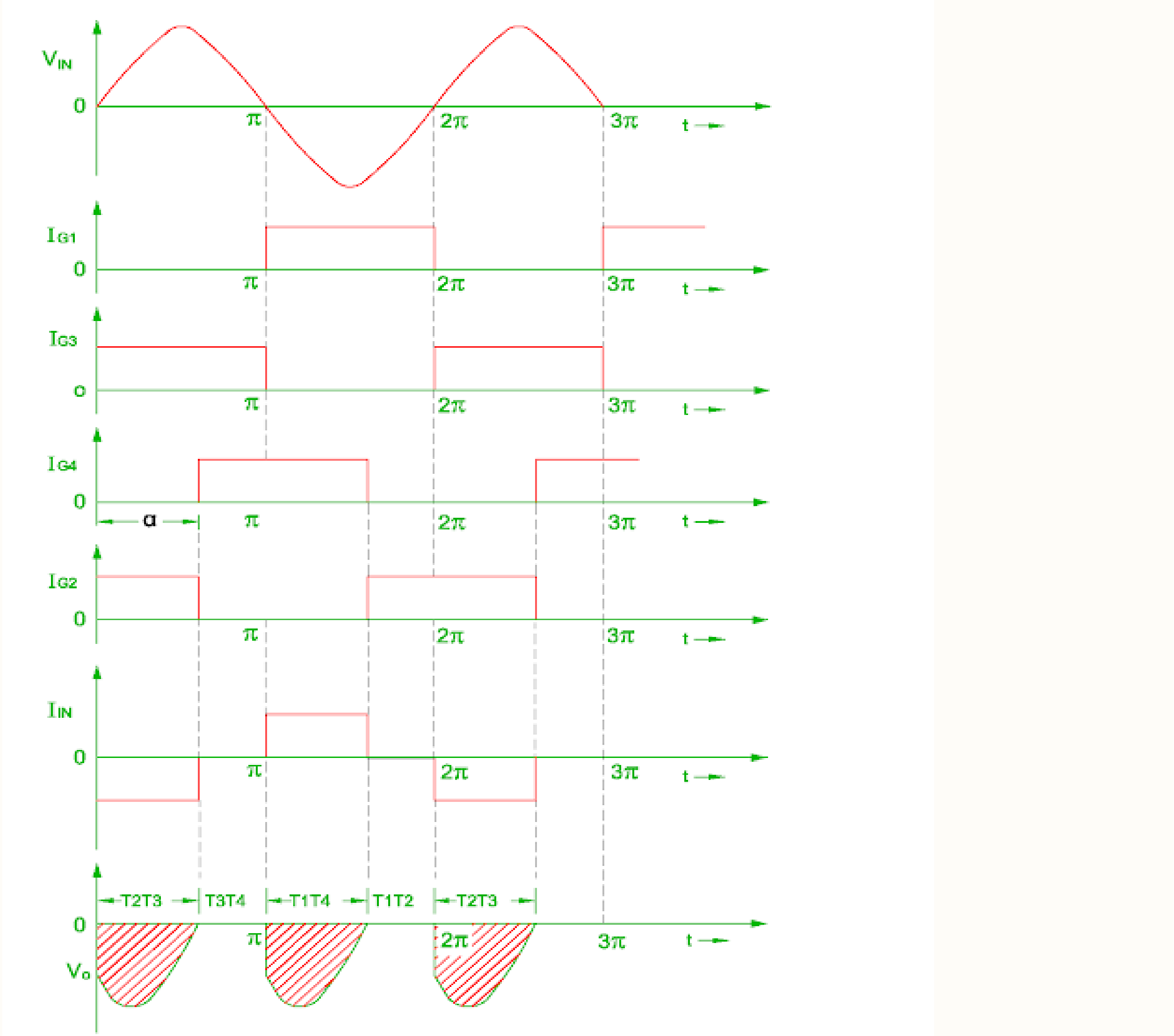


FIG D : WAVEFORMS FOR INVERSION

THYRISTOR TRIGGERING CIRCUITS

TRIGGERING OF THYRISTORS, FIRING OF THYRISTORS.

Turn on methods of SCR

- The name forward break-over voltage is given because at this voltage VBO v-i characteristics break-over and shift to its on state, position with break-over current IBO.
- At this voltage, thyristor changes from off state (high voltage with low leakage current) to on state characterized by low voltage across thyristor with large forward current.

- As other junction J1, J3 are already forward biased, breakdown of junction J2 allows free movement of carriers across three junctions and as a result large forward anode current flows This forward current is limited by load impedance.
- In practice the transition from off state to on state obtained by exceeding VBO is never employed as it may destroy the device

Gate triggering:

- Turning on thyristor by gate triggering is simple, reliable and efficient. It is therefore the most usual method of firing the forward biased SCR.
- A thyristor with forward break-over voltage (say 800 V), higher than the normal voltage (say 400V) is chosen
- This means that thyristor will remain in forward blocking state with normal working voltage across anode and cathodeand with gate open. However, when turn of a thyristor required, a +ve gate voltage between gate and cathode is applied.
- The forward voltage at which the device switches to on state depends on the magnitude of gate current.
- Higher is the gate current lower is the forward break-over voltage. When +ve gate current is applied, gate p layer is flooded with electrons from the cathode, This is because cathode n layer is heavily doped as compared to gate p layer
- As the thyristor is forward biased, some of these electrons reaches, junction J2, As a result width of the depletion layer near junction J2 is reduced.
- This cause the junction J2 to break down at an applied voltage, lower than the forward break-over voltage VBO
- If magnitude of gate current is increased, more electrons would reach junction J2, as a consequence thyristor would get turned on at a much lower forward applied voltage.
- For Ig=0, forward break-over voltage is VBO
- For Ig1,V1<VBO
- For Ig2>Ig1,V2<V1
- Once the SCR is conducting a forward current, reverse biased junction J2 no longer exist.
- No gate current is required for the device to remains in on state. Therefore, if the gate current is removed, the conduction of current from anode to cathode is unaffected
- However, if the gate current reduced to zero before rising the
- anode current attains a value, called latching current, the device will turn off again.
- The latching current may be defined as the minimum value of anode, current which it must attain during turn on process to maintain conduction when gate signal is removed Once the thyristor is conducting, gate loses control, The

thyristor can be turned off only if the forward current falls below a low-level current called holding current.

- The holding current may be defined as the minimum value of anode, current below which it must fall for turning off the thyristor,
- The latching current is higher than holding current
- The latching current is associated with the turn on process, and holding current with turn off process
- Usually latching current is 2 to 3 times the holding current
- In industrial applications, holding current (typically 10 m A) is almost taken as zero

dv/dt triggering

- With forward voltage across anode and cathode, the two outer junctions J1 and J3 are forward biased and inner junction J2 is reverse biased.
- This reverse biased junction J2 has the characteristics of a capacitor due to charges existing across the junction.
- In other words space charge exist in the depletion region near junction J2 and therefore junction J2 behaves like a capacitance.
- If forward voltage is suddenly applied, a charging current through, junction capacitance Cj may turn on the SCR
- Almost the entire suddenly applied forward voltage Va appears across junction J2 the charging current ic.
- As the junction capacitance is almost constant, dCj/dt is zero and current ic,
- If the rise of forward voltage dVa/dt is high, the charging current ic would be more. This charging current plays the role of gate current and turn on the SCR even though gate signal is zero.
- Note that even if Va is small, it is the rate of change of Va that plays the role of turning on the device

Temperature triggering (thermal triggering):

- During forward blocking, most of the applied voltage appears across reverse biased junction J2.
- This voltage across J2, associated with leakage current, would rise the temperature of this junction.
- With increase in temperature, width of depletion layer decreases this further leads to more leakage current and therefore more junction temperature
- With this cumulative process at some high temperature, depletion layer of reverse biased junction vanishes and the device gets turned on

Light triggering:

- For light triggered SCR, a recess is made in the inner p layer
- When this recess is irradiated free charge carriers are generated just like when gate signal is applied between gate and cathode.
- The pulse of light of appropriate wavelength is guided by optical fibers for irradiation
- If the intensity of this light thrown on the recess exceeds a certain value, forward biased SCR is turned on
- Such a thyristor is known as light activated SCR (LASCR)
- LASCR may be triggered with a light source or with a gate
- signal
- Sometimes a combination of both may be used
- Light triggered SCR have now been used in high voltage direct current (HVDC) transmission system.

MODULE 3

DC Choppers

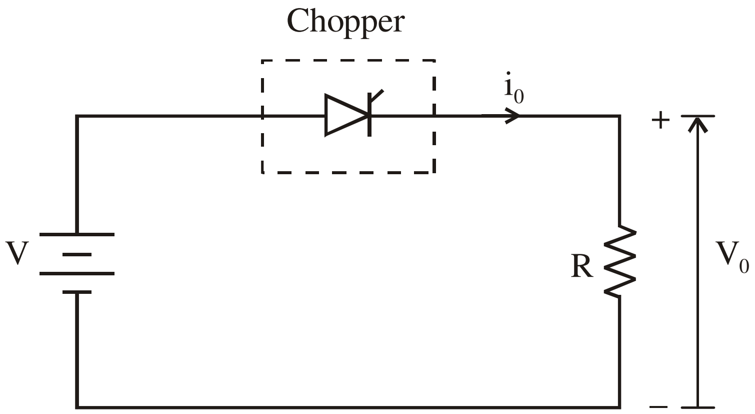
7.1 Introduction

- Chopper is a static device.
- A variable dc voltage is obtained from a constant dc voltage source.
- Also known as dc-to-dc converter.
- Widely used for motor control.
- Also used in regenerative braking.
- Thyristor converter offers greater efficiency, faster response, lower maintenance, smaller size and smooth control.

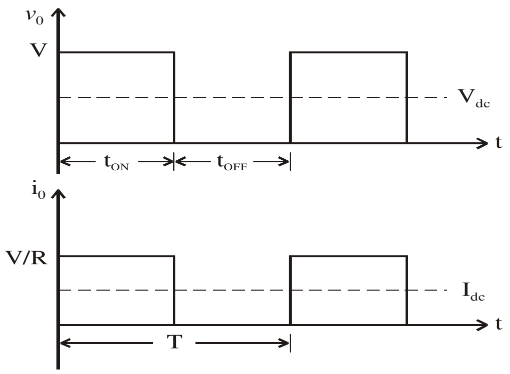
Choppers are of Two Types

- Step-down choppers.
- Step-up choppers.
    - In step down chopper output voltage is less than input voltage.
    - In step up chopper output voltage is more than input voltage.

7.2 Principle of Step-down Chopper



- A step-down chopper with resistive load.
- The thyristor in the circuit acts as a switch.
- When thyristor is ON, supply voltage appears across the load
- When thyristor is OFF, the voltage across the load will be zero.



V_{dc} = Average value of output or load voltage.
I_{dc} = Average value of output or load current.
t_{ON} = Time interval for which SCR conducts.
t_{OFF} = Time interval for which SCR is OFF.
$T = t_{ON} + t_{OFF}$ = Period of switching or chopping period.
$f = \frac{1}{T} =$ Freq. of chopper switching or chopping freq.

Average Output Voltage

$$V_{dc} = V\left(\frac{t_{ON}}{t_{ON} + t_{OFF}}\right)$$

$$V_{dc} = V\left(\frac{t_{ON}}{T}\right) = V.d$$

$$but \left(\frac{t_{ON}}{t}\right) = d = \text{duty cycle}$$

Average Output Current

$$I_{dc} = \frac{V_{dc}}{R}$$

$$I_{dc} = \frac{V}{R}\left(\frac{t_{ON}}{T}\right) = \frac{V}{R}d$$

RMS value of output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} v_o^2 dt}$$

But during t_{ON}, $v_o = V$

Therefore RMS output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt}$$

$$V_O = \sqrt{\frac{V^2}{T} t_{ON}} = \sqrt{\frac{t_{ON}}{T}}.V$$

$$V_O = \sqrt{d}.V$$

Output power $P_O = V_O I_O$

But $I_O = \dfrac{V_O}{R}$

$\therefore$ Output power

$$P_O = \frac{V_O^2}{R}$$

$$P_O = \frac{dV^2}{R}$$

Effective input resistance of chopper

$$R_i = \frac{V}{I_{dc}}$$

$$R_i = \frac{R}{d}$$

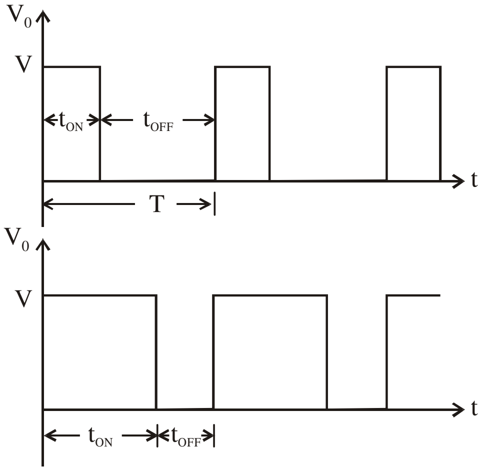
The output voltage can be varied by varying the duty cycle.

Methods of Control

- The output dc voltage can be varied by the following methods.
  - Pulse width modulation control or constant frequency operation.
  - Variable frequency control.
  -

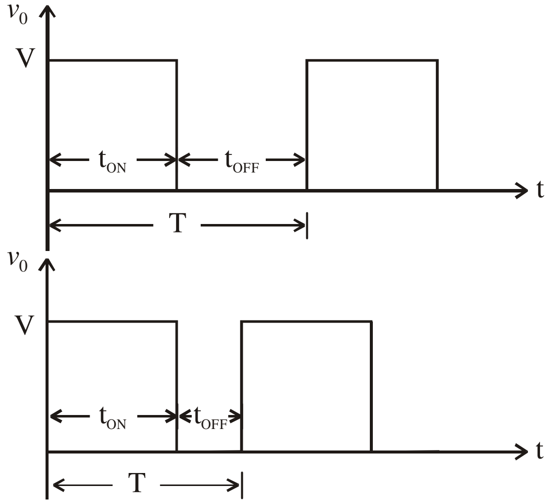
Pulse Width Modulation

- *tON* is varied keeping chopping frequency '*f*' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time t_{ON}

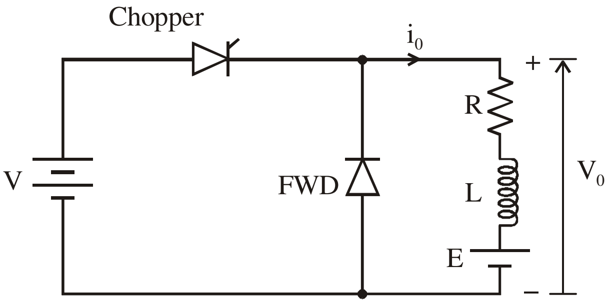


Variable Frequency Control

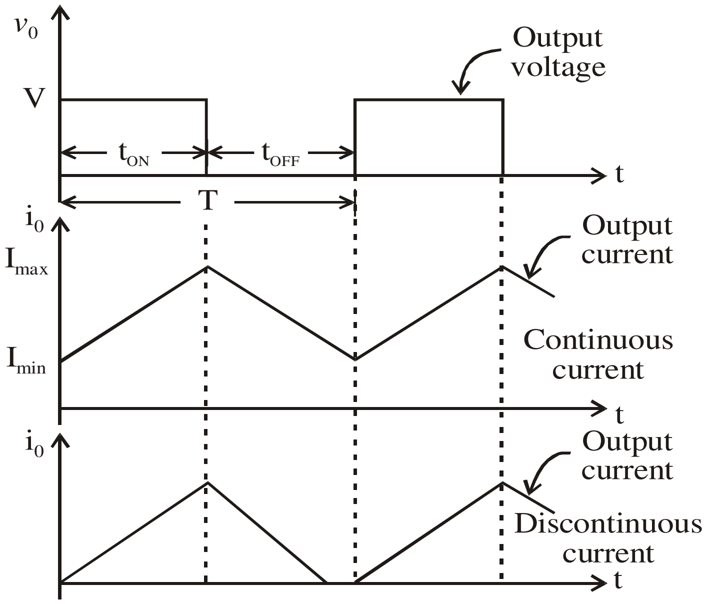
- Chopping frequency '*f*' is varied keeping either t_{ON} or t_{OFF} constant.
- To obtain full output voltage range, frequency has to be varied over a wide range.
- This method produces harmonics in the output and for large *tOFF* load current may become discontinuous



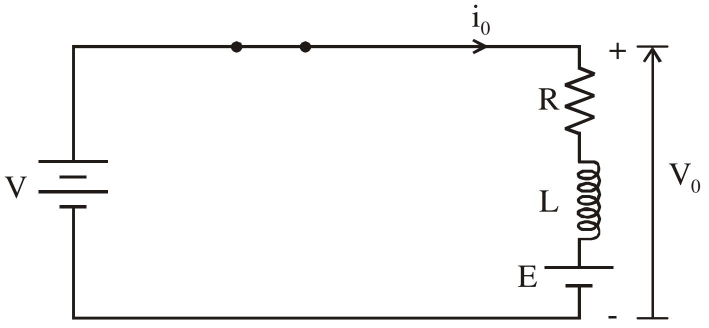
7.2.1 Step-down Chopper with R-L Load



- When chopper is ON, supply is connected across load.
- Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor *'L'*.
- Load current can be continuous or discontinuous depending on the values of *'L'* and duty cycle *'d'*
- For a continuous current operation, load current varies between two limits *Imax* and *Imin*
- When current becomes equal to *Imax* the chopper is turned-off and it is turned-on when current reduces to *Imin.*



Expressions for Load Current *Io* for Continuous Current Operation When Chopper is ON *(0 ≤T ≤ Ton)*



$$V = i_O R + L\frac{di_O}{dt} + E$$

Taking Laplace Transform

$$\frac{V}{S} = RI_O(S) + L\left[S.I_O(S) - i_O(0^-)\right] + \frac{E}{S}$$

At $t = 0$, initial current $i_O(0^-) = I_{\min}$

$$I_O(S) = \frac{V-E}{LS\left(S+\frac{R}{L}\right)} + \frac{I_{\min}}{S+\frac{R}{L}}$$

Taking Inverse Laplace Transform

$$i_O\left(t\right)=\frac{V-E}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right]+I_{\min}e^{-\left(\frac{R}{L}\right)t}$$

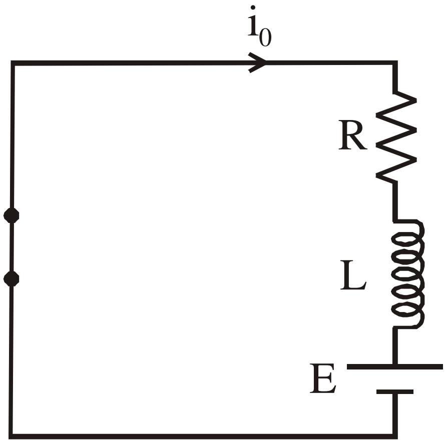
This expression is valid for $0 \le t \le t_{ON}$,

i.e., during the period chopper is ON.

At the instant the chopper is turned off,

load current is $i_O\left(t_{ON}\right)=I_{\max}$

When Chopper is OFF



When Chopper is OFF $\left(0 \le t \le t_{OFF}\right)$

$$0=Ri_O+L\frac{di_O}{dt}+E$$

Talking Laplace transform

$$0=RI_O\left(S\right)+L\left[SI_O\left(S\right)-i_O\left(0^-\right)\right]+\frac{E}{S}$$

Redefining time origin we have at $t=0$,

initial current $i_O\left(0^-\right)=I_{\max}$

$$\therefore I_O\left(S\right)=\frac{I_{\max}}{S+\frac{R}{L}}-\frac{E}{LS\left(S+\frac{R}{L}\right)}$$

Taking Inverse Laplace Transform

$$i_O\left(t\right)=I_{\max}e^{-\frac{R}{L}t}-\frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]$$

The expression is valid for $0 \le t \le t_{OFF}$,
i.e., during the period chopper is OFF

At the instant the chopper is turned ON or at the end of the off period, the load current is

$$i_O\left(t_{OFF}\right) = I_{\min}$$

To Find $I_{\max}$ & $I_{\min}$

From equation

$$i_O\left(t\right) = \frac{V-E}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right] + I_{\min} e^{-\left(\frac{R}{L}\right)t}$$

At $\qquad t = t_{ON} = dT,\ i_O\left(t\right) = I_{\max}$

$$\therefore \qquad I_{\max} = \frac{V-E}{R}\left[1-e^{-\frac{dRT}{L}}\right] + I_{\min} e^{-\frac{dRT}{L}}$$

From equation

$$i_O\left(t\right) = I_{\max} e^{-\frac{R}{L}t} - \frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]$$

At $\qquad t = t_{OFF} = T - t_{ON},\ i_O\left(t\right) = I_{\min}$

$$t = t_{OFF} = \left(1-d\right)T$$

$$\therefore \qquad I_{\min} = I_{\max} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R}\left[1-e^{-\frac{(1-d)RT}{L}}\right]$$

Substituting for $I_{\min}$ in equation

$$I_{\max} = \frac{V-E}{R}\left[1-e^{-\frac{dRT}{L}}\right] + I_{\min} e^{-\frac{dRT}{L}}$$

we get,

$$I_{\max} = \frac{V}{R}\left[\frac{1-e^{-\frac{dRT}{L}}}{1-e^{-\frac{RT}{L}}}\right] - \frac{E}{R}$$

Substituting for $I_{\max}$ in equation

$$I_{\min} = I_{\max} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R}\left[1-e^{-\frac{(1-d)RT}{L}}\right]$$

we get,

$$I_{\min} = \frac{V}{R}\left[\frac{e^{\frac{dRT}{L}}-1}{e^{\frac{RT}{L}}-1}\right] - \frac{E}{R}$$

$\left(I_{\max} - I_{\min}\right)$ is known as the steady state ripple.

Therefore peak-to-peak ripple current

$$\Delta I = I_{\max} - I_{\min}$$

Average output voltage

$$V_{dc} = d.V$$

Average output current

$$I_{dc(approx)} = \frac{I_{\max} + I_{\min}}{2}$$

Assuming load current varies linearly from $I_{\min}$ to $I_{\max}$ instantaneous load current is given by

$$i_O = I_{\min} + \frac{(\Delta I).t}{dT} \; for \; 0 \le t \le t_{ON}\,(dT)$$

$$i_O = I_{\min} + \left(\frac{I_{\max} - I_{\min}}{dT}\right)t$$

RMS value of load current

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT} i_0^2\,dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min} + \frac{(I_{\max} - I_{\min})t}{dT}\right]^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min}^2 + \left(\frac{I_{\max} - I_{\min}}{dT}\right)^2 t^2 + \frac{2I_{\min}(I_{\max} - I_{\min})t}{dT}\right]dt}$$

$$I_{CH} = \sqrt{d}\left[I_{\min}^2 + \frac{(I_{\max} - I_{\min})^2}{3} + I_{\min}(I_{\max} - I_{\min})\right]^{\frac{1}{2}}$$

$$I_{CH} = \sqrt{d}\,I_{O(RMS)}$$

Effective input resistance is

$$R_i = \frac{V}{I_S}$$

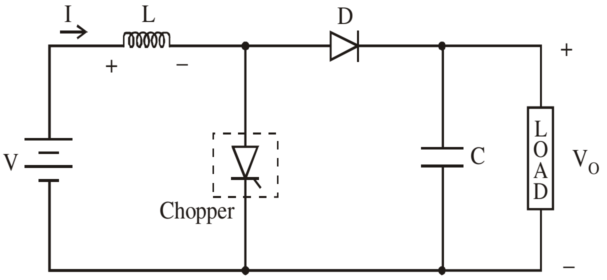
Where

I_S = Average source current

$I_S = dI_{dc}$

$$\therefore \qquad R_i = \frac{V}{dI_{dc}}$$

7.3 Principle of Step-up Chopper



- Step-up chopper is used to obtain a load voltage higher than the input voltage *V*.
- The values of *L* and *C* are chosen depending upon the requirement of output voltage and current.
- When the chopper is *ON,* the inductor *L* is connected across the supply.
- The inductor current *'I'* rises and the inductor stores energy during the *ON* time of the chopper, *tON.*
- When the chopper is off, the inductor current *I* is forced to flow through the diode *D* and load for a period, *tOFF*.
- The current tends to decrease resulting in reversing the polarity of induced EMF in *L*.
- Therefore voltage across load is given by

$$V_O = V + L\frac{dI}{dt} \quad i.e., \quad V_O > V$$

- A large capacitor 'C' connected across the load, will provide a continuous output voltage .
- Diode *D* prevents any current flow from capacitor to the source.
- Step up choppers are used for regenerative braking of dc motors.

(i) Expression For Output Voltage

Assume the average inductor current to be I during ON and OFF time of Chopper.

When Chopper is ON

Voltage across inductor $L = V$

Therefore energy stored in inductor

$$= V.I.t_{ON}$$

Where $t_{ON} = ON$ period of chopper.

When Chopper is OFF

(energy is supplied by inductor to load)

Voltage across $L = V_O - V$

Energy supplied by inductor $L = (V_O - V)\, It_{OFF}$

where $t_{OFF} = OFF$ period of Chopper.

Neglecting losses, energy stored in inductor L = energy supplied by inductor L

$$\therefore \quad VIt_{ON} = (V_O - V)\, It_{OFF}$$

$$V_O = \frac{V\left[t_{ON} + t_{OFF}\right]}{t_{OFF}}$$

$$V_O = V\left(\frac{T}{T - t_{ON}}\right)$$

Where

T = Chopping period or period of switching.

$$T = t_{ON} + t_{OFF}$$

$$V_O = V\left(\frac{1}{1 - \frac{t_{ON}}{T}}\right)$$

$$\therefore \quad V_O = V\left(\frac{1}{1-d}\right)$$

Where $d = \frac{t_{ON}}{T} =$ duty cyle

Performance Parameters

- The thyristor requires a certain minimum time to turn *ON* and turn *OFF*.
- Duty cycle *d* can be varied only between a min. & max. value, limiting the min. and max. value of the output voltage.
- Ripple in the load current depends inversely on the chopping frequency, *f*.
- To reduce the load ripple current, frequency should be as high as possible.

Problem

1. A Chopper circuit is operating on TRC at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

Solution:

$V = 460\text{ V},\ V_{dc} = 350\text{ V},\quad \text{f} = 2\text{ kHz}$

Chopping period $T = \dfrac{1}{f}$

$$T = \frac{1}{2\times10^{-3}} = 0.5\ m\text{sec}$$

Output voltage $V_{dc} = \left(\dfrac{t_{ON}}{T}\right)V$

Conduction period of thyristor

$$t_{ON} = \frac{T\times V_{dc}}{V}$$

$$t_{ON} = \frac{0.5\times10^{-3}\times350}{460}$$

$$t_{ON} = 0.38\text{ msec}$$

Problem

2. Input to the step up chopper is 200 V. The output required is 600 V. If the conducting time of thyristor is 200 µsec. Compute
   - Chopping frequency,
   - If the pulse width is halved for constant frequency of operation, find the new output voltage.

Solution:

$$V = 200\ V,\ \ t_{ON} = 200\mu s,\ \ V_{dc} = 600V$$

$$V_{dc} = V\left(\frac{T}{T - t_{ON}}\right)$$

$$600 = 200\left(\frac{T}{T - 200\times10^{-6}}\right)$$

Solving for T

$$T = 300\mu s$$

Chopping frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{300 \times 10^{-6}} = 3.33 KHz$$

μ

Pulse width is halved

$$\therefore \qquad t_{ON} = \frac{200 \times 10^{-6}}{2} = 100 \quad s$$

Frequency is constant

$$\therefore \qquad f = 3.33 KHz$$

$$T = \frac{1}{f} = 300 \mu s$$

$$\therefore \text{Output voltage} = V\left(\frac{T}{T - t_{ON}}\right)$$

$$= 200\left(\frac{300 \times 10^{-6}}{(300-100)10^{-6}}\right) = 300 \text{ Volts}$$

Problem

3. A dc chopper has a resistive load of 20Ω and input voltage VS = 220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

Solution:

$$V_S = 220V, \; R = 20\Omega, \quad f \; = \; 10 \; kHz$$

$$d = \frac{t_{ON}}{T} = 0.80$$

V_{ch} = Voltage drop across chopper = 1.5 volts

Average output voltage

$$V_{dc} = \left(\frac{t_{ON}}{T}\right)(V_S - V_{ch})$$

$$V_{dc} = 0.80(220 - 1.5) = 174.8 \text{ Volts}$$

Chopper ON time, $t_{ON} = dT$

Chopping period, $T = \dfrac{1}{f}$

$$T = \frac{1}{10\times10^{3}} = 0.1\times10^{-3}\text{ secs} = 100\ \mu\text{secs}$$

Chopper ON time,

$$t_{ON} = dT$$

$$t_{ON} = 0.80\times0.1\times10^{-3}$$

$$t_{ON} = 0.08\times10^{-3} = 80\ \mu\text{secs}$$

Problem

4. In a dc chopper, the average load current is 30 Amps, chopping frequency is 250 Hz, supply voltage is 110 volts. Calculate the ON and OFF periods of the chopper if the load resistance is 2 ohms.

Solution:

$$I_{dc} = 30\ Amps,\ f = 250\ Hz,\ V = 110\ V,\ R = 2\Omega$$

Chopping period, $T = \dfrac{1}{f} = \dfrac{1}{250} = 4\times10^{-3} = 4$ msecs

$$I_{dc} = \frac{V_{dc}}{R}\ \&\ V_{dc} = dV$$

$$\therefore \qquad I_{dc} = \frac{dV}{R}$$

$$d = \frac{I_{dc}R}{V} = \frac{30\times2}{110} = 0.545$$

Chopper ON period,

$$t_{ON} = dT = 0.545\times4\times10^{-3} = 2.18\text{ msecs}$$

Chopper OFF period,

$$t_{OFF} = T - t_{ON}$$

$$t_{OFF} = 4\times10^{-3} - 2.18\times10^{-3}$$

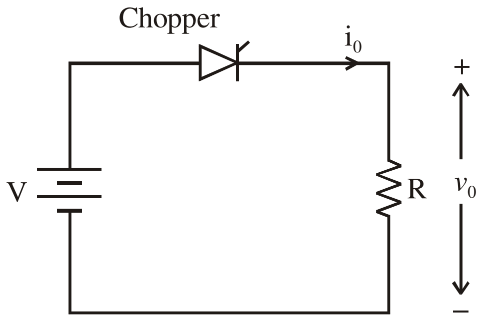
$$t_{OFF} = 1.82\times10^{-3} = 1.82\text{ msec}$$

Problem

5. A dc chopper in figure has a resistive load of R = 10Ω and input voltage of V = 200 V. When chopper is ON, its voltage drop is 2 V and the chopping frequency is 1 kHz. If the duty cycle is 60%, determine

- Average output voltage

- RMS value of output voltage
- Effective input resistance of chopper
- Chopper efficiency.



Solution:

$$V = 200\ V,\ R = 10\Omega,\ \textit{Chopper voltage drop } V_{ch} = 2V$$

$$d = 0.60,\ f = 1\ kHz.$$

Average output voltage

$$V_{dc} = d\left(V - V_{ch}\right)$$

$$V_{dc} = 0.60\left[200 - 2\right] = 118.8 \text{ Volts}$$

RMS value of output voltage

$$V_O = \sqrt{d}\left(V - V_{ch}\right)$$

$$V_O = \sqrt{0.6}\left(200 - 2\right) = 153.37 \text{ Volts}$$

Effective input resistance of chopper is

$$R_i = \frac{V}{I_S} = \frac{V}{I_{dc}}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{118.8}{10} = 11.88 \text{ Amps}$$

$$R_i = \frac{V}{I_S} = \frac{V}{I_{dc}} = \frac{200}{11.88} = 16.83\Omega$$

Output power is

$$P_O = \frac{1}{T}\int_0^{dT} \frac{v_0^2}{R}\,dt = \frac{1}{T}\int_0^{dT} \frac{\left(V - V_{ch}\right)^2}{R}\,dt$$

$$P_O = \frac{d\left(V - V_{ch}\right)^2}{R}$$

$$P_O = \frac{0.6\left[200 - 2\right]^2}{10} = 2352.24 \text{ watts}$$

Input power,

$$P_i = \frac{1}{T}\int_0^{dT} V i_O\,dt$$

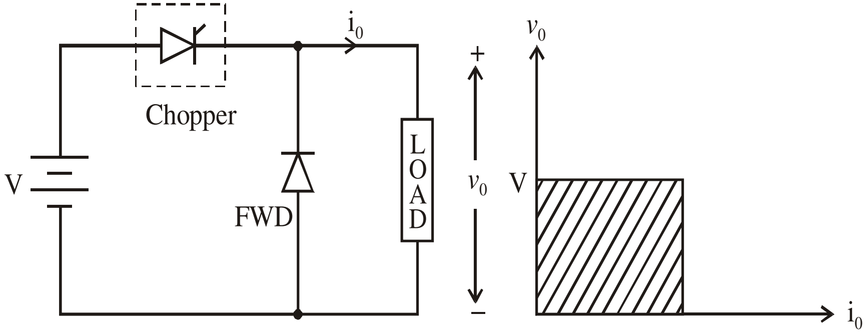
$$P_O = \frac{1}{T}\int_0^{dT} \frac{V\left(V - V_{ch}\right)}{R}\,dt$$

7.4 Classification of Choppers

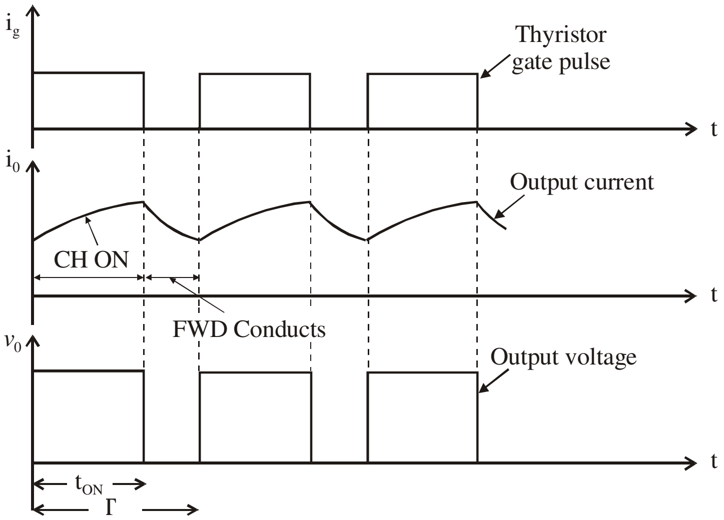
Choppers are classified as

- **Class A Chopper**
- **Class B Chopper**
- **Class C Chopper**
- **Class D Chopper**
- **Class E Chopper**

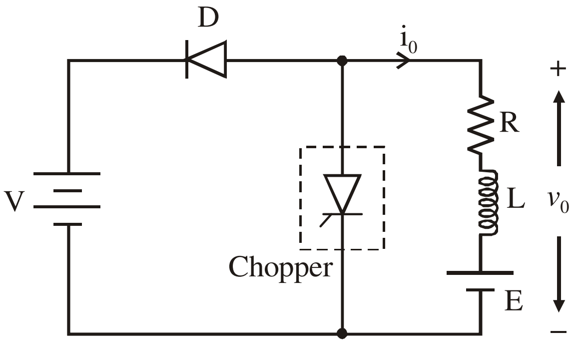
1. Class A Chopper

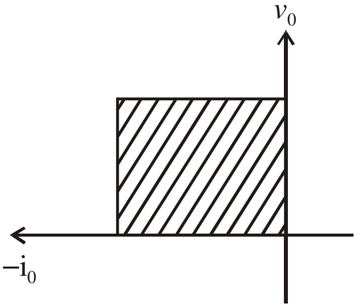


- When chopper is *ON,* supply voltage *V* is connected across the load.
- When chopper is OFF, vO = 0 and the load current continues to flow in the same direction through the FWD.
- The average values of output voltage and current are always positive.
- *Class A Chopper* is a first quadrant chopper .
- *Class A Chopper* is a step-down chopper in which power always flows form source to load.
- It is used to control the speed of dc motor.
- The output current equations obtained in step down chopper with *R-L* load can be used to study the performance of *Class A Chopper*.

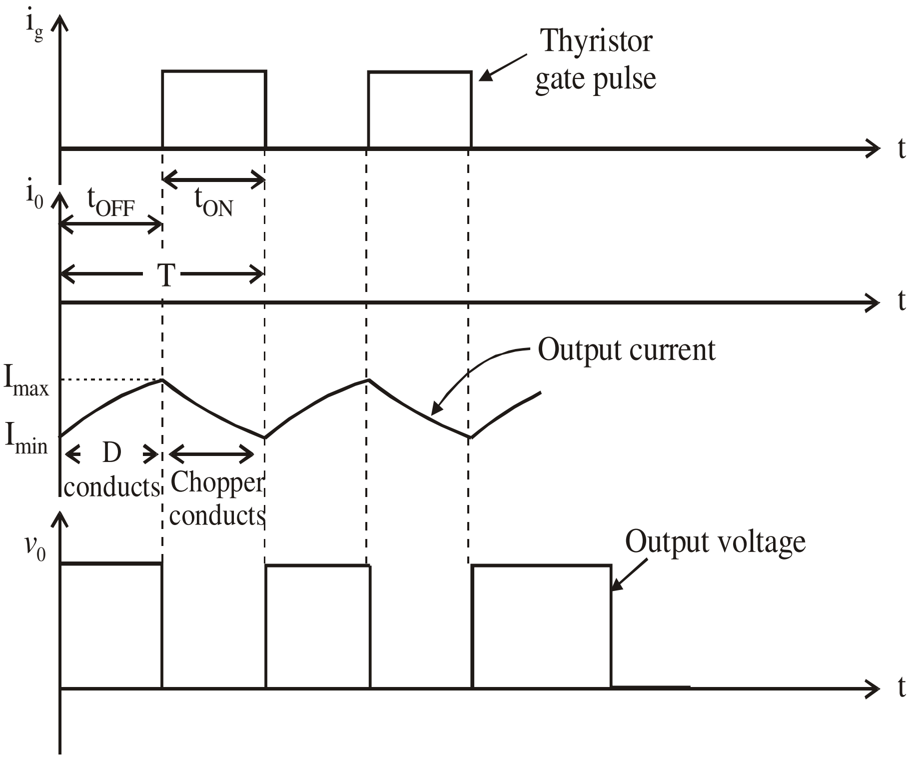


2. Class B Chopper





- When chopper is ON, *E* drives a current through *L* and R in a direction opposite to that shown in figure.
- During the ON period of the chopper, the inductance *L* stores energy.
- When Chopper is OFF, diode *D* conducts, and part of the energy stored in inductor *L* is returned to the supply.
- Average output voltage is positive.
- Average output current is negative.
- Therefore C*lass B Chopper* operates in second quadrant.
- In this chopper, power flows from load to source.
- *Class B Chopper* is used for regenerative braking of dc motor.
- *Class B Chopper* is a step-up chopper.



(i) Expression for Output Current

During the interval diode 'D' conducts voltage equation is given by

$$V = \frac{Ldi_O}{dt} + Ri_O + E$$

For the initial condition i.e.,

$$i_O(t) = I_{\min} \text{ at } t = 0$$

The solution of the above equation is obtained along similar lines as in step-down chopper with R-L load

$$\therefore \quad i_O(t) = \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}t}\right) + I_{\min} e^{-\frac{R}{L}t} \quad 0 < t < t_{OFF}$$

At $t = t_{OFF}$ $\qquad i_{(O)}(t) = I_{\max}$

$$I_{\max} = \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}t_{OFF}}\right) + I_{\min} e^{-\frac{R}{L}t_{OFF}}$$

During the interval chopper is ON voltage equation is given by

$$0 = \frac{Ldi_O}{dt} + Ri_O + E$$

Redefining the time origin, at $t = 0 \quad i_O(t) = I_{\max}$

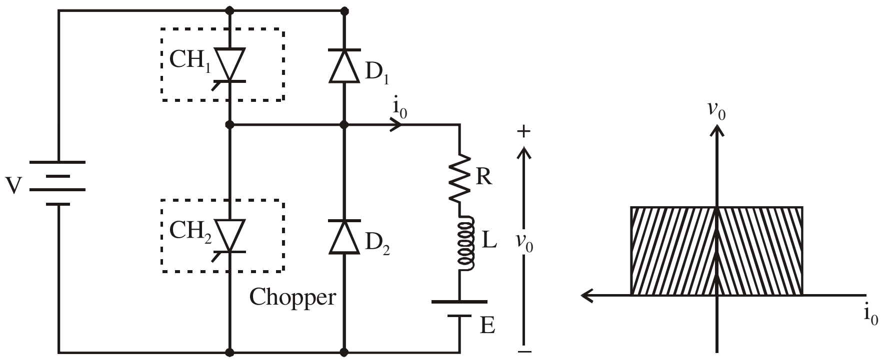
The solution for the stated initial condition is

$$i_O(t) = I_{\max} e^{-\frac{R}{L}t} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad 0 < t < t_{ON}$$

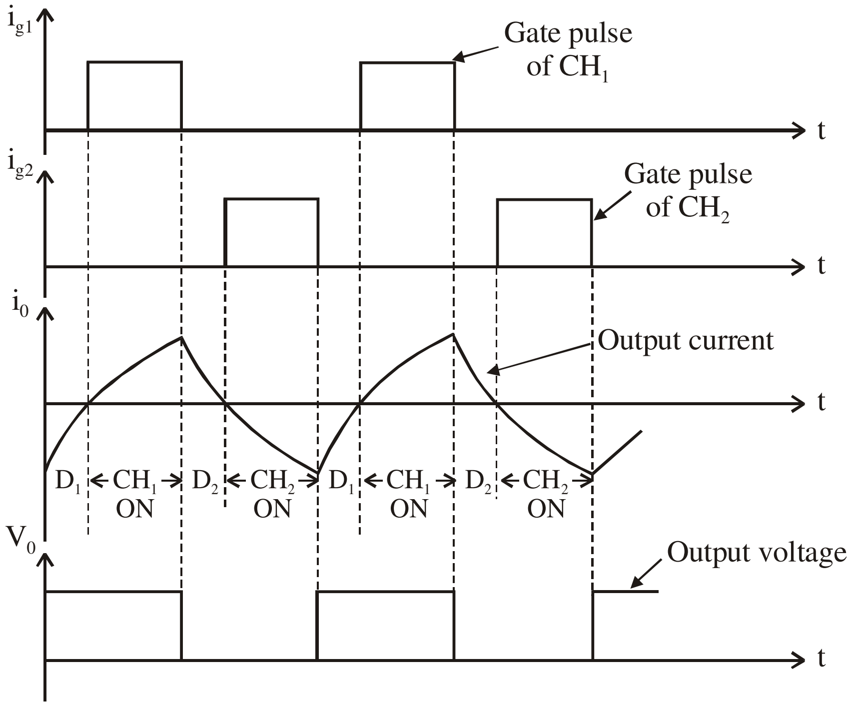
At $t = t_{ON}$ $\qquad i_O(t) = I_{\min}$

$$\therefore \quad I_{\min} = I_{\max} e^{-\frac{R}{L}t_{ON}} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}t_{ON}}\right)$$

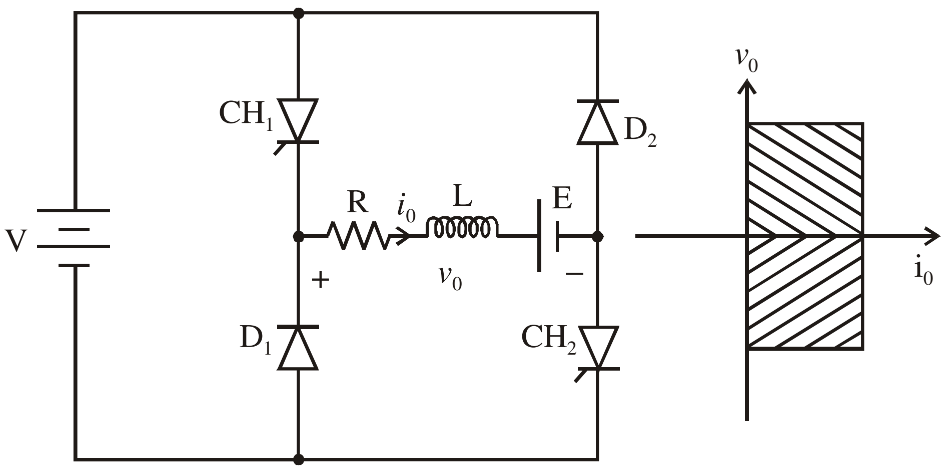
3. Class C Chopper



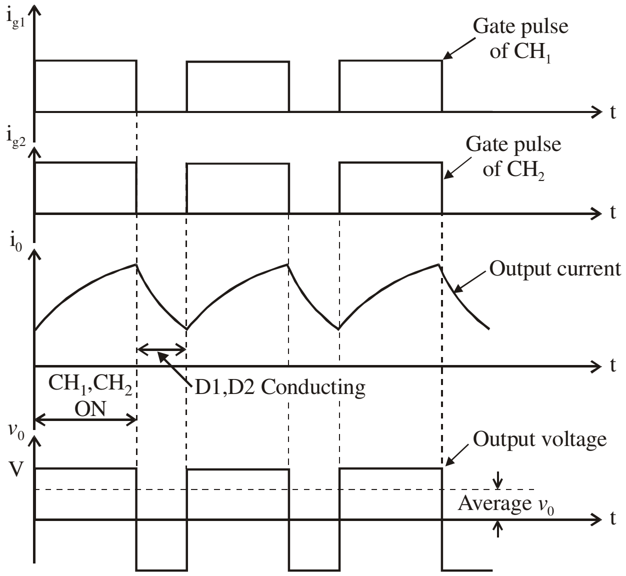
- *Class C Chopper* is a combination of *Class A* and *Class B Choppers*.
- For first quadrant operation, *CH1* is ON or *D2* conducts.
- For second quadrant operation, *CH2* is ON or *D1* conducts.
- When *CH1* is ON, the load current is positive.
- The output voltage is equal to *'V'* & the load receives power from the source.
- When *CH1* is turned OFF, energy stored in inductance *L* forces current to flow through the diode *D2* and the output voltage is zero.
- Current continues to flow in positive direction.
- When *CH2* is triggered, the voltage *E* forces current to flow in opposite direction through L and *CH2* .
- The output voltage is zero.
- On turning OFF *CH2* , the energy stored in the inductance drives current through diode *D1* and the supply
- Output voltage is *V*, the input current becomes negative and power flows from load to source.
- Average output voltage is positive
- Average output current can take both positive and negative values.
- Choppers *CH1 & CH2* should not be turned ON simultaneously as it would result in short circuiting the supply.
- *Class C Chopper* can be used both for dc motor control and regenerative braking of dc motor.
- *Class C Chopper* can be used as a step-up or step-down chopper.

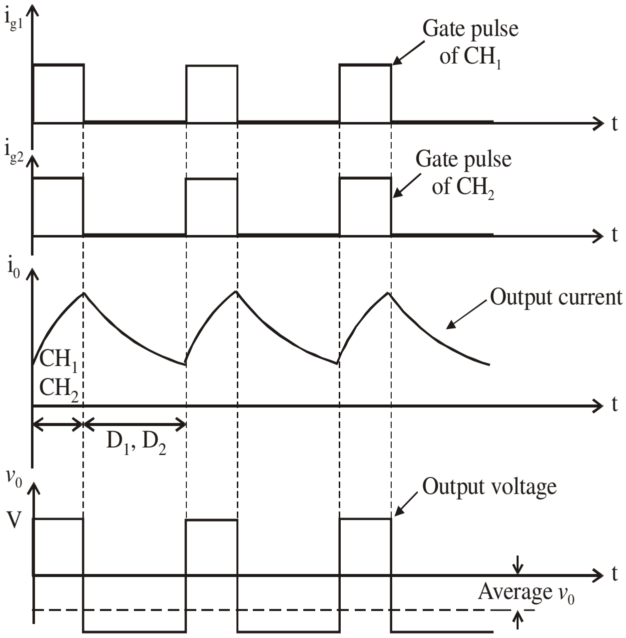


4. Class D Chopper

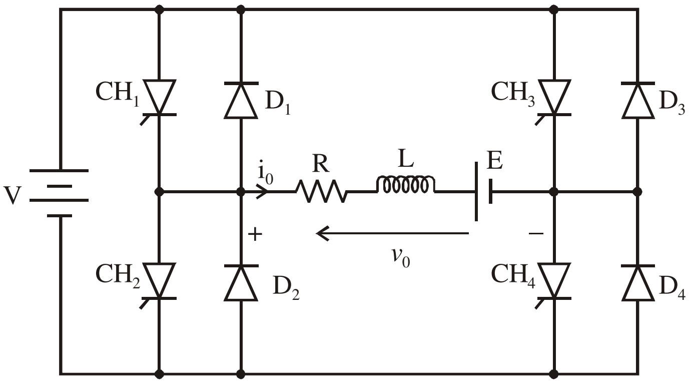


- Class D is a two quadrant chopper.
- When both *CH1* and *CH2* are triggered simultaneously, the output voltage $vO = V$ and output current flows through the load.
- When *CH1* and *CH2* are turned OFF, the load current continues to flow in the same direction through load, *D1* and *D2* , due to the energy stored in the inductor L.
- Output voltage $vO = - V$.
- Average load voltage is positive if chopper ON time is more than the OFF time
- Average output voltage becomes negative if $tON < tOFF$.
- Hence the direction of load current is always positive but load voltage can be positive or negative.

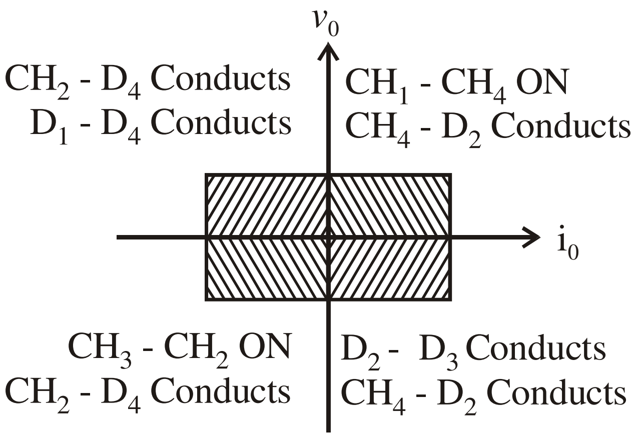




5. Class E Chopper



Four Quadrant Operation



- Class E is a four quadrant chopper
- When *CH1* and *CH4* are triggered, output current i_O flows in positive direction through *CH1* and *CH4*, and with output voltage $v_O = V$.
- This gives the first quadrant operation.
- When both *CH1* and *CH4* are OFF, the energy stored in the inductor L drives i_O through D2 and *D3* in the same direction, but output voltage $v_O = -V$.
- Therefore the chopper operates in the fourth quadrant.
- When *CH2* and *CH3* are triggered, the load current i_O *flows* in opposite direction & output voltage $v_O = -V$.
- Since both i_O and v_O are negative, the chopper operates in third quadrant.
- When both *CH2* and *CH3* are OFF, the load current i_O continues to flow in the same direction *D1* and *D4* and the output voltage $v_O = V$.
- Therefore the chopper operates in second quadrant as v_O is positive but i_O is negative.

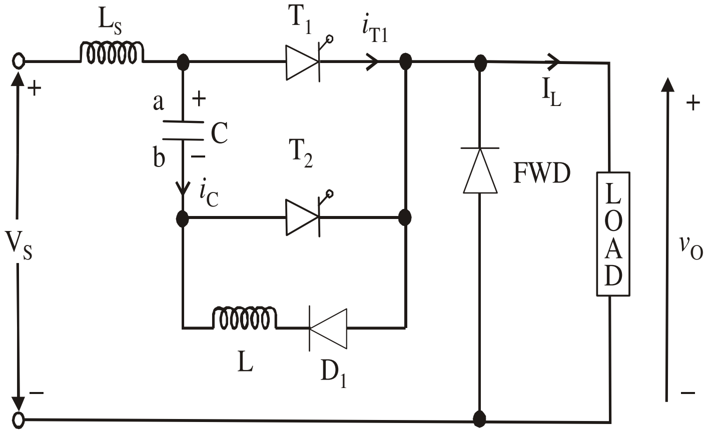
Effect Of Source & Load Inductance

- The source inductance should be as small as possible to limit the transient voltage.
- Also source inductance may cause commutation problem for the chopper.
- Usually an input filter is used to overcome the problem of source inductance.
- The load ripple current is inversely proportional to load inductance and chopping frequency.
- Peak load current depends on load inductance.
- To limit the load ripple current, a smoothing inductor is connected in series with the load.

7. 5 Impulse Commutated Chopper

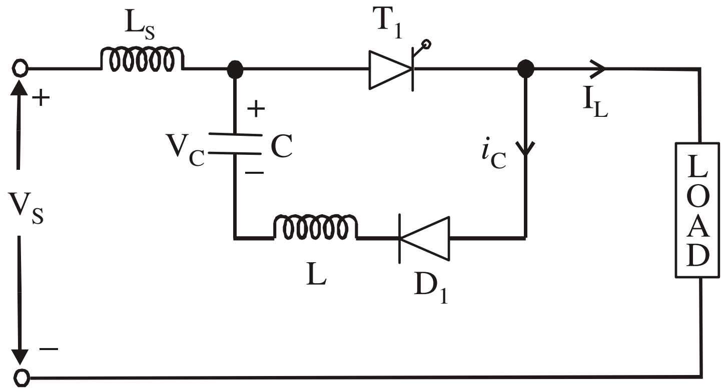
- Impulse commutated choppers are widely used in high power circuits where load fluctuation is not large.
- This chopper is also known as
  - Parallel capacitor turn-off chopper
  - Voltage commutated chopper

- Classical chopper.



- To start the circuit, capacitor *‘C’* is initially charged with polarity (with plate ‘a’ positive) by triggering the thyristor *T2*.
- Capacitor *‘C’* gets charged through *VS, C, T2* and load.
- As the charging current decays to zero thyristor *T2* will be turned-off.
- With capacitor charged with plate ‘a’ positive the circuit is ready for operation.
- Assume that the load current remains constant during the commutation process.
- For convenience the chopper operation is divided into five modes.
  - Mode-1
  - Mode-2
  - Mode-3
  - Mode-4
  - Mode-5

Mode-1 Operation



- Thyristor *T1* is fired at *t = 0.*
- The supply voltage comes across the load.
- Load current *IL* flows through *T1* and load.
- At the same time capacitor discharges through *T1, D1, L1, & ‘C’* and the capacitor reverses its voltage.
- This reverse voltage on capacitor is held constant by diode *D1*.

Capacitor Discharge Current

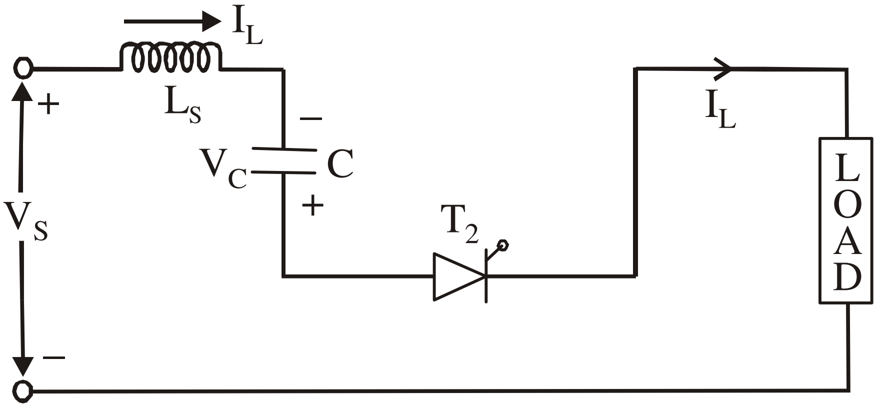
$$i_C(t) = V\sqrt{\frac{C}{L}}\sin\omega t$$

Where $\omega = \frac{1}{\sqrt{LC}}$

& Capacitor Voltage

$$V_C(t) = V\cos\omega t$$

Mode-2 Operation



- Thyristor *T2* is now fired to commutate thyristor *T1*.
- When *T2* is ON capacitor voltage reverse biases *T1* and turns if off.
- The capacitor discharges through the load from $-V$ to *0*.
- Discharge time is known as circuit turn-off time
- Capacitor recharges back to the supply voltage (with plate 'a' positive).
- This time is called the recharging time and is given by

Circuit turn-off time is given by

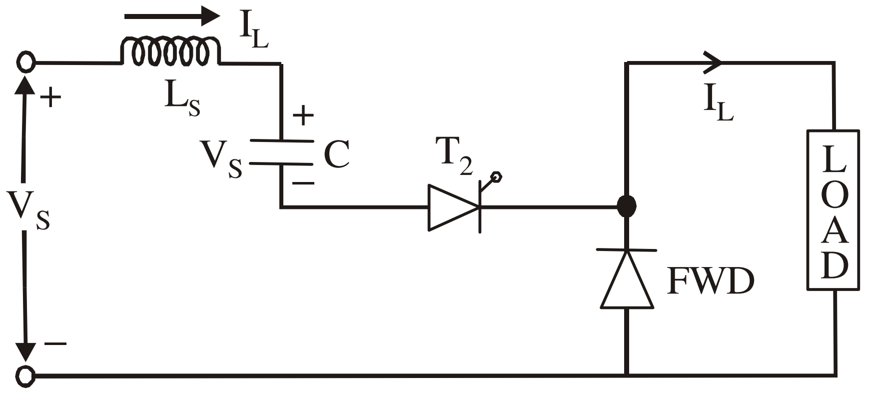
$$t_C = \frac{V_C \times C}{I_L}$$

Where I_L is load current.

t_C depends on load current, it must be designed for the worst case condition which occur at the maximum value of load current and minimum value of capacitor voltage.

- The total time required for the capacitor to discharge and recharge is called the commutation time and it is given by
- At the end of Mode-2 capacitor has recharged to V_S *and* the freewheeling diode starts conducting.

Mode-3 Operation



- *FWD* starts conducting and the load current decays.
- The energy stored in source inductance *LS* is transferred to capacitor.
- Hence capacitor charges to a voltage higher than supply voltage, T2 naturally turns off.

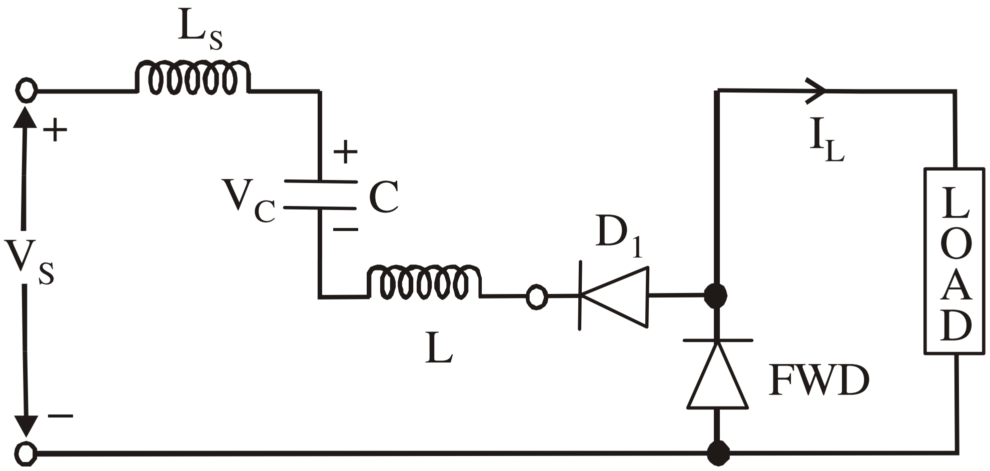
The instantaneous capacitor voltage is

$$V_C(t) = V_S + I_L\sqrt{\frac{L_S}{C}}\sin\omega_S t$$

Where

$$\omega_S = \frac{1}{\sqrt{L_S C}}$$

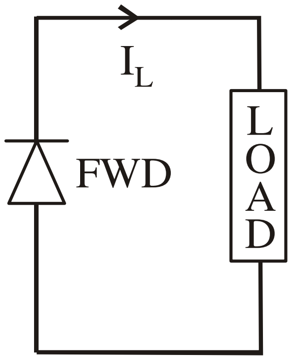
Mode-4 Operation



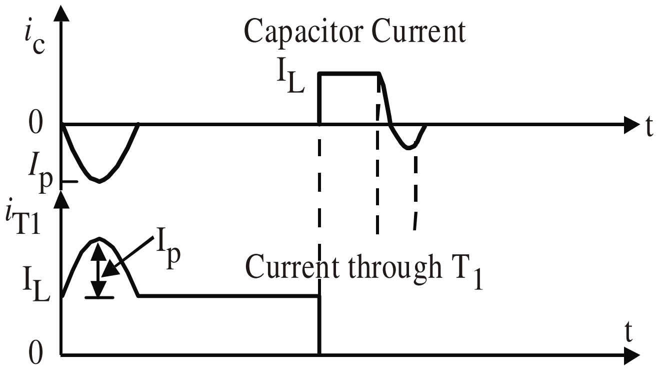
- Capacitor has been overcharged i.e. its voltage is above supply voltage.
- Capacitor starts discharging in reverse direction.
- Hence capacitor current becomes negative.
- The capacitor discharges through *LS, VS, FWD, D1 and L.*

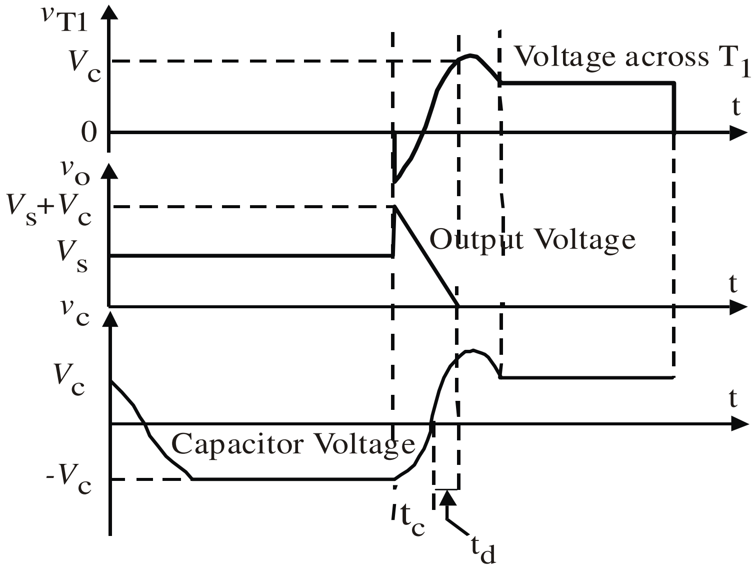
- When this current reduces to zero *D1* will stop conducting and the capacitor voltage will be same as the supply voltage.

Mode-5 Operation



- Both thyristors are off and the load current flows through the FWD.
- This mode will end once thyristor *T1* is fired.





Disadvantages

- A starting circuit is required and the starting circuit should be such that it triggers thyristor *T2* first.
- Load voltage jumps to almost twice the supply voltage when the commutation is initiated.
- The discharging and charging time of commutation capacitor are dependent on the load current and this limits high frequency operation, especially at low load current.
- Chopper cannot be tested without connecting load.

Thyristor *T1* has to carry load current as well as resonant current resulting in increasing its peak current rating.

Recommended questions:

1. Explain the principle of operation of a chopper. Briefly explain time-ratio control and PWM as applied to chopper
2. Explain the working of step down shopper. Determine its performance factors, VA, Vo rms, efficiency and Ri the effective input resistane
3. Explain the working of step done chopper for RLE load. Obtain the expressions for minimum load current I1max load current I2, peak – peak load ripple current di avg value of load current Ia, the rms load current Io and Ri.
4. Give the classification of stem down converters. Explain with the help of circuit diagram one-quadrant and four quadrant converters.
5. The step down chopper has a resistive load of R=10ohm and the input voltage is Vs=220V. When the converter switch remain ON its voltage drop is Vch=2V and the chopping frequency is 1 KHz. If the duty cycle is 50% determine a) the avg output voltage VA, b) the rms output voltage Vo c) the converter efficiency d) the effective input resistance Ri of the converter.
6. Explain the working of step-up chopper. Determine its performance factors.

MODULE 4

MODULE 4 INVERTERS

Voltage source inverters – series- parallel and bridge inverters – PWM inverters – current source inverters.

- Inverters are circuits that convert dc to ac. More precisely, inverters transfer power from a dc source to an ac load.
  The Inverter is the power electronic circuit, which converts the DC voltage into AC voltage. The DC source is normally a battery or output of the controlled rectifier.
- The output voltage waveform of the inverter can be square wave, quasi-square wave or low distorted sine wave.
- The output voltage can be controlled with the help of drives of the switches.
- The pulse width modulation techniques are most commonly used to control the output voltage of inverters. Such inverters are called as PWM inverters.
- The output voltage of the inverter contain harmonics whenever it is not sinusoidal.
- These harmonics can be reduced by using proper control schemes.

Inverters can be broadly classified into two types. They are

1. Voltage Source Inverter (VSI)
2. Current Source Inverter (CSI)

When the DC voltage remains constant, then it is called *Voltage Source Inverter(VSI)* or Voltage Fed Inverter (VFI).

When input current is maintained constant, then it is called *Current Source Inverter (CSI)* or Current Fed Inverter (CFI).

COMPARISON OF VSI AND CSI

VSI	**CSI**
VSI is fed from a DC voltage source having small or negligible impedance.	CSI is fed with adjustable current from a DC voltage source of high impedance.

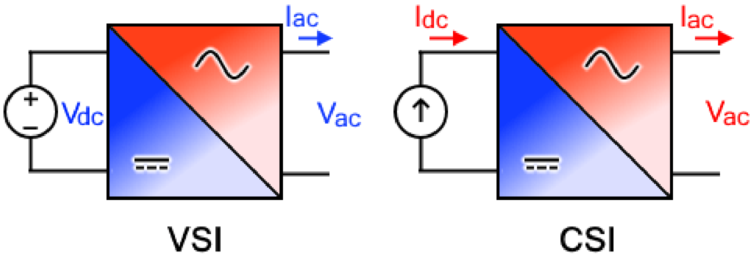
Input voltage is maintained constant	The input current is constant but adjustable.
Output voltage does not dependent on the load	The amplitude of output current is independent of the load.
The waveform of the load current as well as its magnitude depends upon the nature of load impedance.	The magnitude of output voltage and its waveform depends upon the nature of the load impedance.
VSI requires feedback diodes	The CSI does not require any feedback diodes.
The commutation circuit is complicated	Commutation circuit is simple as it contains only capacitors.
Power BJT, Power MOSFET, IGBT, GTO with self commutation can be used in the circuit.	They cannot be used as these devices have to withstand reverse voltage.

Voltage Source Inverter (VSI)

The inverter is known as voltage source inverter when the input of the inverter is a constant DC voltage source. The input to the voltage source inverter has a **stiff DC voltage source**. Stiff DC voltage source means that the impedance of DC voltage source is zero. Practically, DC sources have some negligible impedance. VSI are assumed to be supplied with ideal voltage sources (very low impedance sources). The AC output voltage is completely determined by the states of switching devices in the inverter and the applied DC source.

Current Source Inverter (CSI)

The inverter is known as current source inverter when the input of the inverter is a constant DC current source. Stiff current is supplied to the CSI (current source inverter) from the DC source where the DC source have high impedance. Usually, a large inductor or closed loop-controlled current are used to provide stiff current. The resulting current wave is stiff which is not influenced by the load. The AC output current is completely determined by the states of switching devices in the inverter and the DC applied source.



Voltage Source & Current Source Inverter

The output voltage and current waveform of the inverter circuit, v_o, and i_o respectively, are assumed to be AC quantities. These are stated in terms of RMS values normally while the deviation of these waveforms from their fundamental and sinusoidal components is represented in the terms of THD factors. THD shows the total harmonic distortion.

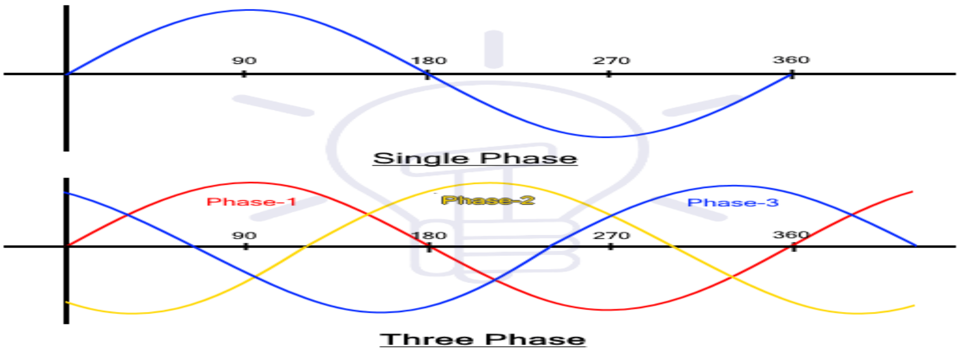
Output Phase Wise Classification

According to the output voltage and current phases, inverters are divided into two main categories. Single-phase inverters and three-phase inverters. These categories are briefly discussed here.

Single Phase Inverters

A single-phase inverter converts DC input into Single phase output. The output voltage/current of single-phase inverter has exactly one phase which has a nominal frequency of 50HZ or 60Hz a nominal voltage. The Nominal voltage is defined as the voltage level at which Electrical system operates. There are different nominal voltages i.e. 120V, 220V, 440V, 690V, 3.3KV, 6.6KV, 11kV, 33kV, 66kV, 132kV, 220kV, 400kV and 765kV

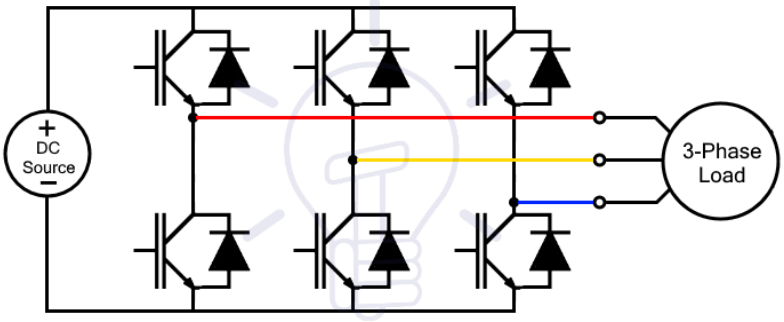
Low nominal voltages can be directly achieved by inverter using an internal transformer or buck-boost circuitry while for high nominal voltages, external step-up transformers are used. Single-phase inverters are used for low loads. There are more losses in single-phase as well as the efficiency of single-phase is low with respect to three-phase inverter. Therefore, 3 phase inverters are preferred for high loads.



Three Phase Inverters

Three-phase inverters convert DC into three-phase power. Three-phase power provides three alternating currents which are uniformly separated in phase angle. Amplitudes and frequencies of all three waves generated at the output are same with slight variations due to load while each wave has a 120^o phase shift from each other.

Basically, a single 3-phase inverter is 3 single-phase inverters, where phases of each inverter are 120 degrees apart and each single-phase inverter is connected to one of the three load terminals.



Typical Three Phase Inverter

There are different topologies for constructing a 3 phase voltage inverter circuit. In case of bridge inverter, operating by 120-degree mode, the Switches of three-phase inverters are operated such that each switch operates T/6 of the total time which creates output waveform that has 6 steps. There is a zero-voltage step between negative and positive voltage levels of the square waveform.

Inverter power ratings can be further increased. For constructing inverters with high power ratings, 2 inverters (three-phase inverters) are connected in series for high voltage rating. For high current rating, 2 six-step three inverters can be connected.

Methods of Commutation Wise Classification

Silicon controlled rectifiers are mainly divided into two main types according to commutation techniques. **Line commutated** and **force commutated** inverters are used commonly while other commutated inverters i.e., Auxiliary commutated inverters and complementary commutated inverters are not used commonly. The two main types are briefly discussed here.

Line Commutated

In these types of inverters, the AC circuits' the line voltage is accessible over the device; The device is turned off when the current in SCR experience zero characteristics. This commutation process is known as line commutation while inverters working on this principle are known as Line commutated Inverters.

Force Commutated

The supply does not experience zero points in this type of commutation. That's why some outside source is needed to commutate the device. This process of commutation is known as force commutation while inverters based on this process are known as Force commutated inverters.

SERIES INVERTERS

The series inverter consists of a pair of thyristors and **RLC** (Resistance, Inductor and capacitor) circuit. One thyristor is connected in parallel with the RLC circuit while one in series between DC source and RLC circuit. this inverter is known as a series inverter because the load is directly connected in series with DC source with the help of T1.

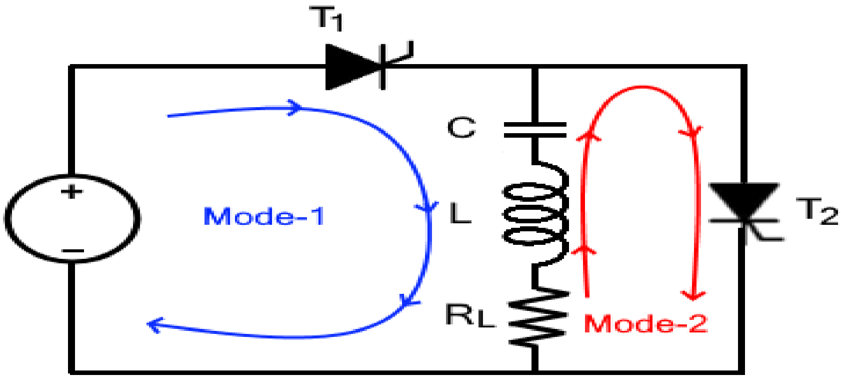
Series inverter is also known as a **self-commutated** inverter because thyristors of this inverter are commutated by their own from the load. Another name for this inverter is "**Load commutated inverter**". This name is given because LCR is the load which provides commutation.

Working of Series Inverter

Two thyristors (T1 & T2) are used for converting DC into AC, including the RLC circuit. Only one thyristor in this circuit will be turned on at a time. T2 will be OFF at the time when

T1 is ON, while T1 will be OFF at the time when T2 is ON. Both thyristors must not turn on at the same time else it will cause short circuit. Turning on both the thyristors at a time will permanently damage the circuit even if it is for a very short interval of time. That is why **time delay** is given to the contrary thyristor. In other words, the contrary thyristor is not turned till the opposite is turned off completely.

Circuit diagram of Series Inverter



Series Inverter

Mode1:

In this mode, T1 is turned on and T2 is turned off. Initially both T1 and T2 are off. As T1 is turned ON, the current starts flowing from DC source into LCR load. In this mode, the current from DC source enters from the capacitor side and leaves from the resistor side. The capacitor starts charging while the inductor discharges itself in this mode.

Mode 2:

In this mode T2 is turned on and T1 is turned off. Before switching from mode1 to mode2, a time delay is provided so that T1 can be switched off completely. Thyristors have specific **reverse recovery time** which is required for a thyristor to turn off completely. After T1 is completely switched OFF, T2 is turned ON. The current will start flowing from the DC power supply through T2 into the load. The current in this mode will enter from the opposite side of the load, which means that the current will enter from the resistor side and will leave from capacitor side.

The alternation of current in the load by switching from mode to mode shows the inverting principle of an inverter. This alternation of current in the load shows that DC current has been successfully converted into AC.

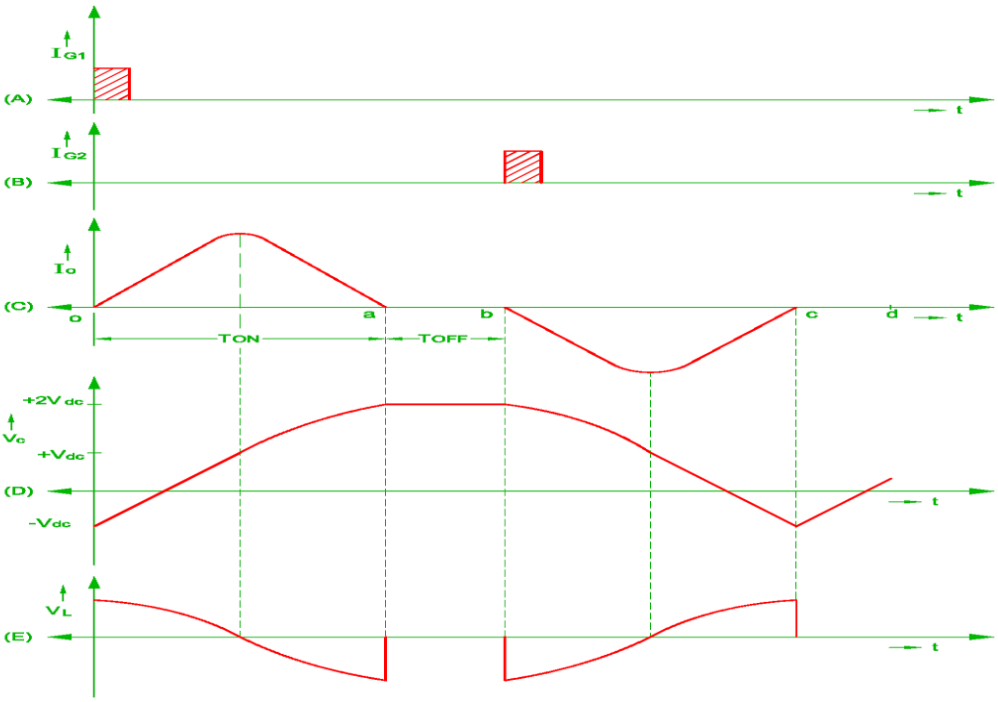


FIGURE D : VOLTAGE AND CURRENT WAVEFORMS OF SERIES INVERTER
(A) Gate pulse for SCR T1 (B) Gate pulse for SCR T2 (C) output current,
(D) Capacitor voltage (E) Load voltage

Advantages

Few advantages of series inverters are given here

- **Induction heating:** series inverters provide high current so, these inverters can be used for induction heaters which require extra current.

- **Florence lighting** these inverters can be used for Florence lighting.

- **High-frequency operation**: These inverters can be utilized at high frequency because these inverters can be functioned from 200 hz to 200khz.

PARALLEL INVERTERS

The parallel inverter consists of two thyristors (T1 & T2), one capacitor, center-tapped transformer, and an inductor. Thyristors are used for providing a path to the flow of current while inductor L is used to make the current source constant.

These thyristors are turned ON and OFF, controlled by commutation capacitor connected between them. The **complementary commutation** method is used for turning ON and turning OFF capacitors.
A complementary commutation means that when T1 is ON, the firing angle is applied to T2, then the capacitor will turn OFF T1. The exact case is when T2 is ON and firing angle is applied to T1, then because of capacitor voltage, the T2 will turn OFF. Output current and voltage are I_o and V_o respectively.

It is known as **Parallel inverters** because in working condition, capacitor C comes in parallel with the load via the transformer. Parallel inverters are also known as center tapped transformers inverter because it has a centre-tapped transformer in between load and driving circuitry. The purpose of transformer is to change DC into AC of the required voltage.

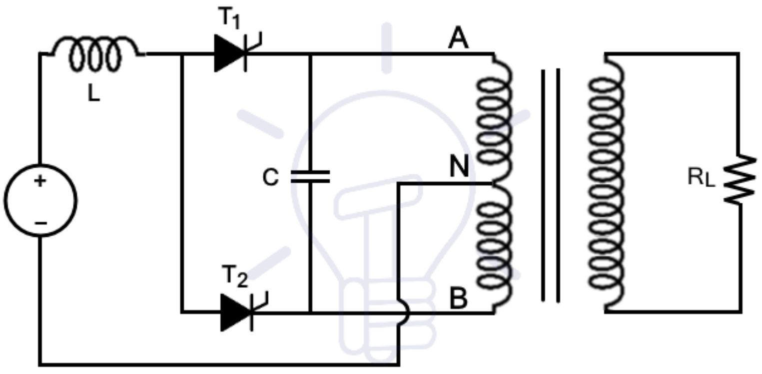
Working of Parallel Inverter
It is operated in simple two modes.

Mode1

When T1 is triggered, the commutated capacitor will turn OFF T2 and the current in primary winding will flow from A to N. such flow of current in primary winding will cause clockwise flow of current in secondary winding.

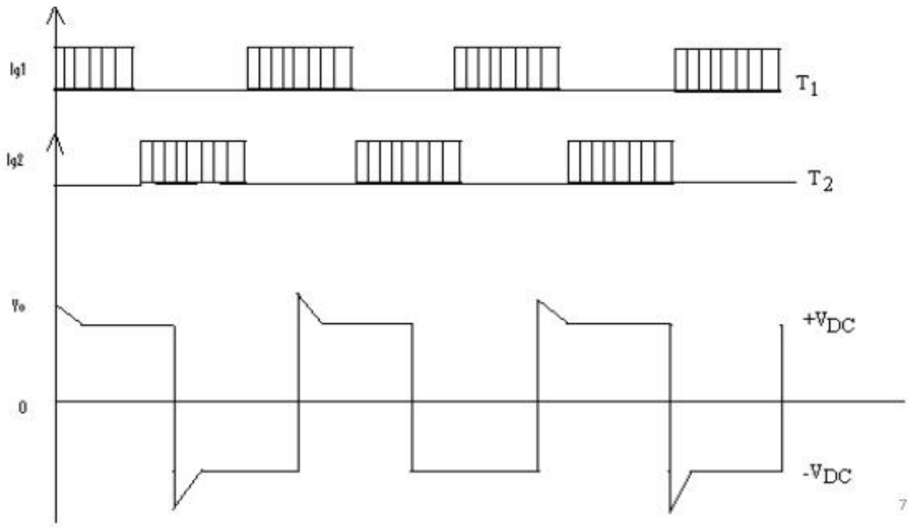
Mode2

By triggering T2, the commutated capacitor will turn off T1. So, the current in the primary winding will flow from B to N. This flow of current in the primary winding will cause an anti-clockwise flow of current in secondary winding.



Parallel Inverter

Wave Form



Circuit Diagram of Parallel Inverter and waveforms

Advantages of Parallel Inverters

Few of the advantages of parallel inverters are given as

- **Stable load voltage**: The waveform of the load voltage is independent of the load while this limitation exists in a series inverter. The output voltage in the series inverter is dependent on load which is not desired.

- **Cheapest circuit:** The circuit of parallel inverter is the cheapest and simplest because it just requires only two switches and a center-tapped transformer.

- **Simple commutation:** these inverters are operated using simple capacitor commutation. In addition, the commutation components do not carry the whole load current which is a very useful aspect of parallel inverter

- **Few control switches:** Just two control switches are required for complete operation while comparing with H-bridge Inverters. The least number of switches required for H-bridge inverters are 4.

BRIDGE TYPE INVERTERS

Half-bridge inverter requires two electronic switches to operate. The switches may be MOSFETs, IJBTs, BJTs or Thyristors. Half bridge with thyristor and BJT switches requires two extra diodes except in pure resistive loads while MOSFETs have a built-in body diode. In simple words, two switches are enough for purely resistive load while other loads (Inductive & capacitive) require two extra diodes. These diodes are known as **feedback diode** or **freewheeling diodes**.

The working principle of half-bridge inverter is the same for all switches but half-bridge with a thyristor switches is discussed here. There are two complementary thyristors which means that one thyristor will be turned on at a time. The circuit work in two modes for resistive load. The switching frequency will decide the output frequency. For 50HZ frequency at output, each thyristor is turned ON for 20ms at a time.

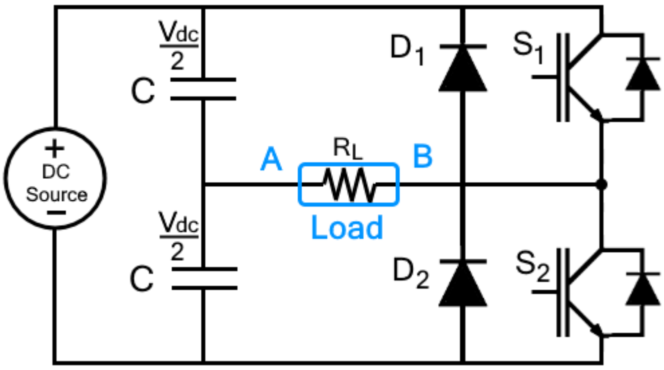
HALF BRIDGE VOLTAGE SOURCE INVERTER.

Half H-bridge is one of the inverter topologies which convert DC into AC. The typical Half-bridge circuit consists of two control switches, 3 wire DC supply, two feedback diodes, and two capacitors connecting the load with the source. Control switch can be any electronic switch i.e. MOSFET, BJT, IGBT, or thyristor, etc.

The circuit is designed in such a way that both switches must not turn-on at a single time & only one of the two switches will conduct. Each switch will operate for half period (T/2), providing half of the applied voltage the load (± $V_{dc}/2$). When both the switches are off, the reserved voltage across the load will be V_{dc} instead of $V_{dc}/2$. This is called a half-bridge inverter.

Some of the conventions in the given circuit are such that

- Current through $\mathbf{S_1}$ is $\mathbf{i_1}$, while the current flowing through $\mathbf{S_2}$ is $\mathbf{i_2}$.
- Output voltage and current are $\mathbf{V_o}$ and $\mathbf{i_o}$
- **T** is the time period and switches are considered unidirectional.



Typical Half H-Bridge Inverter

Operation of Half H-Bridge Inverter with R Load

The operation of half-bridge with pure resistive load is the simplest. A purely resistive load does not have any storage component, so the circuit doesn't need feedback diodes. The circuit with this load will be operated in just two modes.

Mode 1: (0<t<T/2)

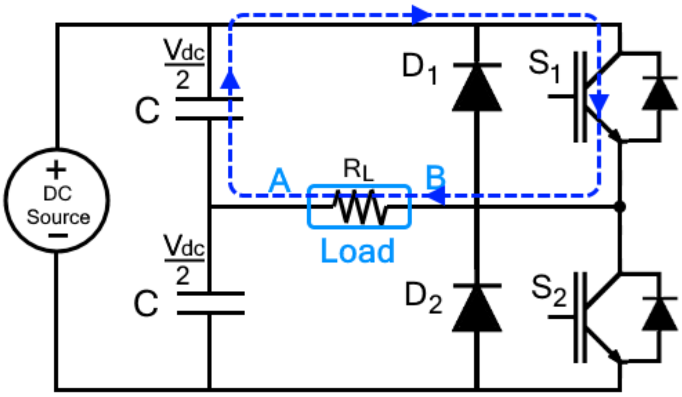
In this mode, S_1 is turned-on from time interval t=0 to t=T/2 while S_2 is turned off. As soon as S_1 is turned on, the voltage across the load will appear. The output voltage across the load will be

$$\mathbf{V_o = V_{dc}/2}$$

The current flowing through the switch $\mathbf{S_1}$ will be

$$\mathbf{I_o = V_{dc}/2R_L}$$

Where $\mathbf{R_L}$ is the load resistance. The current flow in clockwise direction as shown in the figure.



Mode-1 for R-Load

Mode 2: (T/2<t<T)

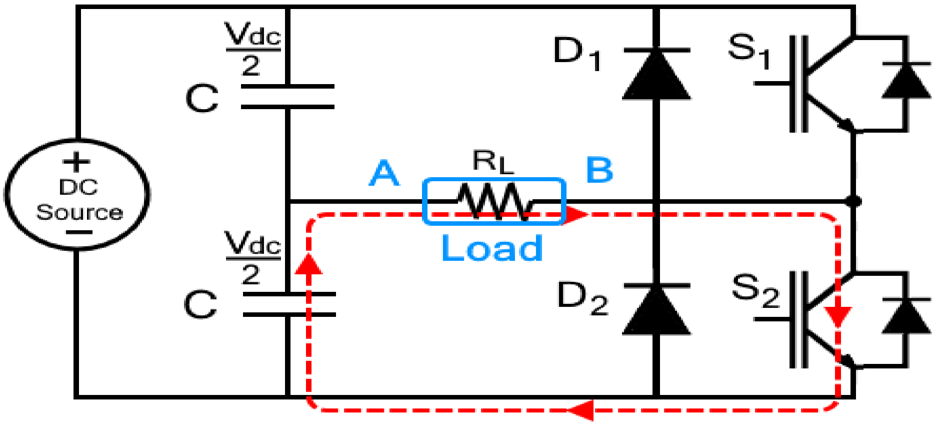
In this mode, switch S_2 is turned-on from the time interval t=T/2 to t=T while S_1 is switched off. Immediate switching of modes is avoided because it causes a short circuit. Due to this reason, S_2 is turned-on with some delay after S_1 is completely turned off. In this case, the output voltage will be negative as the current enters in the load from the opposite direction where output voltage will be

$\mathbf{V_o = -V_{dc}/2}$

The current through S_2 will be

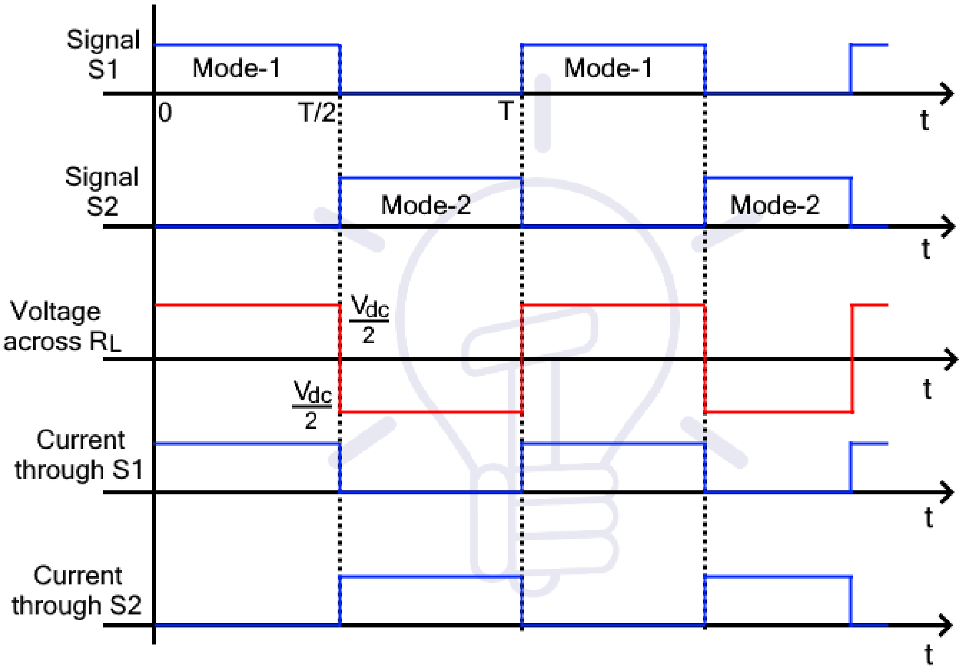
$\mathbf{I_o = V_{dc}/2R_L}$

The current flows in the reverse direction through the load as shown in the figure. Hence it shows that the half H-bridge has converted the applied DC into AC.



Mode-2 for R-Load

Waveform of Half H-Bridge with R Load



Waveform of Half H-Bridge with R Load

The first two waveforms show the pulses applied to the switches where each switch receives the pulse when the complementary switch is off. 3rd graph shows the voltage waveform across the load. This shows that the polarity of voltage changes w.r.t switching. The last two graphs show current through switches S_1 & S_2.

The root-mean-square (RMS) value of the output voltage can be calculated by

$$\sqrt{\left(\frac{1}{T}\int_0^T \frac{V_{dc}\,d\theta}{4}\right)} = \frac{V_{dc}}{2}$$

Fourier transform can be used to express the instantaneous voltage

$$V_o = a_0/2 + \sum_{n=1}^{\infty}(a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

As there is no DC offset so a_o is zero and due to quarter-wave symmetry, all the components in are a_n zero. So, the contribution of b_n will only remain & b_n is given as

$$b_n = \frac{1}{\pi}\left[\int_{\frac{-\pi}{2}}^{0} \frac{-V_{dc}}{2} d(\omega t) + \int_0^{\frac{\pi}{2}} \frac{V_{dc}}{2} d(\omega t)\right] = \frac{2V_{dc}}{n\pi}$$

By putting the value of b_n in Fourier series equation, we get

$$Vo = \sum_{n=1}^{\infty} \left(\frac{V_{dc}}{n\pi} \sin n\omega t \right)$$

$V_o = 0$ for n=2,4,6,8…

ω is the angular frequency of the output voltage. The even harmonics of the output voltage are not present due to quarter-wave symmetry. Hence the result is

$$V_o = \frac{V_{dc}}{\sqrt{2}\,\pi} = 0.45\ V_{dc}$$

From here, **the output voltage is approximately equal to half of the applied voltage.**

The current through the resistive load can be easily calculated out by just dividing the RMS voltage by its resistance.

$$I_L = \frac{1}{R}\left[\sum_{n=1}^{\infty} \left(\frac{V_{dc}}{n\pi} \sin n\omega t \right) \right]$$

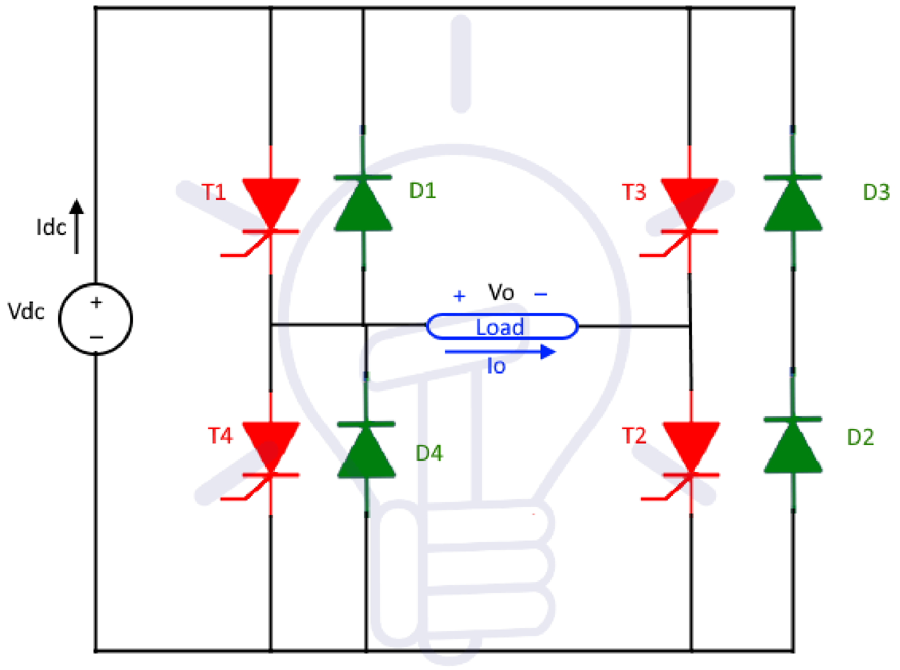
Full Bridge Inverter

Full bridge inverter is a topology of H-bridge inverter used for converting DC power into AC power. The components required for conversion are two times more than that used in single phase Half bridge inverters.

The **circuit of a full bridge inverter** consists of 4 diodes and 4 controlled switches as shown below.

These diodes are known as freewheeling diodes or feedback diodes because these diodes feedback the stored energy in the load back into the DC source. The feedback action happens only when load is other than pure resistive load.

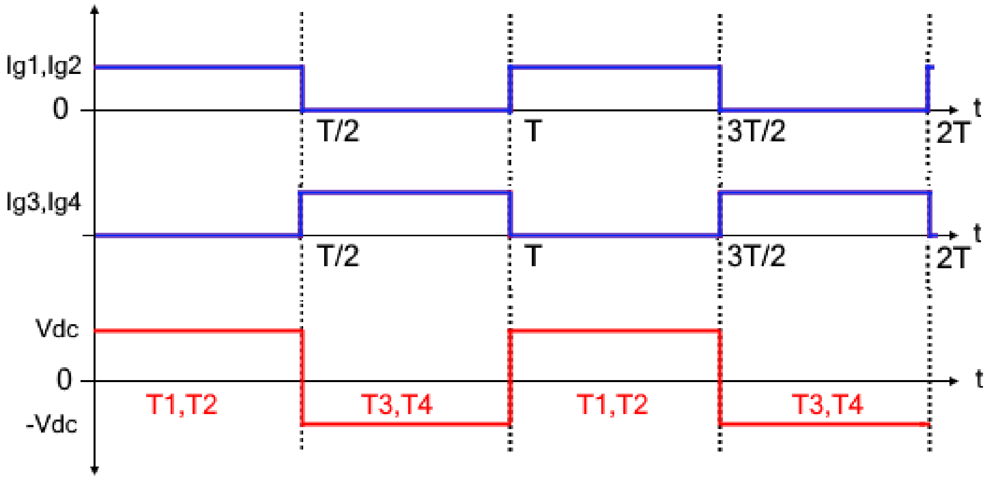
The controlled switches for Full-bridge inverters can be BJT, IGBT, MOSFET or thyristors. Controlled switches considered in this article are thyristors.



Full Bridge Inverter

The general concept of a full bridge inverter is to alternate the polarity of voltage across the load by operating two switches at a time. Positive input voltage will appear across the load by the operation of T_1 and T_2 for a half time period. The polarity of voltage across load will be changed for the other half period by operating T_3 and T_4.

Here thyristors t1,t2 t3 and t4 behave like switches.



Full Bridge Inverter Gate Signals

working operation and waveform of a single-phase full bridge inverter for R load,

Operation of Full Bridge with R Load

The working operation of Full bridge for pure resistive load is simplest as compared to all loads. As there is not any storage component in the load so, only control switches operate while feedback diodes do not operate through the operation of the inverter. Only two modes are enough for understanding the working operation of a full bridge inverter for R load.

Mode 1

Consider all the switches are initially off. By triggering T_1 and T_2, the input DC voltage (+Vdc) will appear across the load. The current flow in clockwise direction from source to the series connected load. The output current across the load will be

$I_o=V_{dc}/R_L$

Where R_L is the load resistance, While the output voltage across the load will be

V_o= Vdc

Mode 2

Thyristors T3 and T4 are triggered immediately after completely commutating T_1 and T_2. The polarity of voltage immediately reverses after switching complementary switches T_1 and T_2 with T_3 and T_4. The DC input voltage across the load appear with the negative voltage which

V_o= -Vdc

While the output appearing current is

$I_o = – V_{dc}/R_L$

The current in anti-lock wise direction flows from source to load through T_3 and T_4 as shown in the figure.

Waveform of Full Bridge with R Load

The current flowing through load and voltage appearing across the load are both in square wave form as shown in the third wave of the figure. The switching pattern is shown in the first two waves. Third wave shows the voltage across the load while the last two waves show the current flowing through the switches.

$$\frac{1}{T}\sqrt{\left(\int_0^{\frac{T}{2}} Vdc^2\,d\theta + \int_0^{\frac{T}{2}} Vdc^2\,d\theta\right)} = Vdc$$

The root-mean-square (RMS) value of the output voltage has been calculated from the equation as given

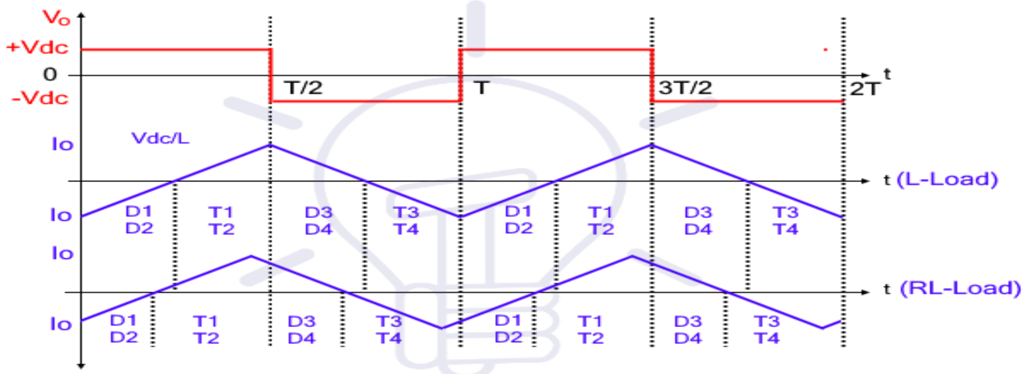
$$Vo = \sum_{n=1,3,5,7\ldots}^{\infty} \left(\frac{4Vdc}{n\pi}\sin nwt\right)$$

Fourier transform is used for expressing instantaneous voltage

As all the even harmonics are absent because the waveform of the output voltage is half-wave symmetric. So, the above equation just shows the odd harmonics. All the even harmonics are absent including average voltage across the load.

Waveform of Full Bridge with L and RL Load

The voltage waveform in both L and RL load is square wave while the current wave in both loads are triangular.



Full Bridge With L & RL load

Advantages of Single Phase Full Bridge Inverter

- Absence of voltage fluctuation in the circuit
- Suitable for high input voltage
- Energy efficient
- The current rating of the power devices is equal to the load current.

Disadvantages of Single Phase Full Bridge Inverter

- The efficiency of the full-bridge inverter (95%) is less than half the bridge inverter (99%).
- Losses are high
- High noise.

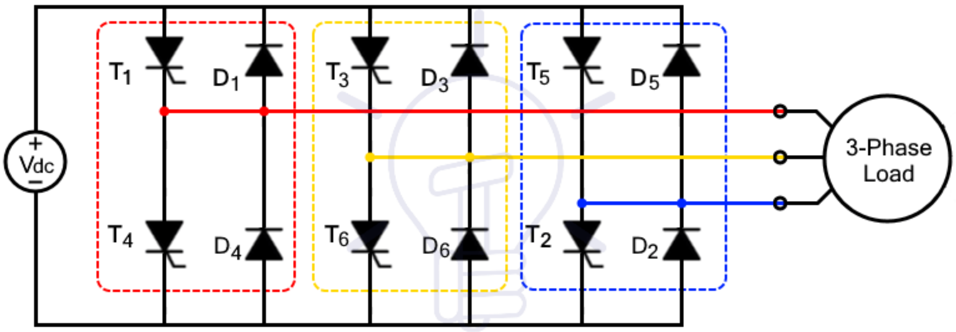
Applications of Single Phase Full Bridge Inverter

- Applicable in applications like low and medium power example square wave / quasi square wave voltage
- A sinusoidal wave which is distorted is used as input in high power applications
- Using high-speed power semiconductor devices, the harmonic contents at the output can be reduced by PWM (Pulse Width Modulation) techniques
- Other applications like AC variable motor, heating induction device, standby power supply.
- Solar Inverters
- Compressors, etc

Three Phase Bridge Inverters:

Industrial and other heavy loads require three-phase power. To run these heavy loads from storage devices or other DC sources, three-phase inverters are required. Three-phase bridge inverters can be used for this purpose.

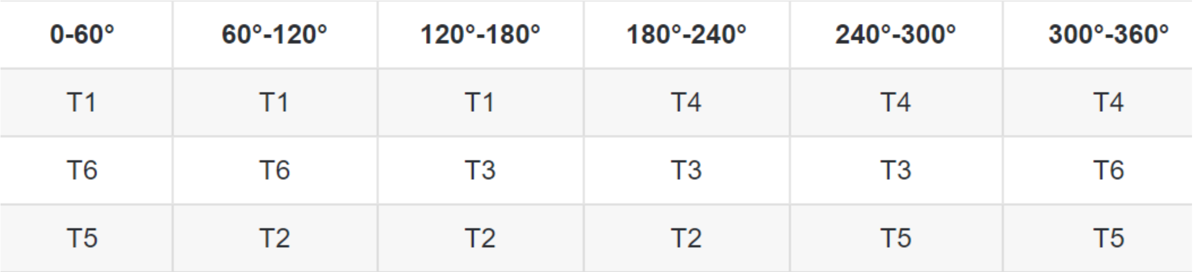
Three-phase bridge inverter is another type of bridge inverter that consists of 6 controlled switches and 6 diodes as shown in the figure. This bridge can be operated in two different modes based on the degree of gate pulses. These modes are known as **180-degree mode** and **120-degree mode** which are defined below.



Three Phase H-Bridge Inverter

180-degree Mode:

In this mode of operation, three thyristors will be in the conduction mode where each out of three thyristors will provide single phases correspondingly. In each leg, one complementary thyristor will be turned ON for half of the time. In other words, one thyristor will be turned ON for half time while other will be closed. In degrees, half time can be represented as 180 degrees. There will be a 120-degree shift between each leg.

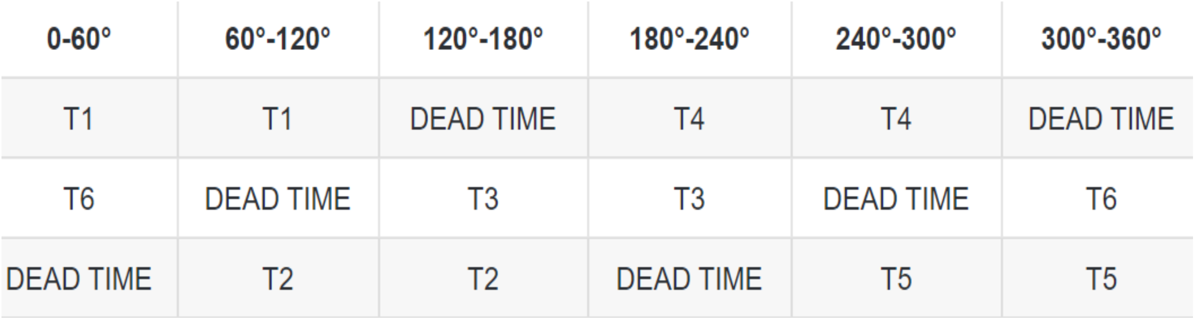


0-60°	60°-120°	120°-180°	180°-240°	240°-300°	300°-360°
T1	T1	T1	T4	T4	T4
T6	T6	T3	T3	T3	T6
T5	T2	T2	T2	T5	T5

The time gaps between the commutation of the complementary thyristor in one leg is zero. This can cause short circuit. To avoid the problem of short-circuiting, 120-degree mode of operation is preferred.

120-degree Mode:

In this mode of operation, only two thyristors out of six will operate at a time where each switch in each leg will conduct for 120^o. There is a time delay of 60^o between the operations of two thyristors in each leg which prevents a short circuit.



0-60°	60°-120°	120°-180°	180°-240°	240°-300°	300°-360°
T1	T1	DEAD TIME	T4	T4	DEAD TIME
T6	DEAD TIME	T3	T3	DEAD TIME	T6
DEAD TIME	T2	T2	DEAD TIME	T5	T5

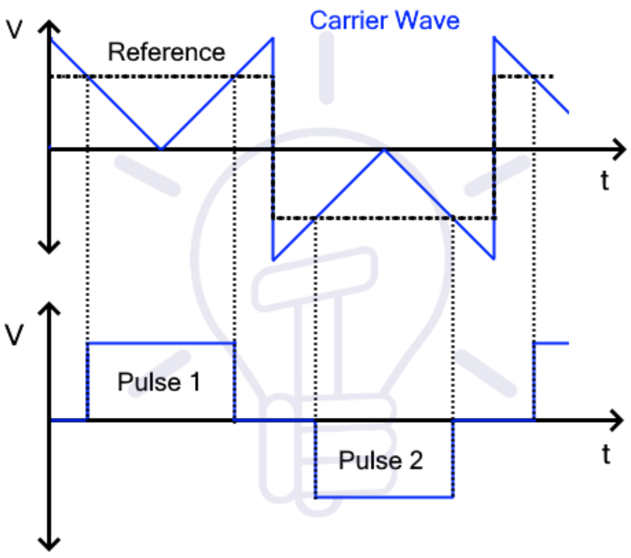
PWM INVERTERS

PWM are used for the internal control of inverter as well as to modify the shapes of output voltage as near to sine wave as possible. Some other reasons using PWM techniques are

- To get rid of lower harmonics in the output voltage
- Minimize the requirement of the filter because low harmonics will be already eliminated using PWM while higher harmonics can be removed easily.
- Easy control of output voltage using various PWM techniques.

Single Pulse Width Modulation (Single PWM)

The gating signal for the switch in single pulse width modulation is generated by comparing a reference pulse with triangular carrier wave. The comparison will produce a single pulse per half cycle of the output wave hence the name Single pulse width modulation. In other words, two pulses are provided for reference where each pulse provide half cycle of the output voltage.



Single Pulse Width Modulation (PWM)

Advantages:

- **Cheaper:** these inverters are relatively cheaper
- **Work for ordinary load:** SPWM inverters work for ordinary loads for example light, bulbs, and fans.

Disadvantages:

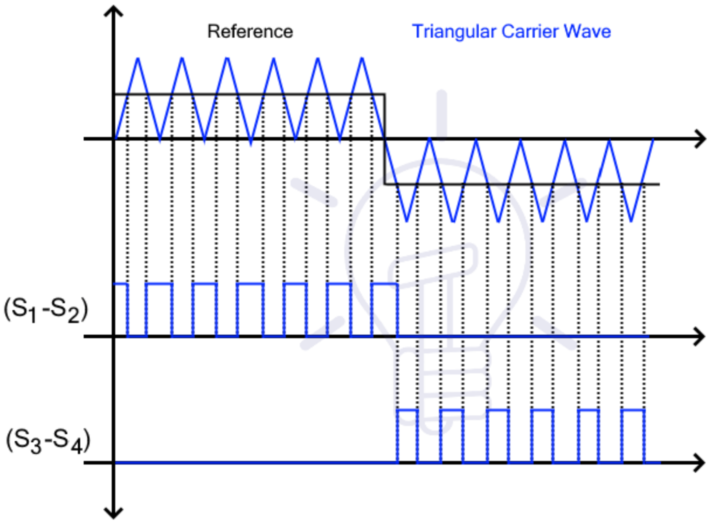
- The main problem in these inverters is that they introduce harmonics in the output.

Multiple Pulse Width Modulation (Multiple PWM)

The limitations of SPWM inverters are overcame by MPWM where multiple reference pulses are compared with high frequency triangular wave for each half-cycle of the output voltage. The number of pulses required for each half can be derived from the equation.

Number of pulses required = $f_c / (2f_o)$

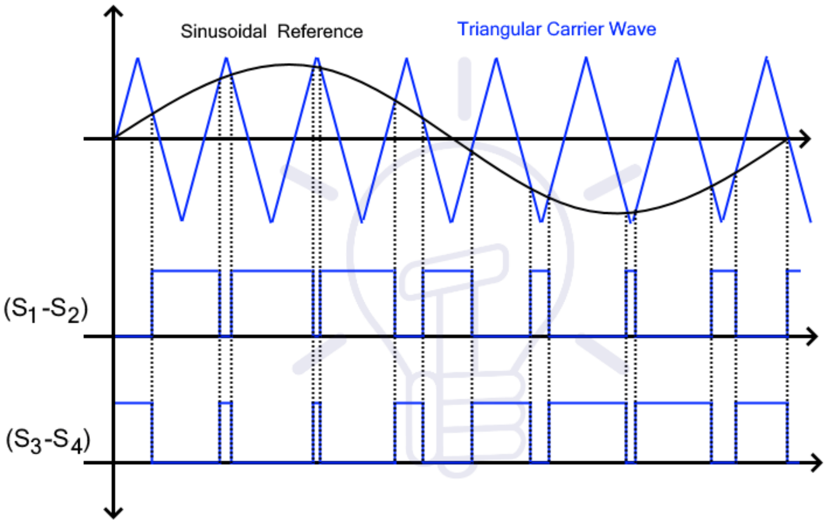
Where f_o is the frequency of the output signal while fc is the carrier frequency.



Multiple Pulse Width Modulation (MPWM)

Sinusoidal Pulse Width Modulation (SPWM)

In this technique, the width of the pulse will wave according to the amplitude of reference sinusoidal wave. This reference signal will decide the output frequency of the voltage wave while modulation index will decide the RMS value of sinusoidal output voltage. The gate pulses generated for the switches are by comparing triangular carrier wave with reference sinusoidal wave. The reference signal used in this technique is a sinusoidal wave so known as Sinusoidal pulse width modulation.



Sinusoidal Pulse Width Modulaion (SPWM)

Several pulses are used for each half cycle of the output voltage but instead of same pulse widths, the width of the pulses increase proportionally to the sine wave. The width of the pulses will increase in the sinusoidal manner. Just like a sinusoidal wave alternates after specific period of time, the resulting pulses will too, as shown in the figure.

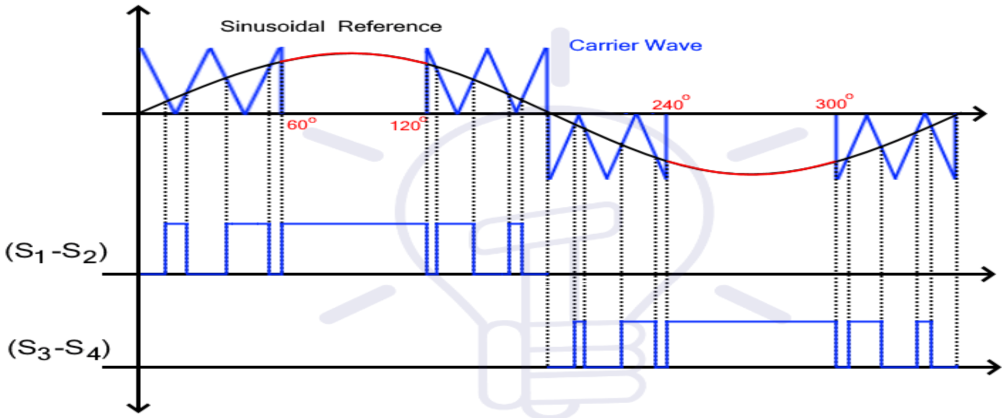
Advantages:

The obtained output voltage is near to sine wave which is required.

There is low harmonic content in the output voltage.

Modified Sinusoidal Pulse Width Modulation (MSPWM)

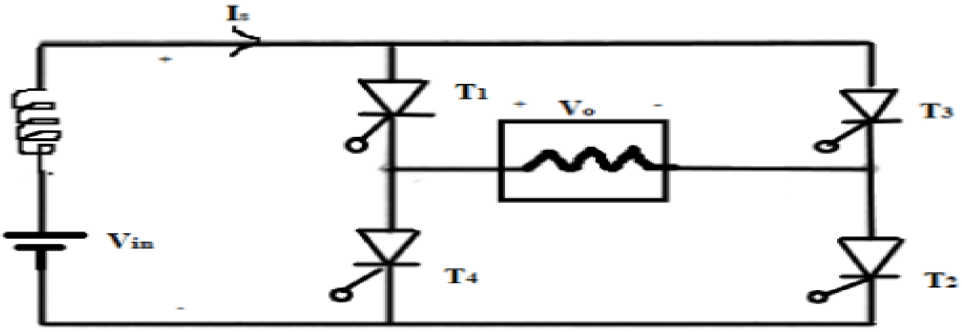
In MSPWM technique the first and last 60 degree of each half wave is used for modulation. This PWM technique will provide a much smoother sine wave as compared to previously discussed techniques.



Modified Sinusoidal Pulse Width Modulation (MSPWM)

CURRENT SOURCE INVERTER WITH R-LOAD

The circuit diagram of the current source inverter with R-load is shown in the below figure.



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Fig,Current Source Inverter with R-Load

The circuit consists of four thyristor switches (T_1, T_2, T_3, T_4), I_S is the input source current which is constant, and you can see that there is no any anti-parallel diode is connected.

The constant current is provided by connecting voltage sources in series with large inductance.

We know that the property of inductance, that it won't allow the sudden change in current, so when we connect voltage source with large inductance then definitely the current produced across it will be constant.

The fundamental dissipation factor of the current source inverter with resistive load is equal to one.

Parameters of the Current Source Inverter with R-Load

If we trigger T_1 and T_2 from 0 to T/2 then the output current and the output voltage is expressed as

$I_0=I_S>0$

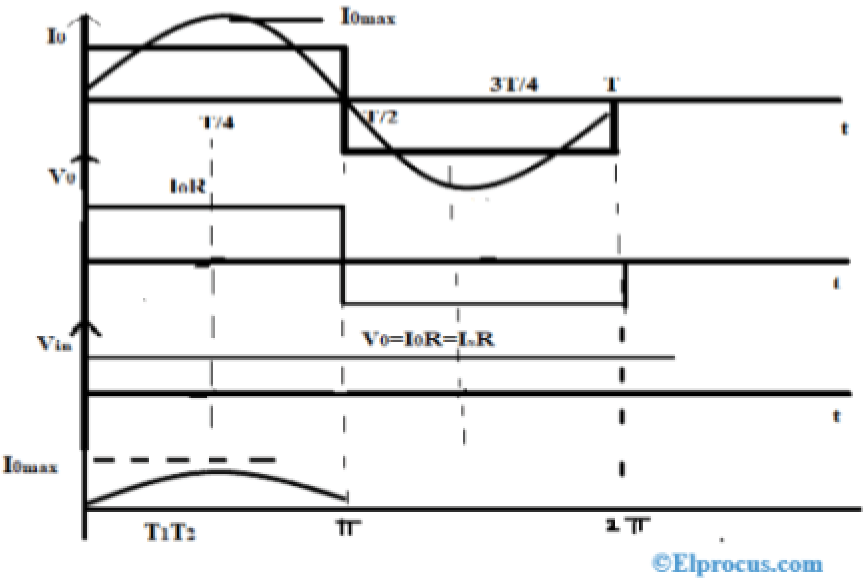
$V_0=I_0R$

If we trigger T_3 and T_4 from T/2 to T then the output current and the output voltage is expressed as

$I_0=-I_S>0$

$V_0=I_0R<0$

The output waveform of the current source inverter with R-load is shown in the below figure



Output Waveform of Current Source Inverter with R-Load

In the case of resistive load, forced commutation is required. From 0 to T/2, T_1 and T_2 are conducting and from T/2 to T, T_3 & T_4 are conducting. So, the conduction angle of each switch will be equal to π and the conduction time of each switch will be equal to T/2.

The input voltage of the resistive load is expressed as

$V_{in}=V_0$ (from 0 to T/2)

$V_{in}=-V_0$ (from T/2 to T)

The RMS output current and the RMS output voltage of the CSI resistive load is expressed as

$I_{0(RMS)}=I_S$

$V_{0(RMS)}= I_{0(RMS)}R$

The average and RMS thyristor current of the CSI with resistive load is

$I_{T(avg)}=I_S/2$

$I_{T(RMS)}=I_S/\sqrt{2}$

The Fourier series of output current and the output voltage of the CSI with resistive load is

$$I_0(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_S}{n\pi} sin n\omega_0 t$$

$$V_0(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_S R}{n\pi} sin n\omega_0 t$$

The fundamental component of the RMS output current is

$I_{01(RMS)}=2\sqrt{2}/\pi * I_S$

The distortion factor of the current source inverter with R-load is

$g=2\sqrt{2}/\pi$

The total harmonic distortion is expressed as

THD=48.43%

The fundamental component of average and RMS thyristor current is

$I_{T01(avg)} = I_{01(max)} / \pi$

$I_{T01(RMS)} = I_{01(max)} / 2$

The fundamental power across the load is expressed as

$V_{01(RMS)} * I_{01(RMS)} * \cos\phi_1$

The total power across the load is expressed as

$I_{0(RMS)}^2 R = V_{0(RMS)}^2 / R$

The input voltage V_{in} is always positive because the power is always delivered from source to load.

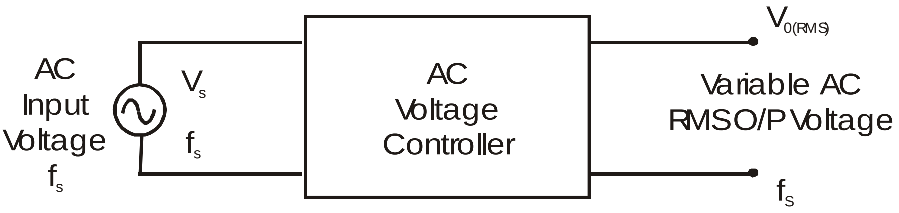
MODULE 5

AC VOLTAGE CONTROLLER CIRCUITS

(RMS VOLTAGE CONTROLLERS)

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors in the ac voltage controller circuits.

In brief, an ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle 'α'



There are two different types of thyristor control used in practice to control the ac power flow

- On-Off control
- Phase control

These are the two ac output voltage control techniques.

In On-Off control technique Thyristors are used as switches to connect the load circuit to the ac supply (source) for a few cycles of the input ac supply and then to disconnect it for few input cycles. The Thyristors thus act as a high speed contactor (or high speed ac switch).

PHASE CONTROL

In phase control the Thyristors are used as switches to connect the load circuit to the input ac supply, for a part of every input cycle. That is the ac supply voltage is chopped using Thyristors during a part of each input cycle.

The thyristor switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.

By controlling the phase angle or the trigger angle 'α' (delay angle), the output RMS voltage across the load can be controlled.

The trigger delay angle 'α' is defined as the phase angle (the value of ωt) at which the thyristor turns on and the load current begins to flow.

Thyristor ac voltage controllers use ac line commutation or ac phase commutation. Thyristors in ac voltage controllers are line commutated (phase commutated) since the input supply is ac. When the input ac voltage reverses and becomes negative during the negative half cycle the current flowing through the conducting thyristor decreases and falls to zero. Thus the ON thyristor naturally turns off, when the device current falls to zero.

Phase control Thyristors which are relatively inexpensive, converter grade Thyristors which are slower than fast switching inverter grade Thyristors are normally used.

For applications upto 400Hz, if Triacs are available to meet the voltage and current ratings of a particular application, Triacs are more commonly used.

Due to ac line commutation or natural commutation, there is no need of extra commutation circuitry or components and the circuits for ac voltage controllers are very simple.

Due to the nature of the output waveforms, the analysis, derivations of expressions for performance parameters are not simple, especially for the phase controlled ac voltage controllers with RL load. But however most of the practical loads are of the RL type and hence RL load should be considered in the analysis and design of ac voltage controller circuits.

TYPE OF AC VOLTAGE CONTROLLERS

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

- Single Phase AC Controllers.
- Three Phase AC Controllers.

Single phase ac controllers operate with single phase ac supply voltage of 230V RMS at 50Hz in our country. Three phase ac controllers operate with 3 phase ac supply of 400V RMS at 50Hz supply frequency.

Each type of controller may be sub divided into

- Uni-directional or half wave ac controller.
- Bi-directional or full wave ac controller.

In brief different types of ac voltage controllers are

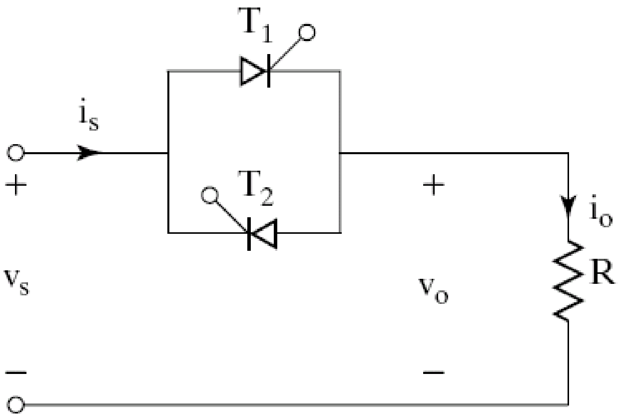
- Single phase half wave ac voltage controller (uni-directional controller).
- Single phase full wave ac voltage controller (bi-directional controller).
- Three phase half wave ac voltage controller (uni-directional controller).
- Three phase full wave ac voltage controller (bi-directional controller).

APPLICATIONS OF AC VOLTAGE CONTROLLERS

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors (single phase and poly phase ac induction motor control).
- AC magnet controls.

PRINCIPLE OF ON-OFF CONTROL TECHNIQUE (INTEGRAL CYCLE CONTROL)

The basic principle of on-off control technique is explained with reference to a single phase full wave ac voltage controller circuit shown below. The thyristor switches T_1 and T_2 are turned on by applying appropriate gate trigger pulses to connect the input ac supply to the load for 'n' number of input cycles during the time interval t_{ON}. The thyristor switches T_1 and T_2 are turned off by blocking the gate trigger pulses for ‘m’ number of input cycles during the time interval t_{OFF}. The ac controller ON time t_{ON} usually consists of an integral number of input cycles.



$R = R_L$ **= Load Resistance**

Fig.: Single phase full wave AC voltage controller circuit

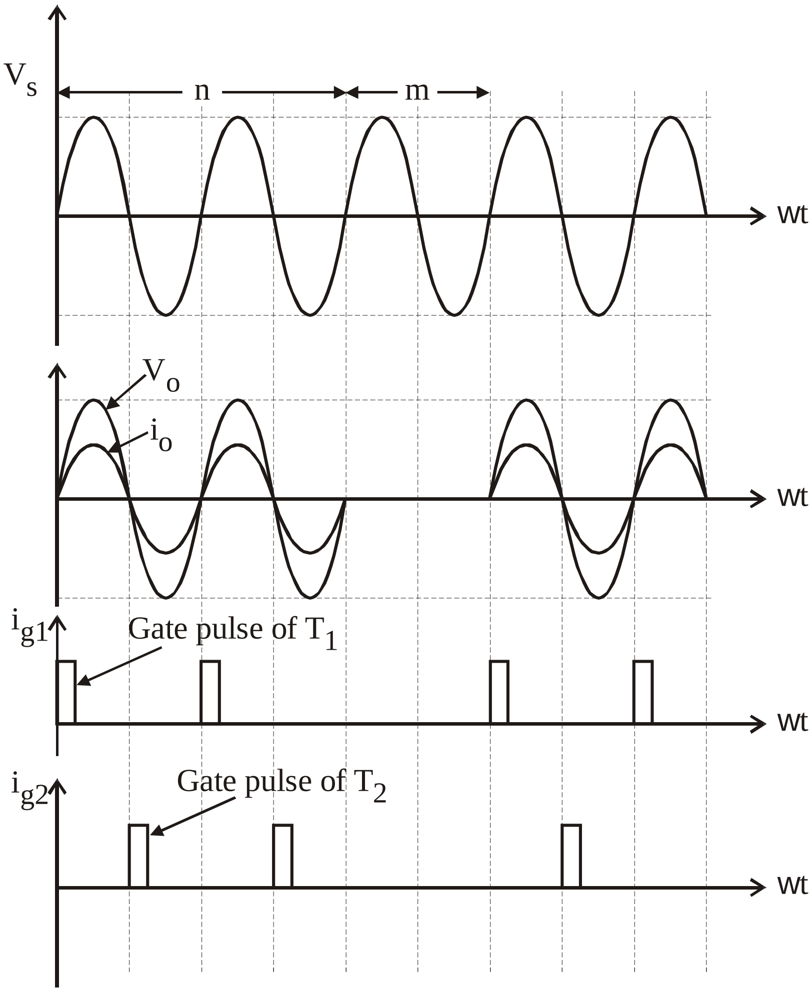


Fig.: Waveforms

Example

Referring to the waveforms of ON-OFF control technique in the above diagram,

$n =$ Two input cycles. Thyristors are turned ON during t_{ON} for two input cycles.

$m =$ One input cycle. Thyristors are turned OFF during t_{OFF} for one input cycle

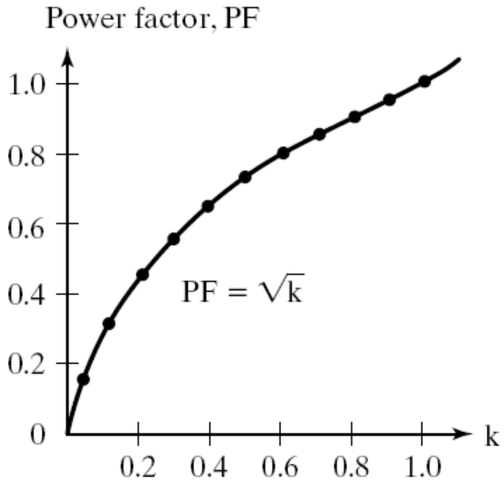


Fig.: Power Factor

Thyristors are turned ON precisely at the zero voltage crossings of the input supply. The thyristor T_1 is turned on at the beginning of each positive half cycle by applying the gate trigger pulses to T_1 as shown, during the ON time t_{ON} . The load current flows in the positive direction, which is the downward direction as shown in the circuit diagram when T_1 conducts. The thyristor T_2 is turned on at the beginning of each negative half cycle, by applying gating signal to the gate of T_2, during t_{ON}. The load current flows in the reverse direction, which is the upward direction when T_2 conducts. Thus we obtain a bi-directional load current flow (alternating load current flow) in a ac voltage controller circuit, by triggering the thyristors alternately.

This type of control is used in applications which have high mechanical inertia and high thermal time constant (Industrial heating and speed control of ac motors). Due to zero voltage and zero current switching of Thyristors, the harmonics generated by switching actions are reduced.

For a sine wave input supply voltage,

$$v_s = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t$$

$V_S =$ RMS value of input ac supply = $\dfrac{V_m}{\sqrt{2}}$ = RMS phase supply voltage.

If the input ac supply is connected to load for 'n' number of input cycles and disconnected for 'm' number of input cycles, then

$$t_{ON} = n \times T, \qquad t_{OFF} = m \times T$$

Where $T = \dfrac{1}{f}$ = input cycle time (time period) and

f = input supply frequency.

t_{ON} = controller on time = $n \times T$.

t_{OFF} = controller off time = $m \times T$.

T_O = Output time period = $\left(t_{ON} + t_{OFF}\right) = \left(nT + mT\right)$.

We can show that,

Output RMS voltage $V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$

Where $V_{i(RMS)}$ is the RMS input supply voltage = V_S.

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE, FOR ON-OFF CONTROL METHOD.

Output RMS voltage $V_{O(RMS)} = \sqrt{\frac{1}{\omega T_O}\int\limits_{\omega t=0}^{\omega t_{ON}} V_m{}^2 Sin^2\omega t.d\left(\omega t\right)}$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{\omega T_O}\int\limits_0^{\omega t_{ON}} Sin^2\omega t.d\left(\omega t\right)}$$

Substituting for $Sin^2\theta = \frac{1-Cos2\theta}{2}$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{\omega T_O}\int\limits_0^{\omega t_{ON}}\left[\frac{1-Cos2\omega t}{2}\right]d\left(\omega t\right)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\omega T_O}\left[\int\limits_0^{\omega t_{ON}} d\left(\omega t\right) - \int\limits_0^{\omega t_{ON}} Cos2\omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\omega T_O}\left[\left(\omega t\right)\Big/_0^{\omega t_{ON}} - \frac{Sin2\omega t}{2}\Big/_0^{\omega t_{ON}}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\omega T_O}\left[\left(\omega t_{ON} - 0\right) - \frac{\sin 2\omega t_{ON} - \sin 0}{2}\right]}$$

Now t_{ON} = An integral number of input cycles; Hence

$t_{ON} = T, 2T, 3T, 4T, 5T, ...$ & $\omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, ...$

Where T is the input supply time period (T = input cycle time period). Thus we note that $\sin 2\omega t_{ON} = 0$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2\cancel{\omega}t_{ON}}{2\cancel{\omega}T_O}} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{t_{ON}}{T_O}}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$$

Where $V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_S =$ RMS value of input supply voltage;

$$\frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON}+t_{OFF}} = \frac{nT}{nT+mT} = \frac{n}{(n+m)} = k$$ = duty cycle (d).

$$V_{O(RMS)} = V_S\sqrt{\frac{n}{(m+n)}} = V_S\sqrt{k}$$

PERFORMANCE PARAMETERS OF AC VOLTAGE CONTROLLERS

- **RMS Output (Load) Voltage**

$$V_{O(RMS)} = \left[\frac{n}{2\pi(n+m)}\int_0^{2\pi} V_m{}^2 \sin^2 \omega t . d(\omega t)\right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{n}{(m+n)}} = V_{i(RMS)}\sqrt{k} = V_S\sqrt{k}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{k} = V_S\sqrt{k}$$

Where $V_S = V_{i(RMS)}$ = RMS value of input supply voltage.

- **Duty Cycle**

$$k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{(t_{ON}+t_{OFF})} = \frac{nT}{(m+n)T}$$

Where, $k = \frac{n}{(m+n)}$ = duty cycle (d).

- **RMS Load Current**

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L};$$ for a resistive load $Z = R_L$.

- **Output AC (Load) Power**

$$P_O = I^2_{O(RMS)} \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V_S I_S}$$

$$PF = \frac{I_{O(RMS)}^2 \times R_L}{V_{i(RMS)} \times I_{in(RMS)}}; \qquad I_S = I_{in(RMS)} = \text{RMS input supply current.}$$

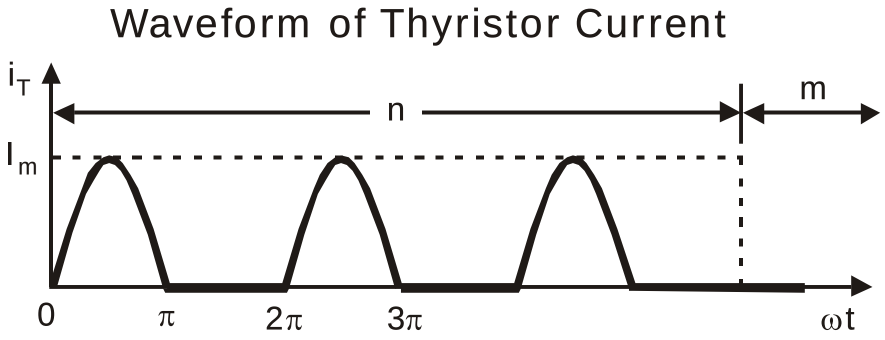
The input supply current is same as the load current $I_{in} = I_O = I_L$

Hence, RMS supply current = RMS load current; $I_{in(RMS)} = I_{O(RMS)}$.

$$PF = \frac{I_{O(RMS)}^2 \times R_L}{V_{i(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{i(RMS)}} = \frac{V_{i(RMS)}\sqrt{k}}{V_{i(RMS)}} = \sqrt{k}$$

$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

- **The Average Current of Thyristor** $I_{T(Avg)}$



$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int_0^{\pi} I_m \sin \omega t . d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \int_0^{\pi} \sin \omega t . d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-\cos \omega t \Big/_0^{\pi} \right]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-\cos \pi + \cos 0 \right]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi\left(m+n\right)}\left[-\left(-1\right)+1\right]$$

$$I_{T(Avg)} = \frac{n}{2\pi\left(m+n\right)}\left[2I_m\right]$$

$$I_{T(Avg)} = \frac{I_m n}{\pi\left(m+n\right)} = \frac{k.I_m}{\pi}$$

$$k = \text{duty cycle} = \frac{t_{ON}}{\left(t_{ON}+t_{OFF}\right)} = \frac{n}{\left(n+m\right)}$$

$$I_{T(Avg)} = \frac{I_m n}{\pi\left(m+n\right)} = \frac{k.I_m}{\pi},$$

Where $I_m = \frac{V_m}{R_L}$ = maximum or peak thyristor current.

- **RMS Current of Thyristor** $I_{T(RMS)}$

$$I_{T(RMS)} = \left[\frac{n}{2\pi\left(n+m\right)}\int_0^{\pi} I_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi\left(n+m\right)}\int_0^{\pi} \sin^2 \omega t.d\left(\omega t\right)\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi\left(n+m\right)}\int_0^{\pi} \frac{\left(1-\cos 2\omega t\right)}{2} d\left(\omega t\right)\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi\left(n+m\right)}\left\{\int_0^{\pi} d\left(\omega t\right) - \int_0^{\pi} \cos 2\omega t.d\left(\omega t\right)\right\}\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi\left(n+m\right)}\left\{\left(\omega t\right)\Big/_0^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_0^{\pi}\right\}\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi\left(n+m\right)}\left\{\left(\pi-0\right) - \left(\frac{\sin 2\pi - \sin 0}{2}\right)\right\}\right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)} \{\pi - 0 - 0\} \right]^{1/2}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2 \pi}{4\pi(n+m)} \right]^{1/2} = \left[\frac{nI_m^2}{4(n+m)} \right]^{1/2}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2} \sqrt{k}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{k}$$

PROBLEM

1. *A single phase full wave ac voltage controller working on ON-OFF control technique has supply voltage of 230V, RMS 50Hz, load = 50Ω. The controller is ON for 30 cycles and off for 40 cycles. Calculate*
    - *ON & OFF time intervals.*
    - *RMS output voltage.*
    - *Input P.F.*
    - *Average and RMS thyristor currents.*

$V_{in(RMS)} = 230V$, $\quad V_m = \sqrt{2} \times 230V = 325.269\,\text{V}$, $\quad V_m = 325.269V$,

$T = \frac{1}{f} = \frac{1}{50Hz} = 0.02\,\text{sec}$, $\quad T = 20ms$.

n = number of input cycles during which controller is ON; $n = 30$.

m = number of input cycles during which controller is OFF; $m = 40$.

$$t_{ON} = n \times T = 30 \times 20ms = 600ms = 0.6\,\text{sec}$$

$t_{ON} = n \times T = 0.6\,\text{sec}$ = controller ON time.

$$t_{OFF} = m \times T = 40 \times 20ms = 800ms = 0.8\,\text{sec}$$

$t_{OFF} = m \times T = 0.8\,\text{sec}$ = controller OFF time.

Duty cycle $k = \frac{n}{(m+n)} = \frac{30}{(40+30)} = 0.4285$

RMS output voltage

$$V_{O(RMS)} = V_{i(RMS)} \times \sqrt{\frac{n}{(m+n)}}$$

$$V_{O(RMS)} = 230V \times \sqrt{\frac{30}{(30+40)}} = 230\sqrt{\frac{3}{7}}$$

$$V_{O(RMS)} = 230V\sqrt{0.42857} = 230 \times 0.65465$$

$$V_{O(RMS)} = 150.570V$$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L} = \frac{150.570V}{50\Omega} = 3.0114A$$

$$P_O = I_{O(RMS)}^2 \times R_L = 3.0114^2 \times 50 = 453.426498W$$

Input Power Factor $P.F = \sqrt{k}$

$$PF = \sqrt{\frac{n}{(m+n)}} = \sqrt{\frac{30}{70}} = \sqrt{0.4285}$$

$$PF = 0.654653$$

Average Thyristor Current Rating

$$I_{T(Avg)} = \frac{I_m}{\pi} \times \left(\frac{n}{m+n}\right) = \frac{k \times I_m}{\pi}$$

where $$I_m = \frac{V_m}{R_L} = \frac{\sqrt{2} \times 230}{50} = \frac{325.269}{50}$$

$I_m = 6.505382A$ = Peak (maximum) thyristor current.

$$I_{T(Avg)} = \frac{6.505382}{\pi} \times \left(\frac{3}{7}\right)$$

$$I_{T(Avg)} = 0.88745A$$

RMS Current Rating of Thyristor

$$I_{T(RMS)} = \frac{I_m}{2}\sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2}\sqrt{k} = \frac{6.505382}{2} \times \sqrt{\frac{3}{7}}$$

$$I_{T(RMS)} = 2.129386A$$

PRINCIPLE OF AC PHASE CONTROL

The basic principle of ac phase control technique is explained with reference to a single phase half wave ac voltage controller (unidirectional controller) circuit shown in the below figure.

The half wave ac controller uses one thyristor and one diode connected in parallel across each other in opposite direction that is anode of thyristor T_1 is connected to the cathode of diode D_1 and the cathode of T_1 is connected to the anode of D_1. The output voltage across the load resistor 'R' and hence the ac power flow to the load is controlled by varying the trigger angle 'α'.

The trigger angle or the delay angle 'α' refers to the value of ωt or the instant at which the thyristor T_1 is triggered to turn it ON, by applying a suitable gate trigger pulse between the gate and cathode lead.

The thyristor T_1 is forward biased during the positive half cycle of input ac supply. It can be triggered and made to conduct by applying a suitable gate trigger pulse only during the positive half cycle of input supply. When T_1 is triggered it conducts and the load current flows through the thyristor T_1, the load and through the transformer secondary winding.

By assuming T_1 as an ideal thyristor switch it can be considered as a closed switch when it is ON during the period $\omega t = \alpha$ to π radians. The output voltage across the load follows the input supply voltage when the thyristor T_1 is turned-on and when it conducts from $\omega t = \alpha$ to π radians. When the input supply voltage decreases to zero at $\omega t = \pi$, for a resistive load the load current also falls to zero at $\omega t = \pi$ and hence the thyristor T_1 turns off at $\omega t = \pi$. Between the time period $\omega t = \pi$ to 2π, when the supply voltage reverses and becomes negative the diode D_1 becomes forward biased and hence turns ON and conducts. The load current flows in the opposite direction during $\omega t = \pi$ to 2π radians when D_1 is ON and the output voltage follows the negative half cycle of input supply.

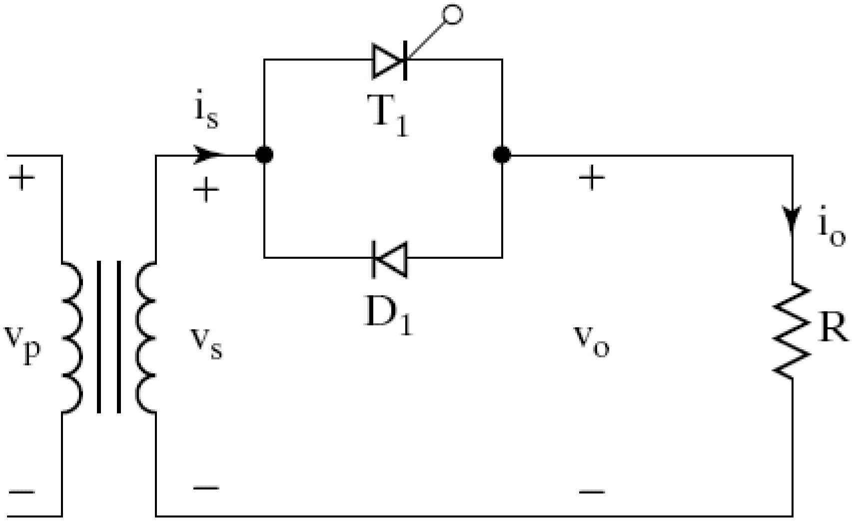
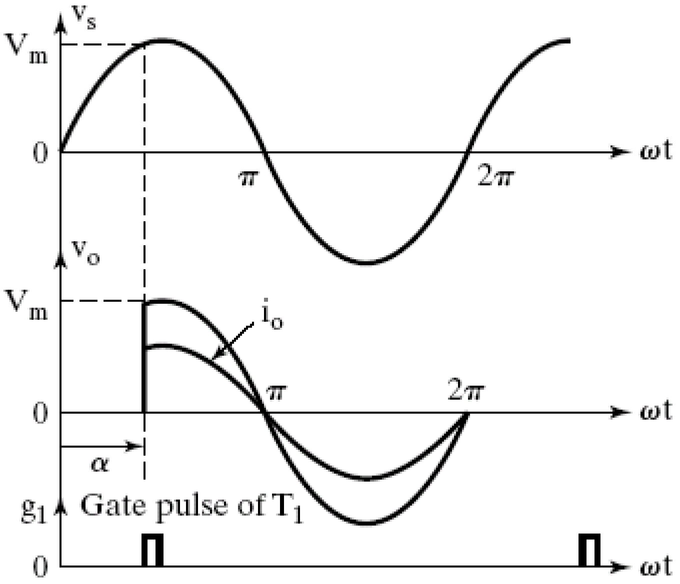


Fig.: Halfwave AC phase controller (Unidirectional Controller)



Equations

Input AC Supply Voltage across the Transformer Secondary Winding.

$$v_s = V_m \sin \omega t$$

$V_S = V_{in(RMS)} = \frac{V_m}{\sqrt{2}}$ = RMS value of secondary supply voltage.

Output Load Voltage

$v_o = v_L = 0$; for $\omega t = 0$ to α

$v_o = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to 2π .

Output Load Current

$i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L}$; for $\omega t = \alpha$ to 2π .

$i_o = i_L = 0$; for $\omega t = 0$ to α .

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m{}^2 \sin^2 \omega t . d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{2\pi}\left[\int_{\alpha}^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) . d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m{}^2}{4\pi}\left[\int_{\alpha}^{2\pi}\left(1-\cos 2\omega t\right).d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d\left(\omega t\right) - \int_{\alpha}^{2\pi}\cos 2\omega t.d\omega t\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\left(\omega t\right)\Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left\{\frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2}\right\}} \qquad ;\sin 4\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

Where, $V_{i(RMS)} = V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage (across the transformer secondary winding).

Note: Output RMS voltage across the load is controlled by changing $'\alpha'$ as indicated by the expression for $V_{O(RMS)}$

PLOT OF $V_{O(RMS)}$ VERSUS TRIGGER ANGLE α FOR A SINGLE PHASE HALF-WAVE AC VOLTAGE CONTROLLER (UNIDIRECTIONAL CONTROLLER)

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right)+\frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right)+\frac{\sin 2\alpha}{2}\right]}$$

By using the expression for $V_{O(RMS)}$ we can obtain the control characteristics, which is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α . A typical control characteristic of single phase half-wave phase controlled ac voltage controller is as shown below

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$
0	0	$V_S = \frac{V_m}{\sqrt{2}}$
30^0	$\pi/6$; $\left(1\pi/6\right)$	0.992765 V_S
60^0	$\pi/3$; $\left(2\pi/6\right)$	0.949868 V_S
90^0	$\pi/2$; $\left(3\pi/6\right)$	0.866025 V_S
120^0	$2\pi/3$; $\left(4\pi/6\right)$	0.77314 V_S
150^0	$5\pi/6$; $\left(5\pi/6\right)$	0.717228 V_S
180^0	π ; $\left(6\pi/6\right)$	0.707106 V_S

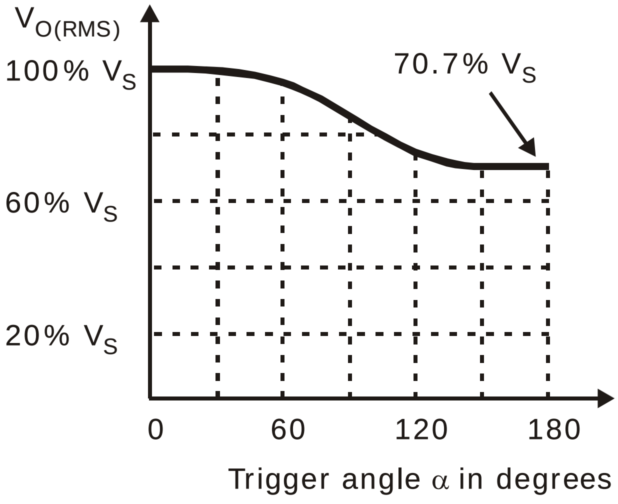


Fig.: Control characteristics of single phase half-wave phase controlled ac voltage ontroller

Note: We can observe from the control characteristics and the table given above that the

range of RMS output voltage control is from 100% of V_S to 70.7% of V_S when we vary the trigger angle α from zero to 180 degrees. Thus the half wave ac controller has the draw back of limited range RMS output voltage control.

TO CALCULATE THE AVERAGE VALUE (DC VALUE) OF OUTPUT VOLTAGE

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m \sin \omega t.d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_{\alpha}^{2\pi} \sin \omega t.d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos \omega t \Big/_{\alpha}^{2\pi} \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos 2\pi + \cos \alpha \right] \qquad ; \ \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi} \left[\cos \alpha - 1 \right] \quad ; \ V_m = \sqrt{2} V_S$$

Hence $V_{dc} = \dfrac{\sqrt{2} V_S}{2\pi} \left(\cos \alpha - 1 \right)$

When $'\alpha'$ is varied from 0 to π. V_{dc} varies from 0 to $\dfrac{-V_m}{\pi}$

DISADVANTAGES OF SINGLE PHASE HALF WAVE AC VOLTAGE CONTROLLER.

- The output load voltage has a DC component because the two halves of the output voltage waveform are not symmetrical with respect to ‘0’ level. The input supply current waveform also has a DC component (average value) which can result in the problem of core saturation of the input supply transformer.
- The half wave ac voltage controller using a single thyristor and a single diode provides control on the thyristor only in one half cycle of the input supply. Hence ac power flow to the load can be controlled only in one half cycle.
- Half wave ac voltage controller gives limited range of RMS output voltage control. Because the RMS value of ac output voltage can be varied from a maximum of 100% of V_S at a trigger angle $\alpha = 0$ to a low of 70.7% of V_S at $\alpha = \pi$ Radians.

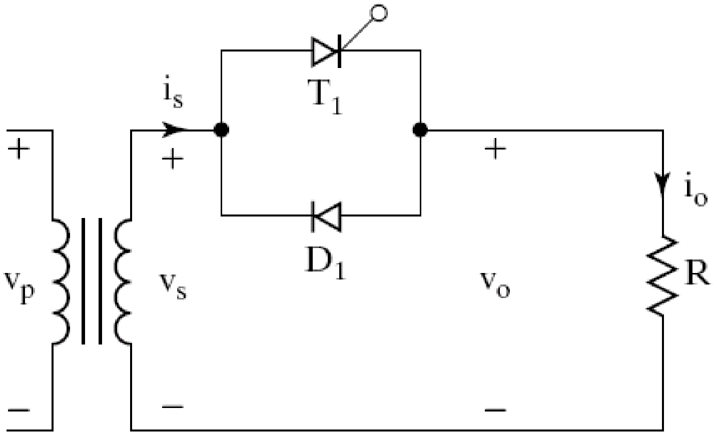
These drawbacks of single phase half wave ac voltage controller can be over come by using a single phase full wave ac voltage controller.

APPLICATIONS OF RMS VOLTAGE CONTROLLER

- Speed control of induction motor (polyphase ac induction motor).
- Heater control circuits (industrial heating).
- Welding power control.
- Induction heating.
- On load transformer tap changing.
- Lighting control in ac circuits.
- Ac magnet controls.

Problem

1. *A single phase half-wave ac voltage controller has a load resistance* $R = 50\Omega$*, input ac supply voltage is 230V RMS at 50Hz. The input supply transformer has a turns ratio of 1:1. If the thyristor* T_1 *is triggered at* $\alpha = 60^0$*. Calculate*
    - *RMS output voltage.*
    - *Output power.*
    - *RMS load current and average load current.*
    - *Input power factor.*
    - *Average and RMS thyristor current.*



Given,

$V_p = 230V, RMS$ primary supply voltage.

$f =$ Input supply frequency = 50Hz.

$R_L = 50\Omega$

$\alpha = 60^0 = \frac{\pi}{3}$ radians.

$V_S =$ RMS secondary voltage.

$$\frac{V_p}{V_S} = \frac{N_p}{N_S} = \frac{1}{1} = 1$$

Therefore $V_p = V_S = 230V$

Where, N_p = Number of turns in the primary winding.

N_S = Number of turns in the secondary winding.

- **RMS Value of Output (Load) Voltage** $V_{O(RMS)}$

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m^2 \sin^2 \omega t.d\left(\omega t\right)}$$

We have obtained the expression for $V_{O(RMS)}$ as

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right)\right]+\frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = 230\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\frac{\pi}{3}\right)\right]+\frac{\sin 120^0}{2}}$$

$$V_{O(RMS)} = 230\sqrt{\frac{1}{2\pi}\left[5.669\right]} = 230\times 0.94986$$

$$V_{O(RMS)} = 218.4696\ V \approx 218.47\ V$$

- **RMS Load Current** $I_{O(RMS)}$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L} = \frac{218.46966}{50} = 4.36939\ Amps$$

- **Output Load Power** P_O

$$P_O = I_{O(RMS)}^2 \times R_L = \left(4.36939\right)^2 \times 50 = 954.5799\ Watts$$

$$P_O = 0.9545799\ KW$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S}$$

V_S = RMS secondary supply voltage = 230V.
I_S = RMS secondary supply current = RMS load current.

$$\therefore I_S = I_{O(RMS)} = 4.36939\ Amps$$

$$\therefore PF = \frac{954.5799\ \text{W}}{\left(230\times 4.36939\right)\ \text{W}} = 0.9498$$

- **Average Output (Load) Voltage**

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m \sin\omega t.d\left(\omega t\right)\right]$$

We have obtained the expression for the average / DC output voltage as,

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[\cos\alpha - 1\right]$$

$$V_{O(dc)} = \frac{\sqrt{2}\times 230}{2\pi}\left[\cos\left(60^0\right) - 1\right] = \frac{325.2691193}{2\pi}\left[0.5 - 1\right]$$

$$V_{O(dc)} = \frac{325.2691193}{2\pi}\left[-0.5\right] = -25.88409 \text{ Volts}$$

- **Average DC Load Current**

$$I_{O(dc)} = \frac{V_{O(dc)}}{R_L} = \frac{-25.884094}{50} = -0.51768 \text{ Amps}$$

- **Average & RMS Thyristor Currents**

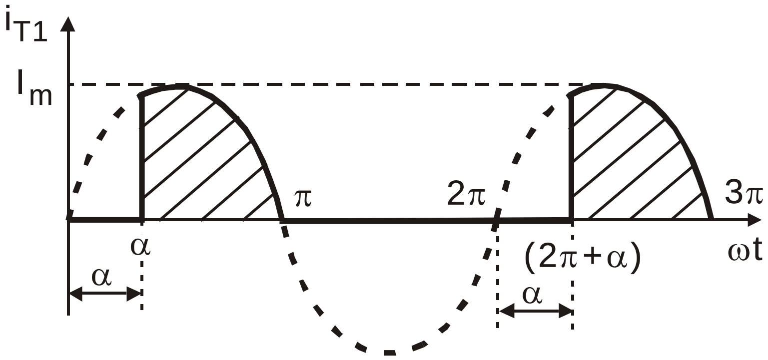


Fig.: Thyristor Current Waveform

Referring to the thyristor current waveform of a single phase half-wave ac voltage controller circuit, we can calculate the average thyristor current $I_{T(Avg)}$ as

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m \sin\omega t.d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[\int_{\alpha}^{\pi} \sin\omega t.d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[\left(-\cos\omega t\right)\Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos(\pi)+\cos\alpha\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[1+\cos\alpha\right]$$

Where, $I_m = \frac{V_m}{R_L}$ = Peak thyristor current = Peak load current.

$$I_m = \frac{\sqrt{2}\times 230}{50}$$

$$I_m = 6.505382 \text{ Amps}$$

$$I_{T(Avg)} = \frac{V_m}{2\pi R_L}\left[1+\cos\alpha\right]$$

$$I_{T(Avg)} = \frac{\sqrt{2}\times 230}{2\pi\times 50}\left[1+\cos\left(60^0\right)\right]$$

$$I_{T(Avg)} = \frac{\sqrt{2}\times 230}{100\pi}\left[1+0.5\right]$$

$$I_{T(Avg)} = 1.5530 \text{ Amps}$$

- RMS thyristor current $I_{T(RMS)}$ can be calculated by using the expression

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t.d\left(\omega t\right)\right]}$$

$$I_{T(RMS)} = \sqrt{\frac{I_m^2}{2\pi}\left[\int_{\alpha}^{\pi}\frac{\left(1-\cos 2\omega t\right)}{2}.d\left(\omega t\right)\right]}$$

$$I_{T(RMS)} = \sqrt{\frac{I_m^2}{4\pi}\left[\int_{\alpha}^{\pi} d\left(\omega t\right)-\int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right)\right]}$$

$$I_{T(RMS)} = I_m\sqrt{\frac{1}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi}-\left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right]}$$

$$I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi}\left[(\pi-\alpha)-\left\{\frac{\sin 2\pi - \sin 2\alpha}{2}\right\}\right]}$$

$$I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$I_{T(RMS)} = \frac{6.50538}{\sqrt{2}} \sqrt{\frac{1}{2\pi}\left[\left(\pi-\frac{\pi}{3}\right)+\frac{\sin\left(120^0\right)}{2}\right]}$$

$$I_{T(RMS)} = 4.6\sqrt{\frac{1}{2\pi}\left[\left(\frac{2\pi}{3}\right)+\frac{0.8660254}{2}\right]}$$

$$I_{T(RMS)} = 4.6 \times 0.6342 = 2.91746A$$

$$I_{T(RMS)} = 2.91746 \text{ Amps}$$

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (AC REGULATOR) OR RMS VOLTAGE CONTROLLER WITH RESISTIVE LOAD

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle $'\alpha'$.

The RMS value of load voltage can be varied by varying the trigger angle $'\alpha'$. The input supply current is alternating in the case of a full wave ac voltage controller and due to the symmetrical nature of the input supply current waveform there is no dc component of input supply current i.e., the average value of the input supply current is zero.

A single phase full wave ac voltage controller with a resistive load is shown in the figure below. It is possible to control the ac power flow to the load in both the half cycles by adjusting the trigger angle $'\alpha'$. Hence the full wave ac voltage controller is also referred to as to a bi-directional controller.

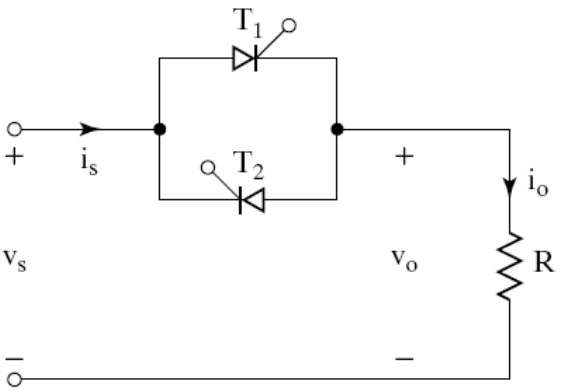


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using SCRs

The thyristor T_1 is forward biased during the positive half cycle of the input supply voltage. The thyristor T_1 is triggered at a delay angle of $'\alpha'$ $\left(0 \le \alpha \le \pi \text{ radians}\right)$. Considering the ON thyristor T_1 as an ideal closed switch the input supply voltage appears across the load resistor R_L and the output voltage $v_O = v_S$ during $\omega t = \alpha$ to π radians. The load current flows through the ON thyristor T_1 and through the load resistor R_L in the downward direction during the conduction time of T_1 from $\omega t = \alpha$ to π radians.

At $\omega t = \pi$, when the input voltage falls to zero the thyristor current (which is flowing through the load resistor R_L) falls to zero and hence T_1 naturally turns off . No current flows in the circuit during $\omega t = \pi$ to $\left(\pi + \alpha\right)$.

The thyristor T_2 is forward biased during the negative cycle of input supply and when thyristor T_2 is triggered at a delay angle $\left(\pi + \alpha\right)$, the output voltage follows the negative halfcycle of input from $\omega t = \left(\pi + \alpha\right)$ to 2π . When T_2 is ON, the load current flows in the reverse direction (upward direction) through T_2 during $\omega t = \left(\pi + \alpha\right)$ to 2π radians. The time interval (spacing) between the gate trigger pulses of T_1 and T_2 is kept at π radians or 180^0. At $\omega t = 2\pi$ the input supply voltage falls to zero and hence the load current also falls to zero and thyristor T_2 turn off naturally.

Instead of using two SCR's in parallel, a Triac can be used for full wave ac voltage control.

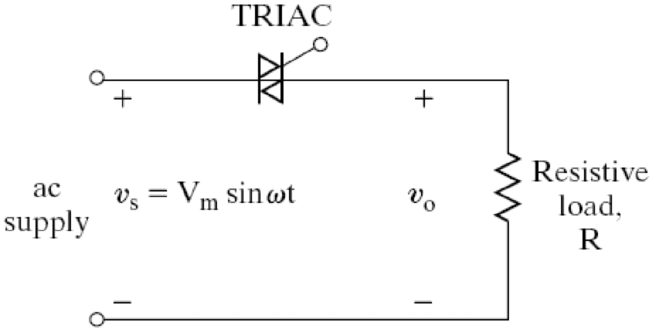


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC

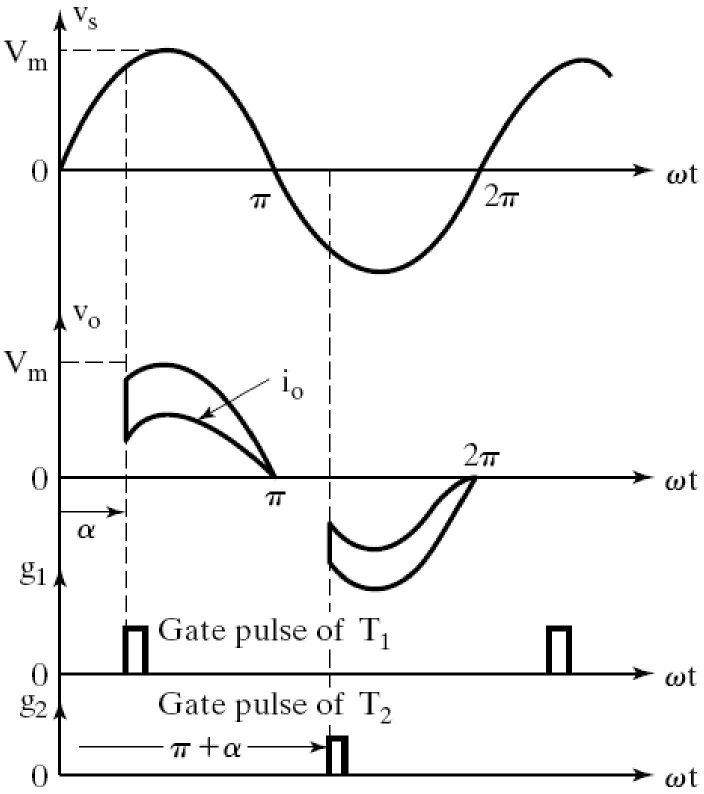


Fig: Waveforms of single phase full wave ac voltage controller

EQUATIONS

Input supply voltage

$$v_S = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t\ ;$$

Output voltage across the load resistor R_L;

$$v_O = v_L = V_m \sin \omega t\ ;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Output load current

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t\ ;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT (LOAD) VOLTAGE

The RMS value of output voltage (load voltage) can be found using the expression

$$V_{O(RMS)}^2 = V^2{}_{L(RMS)} = \frac{1}{2\pi} \int_0^{2\pi} v_L{}^2 d(\omega t)\ ;$$

For a full wave ac voltage controller, we can see that the two half cycles of output voltage waveforms are symmetrical and the output pulse time period (or output pulse repetition time) is π radians. Hence we can also calculate the RMS output voltage by using the expression given below.

$$V^2{}_{L(RMS)} = \frac{1}{\pi}\int_0^{\pi} V_m{}^2 \sin^2 \omega t.d\omega t$$

$$V^2{}_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L{}^2 .d\left(\omega t\right) \ ;$$

$v_L = v_O = V_m \sin \omega t$; For $\omega t = \alpha$ to π and $\omega t = \left(\pi + \alpha\right)$ to 2π

Hence,

$$V_{L(RMS)}^2 = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi}\left(V_m \sin \omega t\right)^2 d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\left(V_m \sin \omega t\right)^2 d\left(\omega t\right)\right]$$

$$= \frac{1}{2\pi}\left[V_m{}^2 \int_{\alpha}^{\pi} \sin^2 \omega t.d\left(\omega t\right) + V_m{}^2 \int_{\pi+\alpha}^{2\pi} \sin^2 \omega t.d\left(\omega t\right)\right]$$

$$= \frac{V_m{}^2}{2\pi}\left[\int_{\alpha}^{\pi}\frac{1-\cos 2\omega t}{2} d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\frac{1-\cos 2\omega t}{2} d\left(\omega t\right)\right]$$

$$= \frac{V_m{}^2}{2\pi \times 2}\left[\int_{\alpha}^{\pi} d\left(\omega t\right) - \int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi} d\left(\omega t\right) - \int_{\pi+\alpha}^{2\pi}\cos 2\omega t.d\left(\omega t\right)\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi} + \left(\omega t\right)\Big/_{\pi+\alpha}^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[\left(\pi - \alpha\right) + \left(\pi - \alpha\right) - \frac{1}{2}\left(\sin 2\pi - \sin 2\alpha\right) - \frac{1}{2}\left(\sin 4\pi - \sin 2\left(\pi + \alpha\right)\right)\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2\left(\pi - \alpha\right) - \frac{1}{2}\left(0 - \sin 2\alpha\right) - \frac{1}{2}\left(0 - \sin 2\left(\pi + \alpha\right)\right)\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2} + \frac{\sin 2\left(\pi + \alpha\right)}{2}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2\left(\pi - \alpha\right) + \frac{\sin 2\alpha}{2} + \frac{\sin\left(2\pi + 2\alpha\right)}{2}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\frac{\sin 2\alpha}{2}+\frac{1}{2}(\sin 2\pi.\cos 2\alpha+\cos 2\pi.\sin 2\alpha)\right]$$

$$\sin 2\pi = 0 \quad \& \quad \cos 2\pi = 1$$

Therefore,

$$V^2_{L(RMS)} = \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\alpha}{2}\right]$$

$$= \frac{V_m{}^2}{4\pi}\left[2(\pi-\alpha)+\sin 2\alpha\right]$$

$$V^2{}_{L(RMS)} = \frac{V_m{}^2}{4\pi}\left[(2\pi-2\alpha)+\sin 2\alpha\right]$$

Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[(2\pi-2\alpha)+\sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left[(2\pi-2\alpha)+\sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(2\pi-2\alpha)+\sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_{i(RMS)}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-0)+\frac{\sin 2\times 0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi)+\frac{0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

The output control characteristic for a single phase full wave ac voltage controller with resistive load can be obtained by plotting the equation for $V_{O(RMS)}$

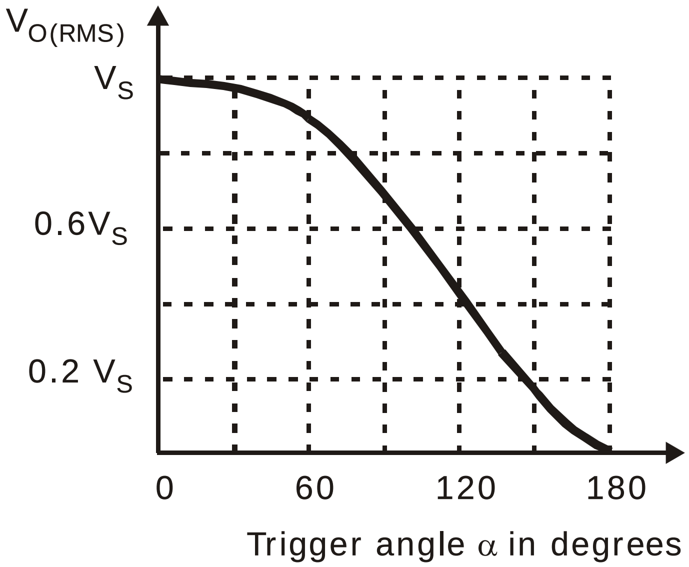
CONTROL CHARACTERISTIC OF SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

The control characteristic is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α; which can be obtained by using the expression for the RMS output voltage of a full-wave ac controller with resistive load.

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]} \quad ;$$

Where $V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input supply voltage

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$	**%**
0	0	V_S	100% V_S
30^0	$\pi/6$; $(1\pi/6)$	0.985477 V_S	98.54% V_S
60^0	$\pi/3$; $(2\pi/6)$	0.896938 V_S	89.69% V_S
90^0	$\pi/2$; $(3\pi/6)$	0.7071 V_S	70.7% V_S
120^0	$2\pi/3$; $(4\pi/6)$	0.44215 V_S	44.21% V_S
150^0	$5\pi/6$; $(5\pi/6)$	0.1698 V_S	16.98% V_S
180^0	π ; $(6\pi/6)$	0 V_S	0 V_S



We can notice from the figure, that we obtain a much better output control characteristic by using a single phase full wave ac voltage controller. The RMS output voltage can be varied from a maximum of 100% V_S at $\alpha = 0$ to a minimum of '0' at $\alpha = 180^0$. Thus we get a full range output voltage control by using a single phase full wave ac voltage controller.

Need For Isolation

In the single phase full wave ac voltage controller circuit using two SCRs or Thyristors T_1 and T_2 in parallel, the gating circuits (gate trigger pulse generating circuits) of Thyristors T_1 and T_2 must be isolated. Figure shows a pulse transformer with two separate windings to provide isolation between the gating signals of T_1 and T_2.

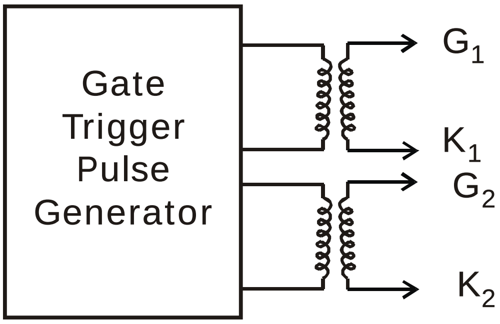


Fig.: Pulse Transformer

SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH COMMON CATHODE

It is possible to design a single phase full wave ac controller with a common cathode configuration by having a common cathode point for T_1 and T_2 & by adding two diodes in a full wave ac controller circuit as shown in the figure below

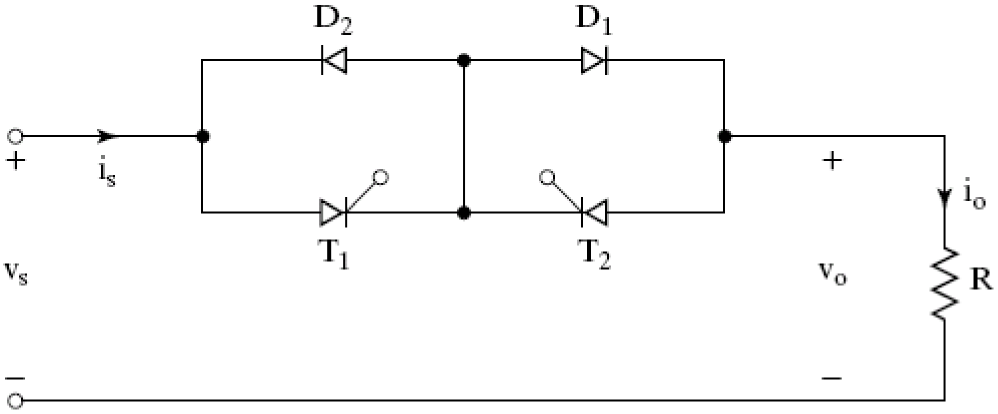


Fig.: Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)

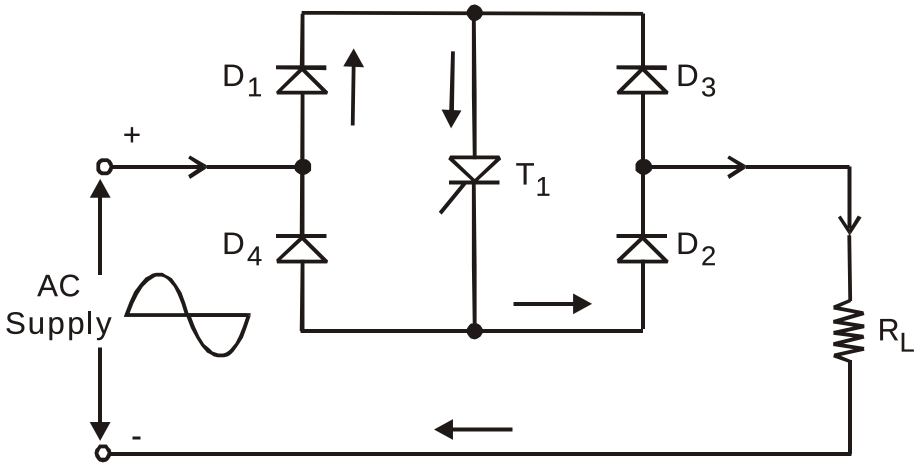
Thyristor T_1 and diode D_1 are forward biased during the positive half cycle of input supply. When thyristor T_1 is triggered at a delay angle α, Thyristor T_1 and diode D_1 conduct together from $\omega t = \alpha$ to π during the positive half cycle.

The thyristor T_2 and diode D_2 are forward biased during the negative half cycle of input supply, when trigged at a delay angle α, thyristor T_2 and diode D_2 conduct together during the negative half cycle from $\omega t = (\pi + \alpha)$ to 2π .

In this circuit as there is one single common cathode point, routing of the gate trigger pulses to the thyristor gates of T_1 and T_2 is simpler and only one isolation circuit is required.

But due to the need of two power diodes the costs of the devices increase. As there are two power devices conducting at the same time the voltage drop across the ON devices increases and the ON state conducting losses of devices increase and hence the efficiency decreases.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER USING A SINGLE THYRISTOR



A single phase full wave ac controller can also be implemented with one thyristor and four diodes connected in a full wave bridge configuration as shown in the above figure. The four diodes act as a bridge full wave rectifier. The voltage across the thyristor T_1 and current through thyristor T_1 are always unidirectional. When T_1 is triggered at $\omega t = \alpha$, during the positive half cycle $\left(0 \le \alpha \le \pi\right)$, the load current flows through D_1, T_1, diode D_2 and through the load. With a resistive load, the thyristor current (flowing through the ON thyristor T_1) , the load current falls to zero at $\omega t = \pi$, when the input supply voltage decreases to zero at $\omega t = \pi$, the thyristor naturally turns OFF.

In the negative half cycle, diodes $D_3 \,\&\, D_4$ are forward biased during $\omega t = \pi$ to 2π radians. When T_1 is triggered at $\omega t = \left(\pi + \alpha\right)$, the load current flows in the opposite direction (upward direction) through the load, through D_3, T_1 and D_4. Thus D_3, D_4 and T_1 conduct together during the negative half cycle to supply the load power. When the input supply voltage becomes zero at $\omega t = 2\pi$, the thyristor current (load current) falls to zero at $\omega t = 2\pi$ and the thyristor T_1 naturally turns OFF. The waveforms and the expression for the RMS output voltage are the same as discussed earlier for the single phase full wave ac controller.

But however if there is a large inductance in the load circuit, thyristor T_1 may not be turned OFF at the zero crossing points, in every half cycle of input voltage and this may result in a loss of output control. This would require detection of the zero crossing of the load current waveform in order to ensure guaranteed turn off of the conducting thyristor before triggering the thyristor in the next half cycle, so that we gain control on the output voltage.

In this full wave ac controller circuit using a single thyristor, as there are three power devices conducting together at the same time there is more conduction voltage drop and an increase in the ON state conduction losses and hence efficiency is also reduced.

The diode bridge rectifier and thyristor (or a power transistor) act together as a bidirectional switch which is commercially available as a single device module and it has relatively low ON state conduction loss. It can be used for bidirectional load current control and for controlling the RMS output voltage.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (BIDIRECTIONAL CONTROLLER) WITH RL LOAD

In this section we will discuss the operation and performance of a single phase full wave ac voltage controller with RL load. In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors T_1 and T_2 (T_1 and T_2 are two SCRs) connected in parallel is shown in the figure below. In place of two thyristors a single Triac can be used to implement a full wave ac controller, if a suitable Traic is available for the desired RMS load current and the RMS output voltage ratings.

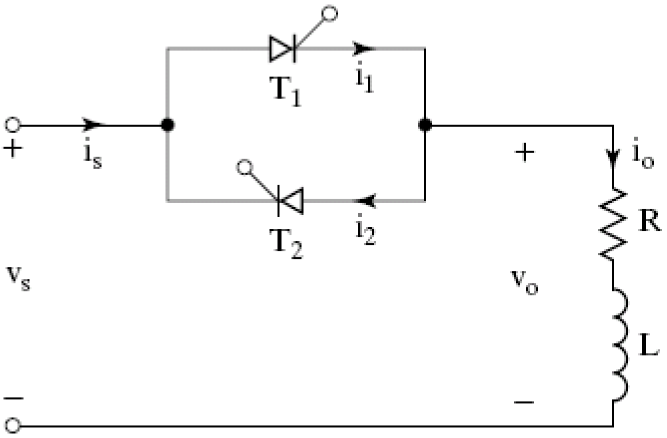


Fig: Single phase full wave ac voltage controller with RL load

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulses of T_1 and T_2, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

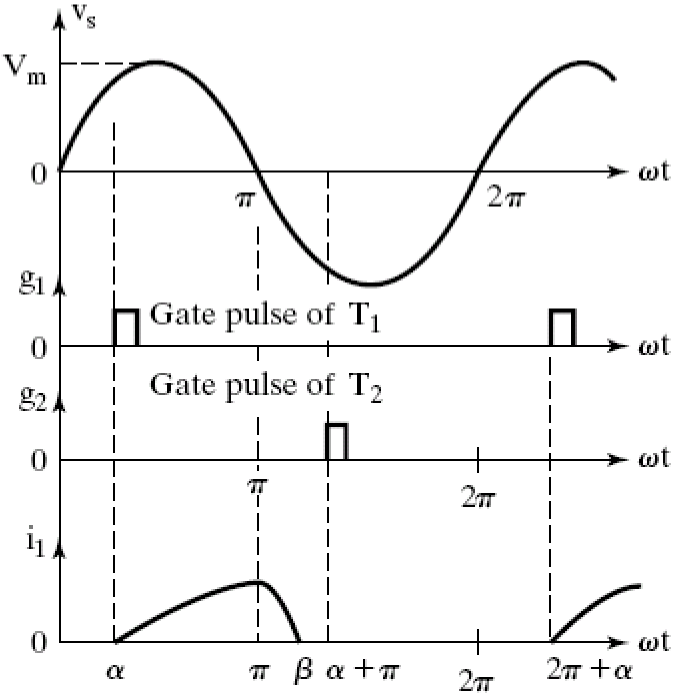


Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

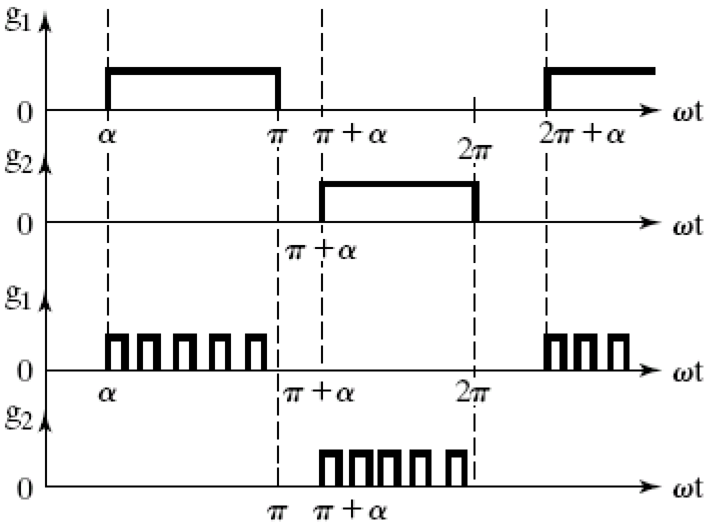


Fig.: Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \phi$
Discontinuous load current operation occurs for $\alpha > \phi$ and $\beta < (\pi + \alpha)$;
i.e., $(\beta - \alpha) < \pi$, conduction angle $< \pi$.

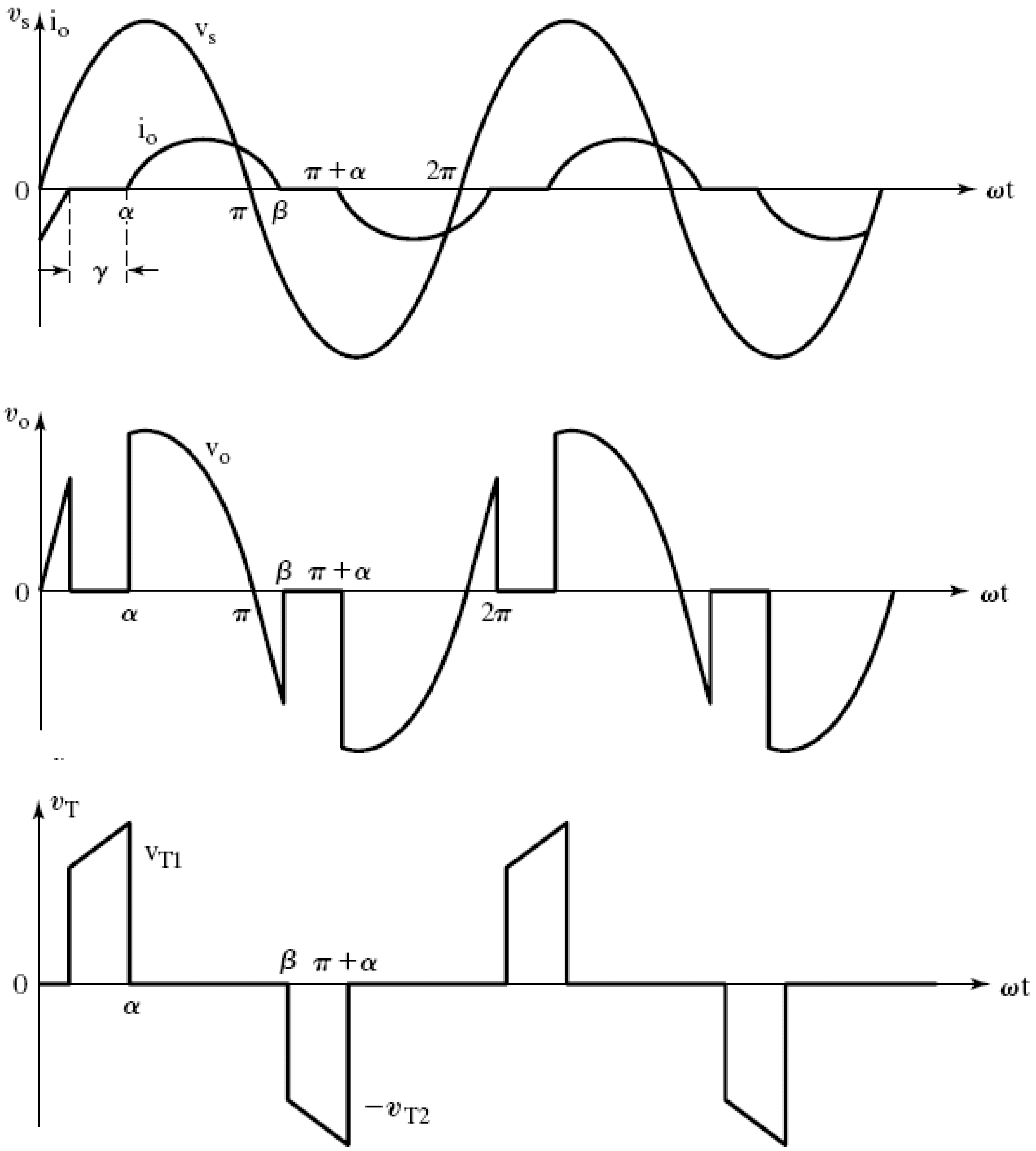


Fig.: Waveforms of Input supply voltage, Load Current, Load Voltage and Thyristor Voltage across T_1

Note

- The RMS value of the output voltage and the load current may be varied by varying the trigger angle α .
- This circuit, AC RMS voltage controller can be used to regulate the RMS voltage across the terminals of an ac motor (induction motor). It can be used to control the temperature of a furnace by varying the RMS output voltage.

- For very large load inductance 'L' the SCR may fail to commutate, after it is triggered and the load voltage will be a full sine wave (similar to the applied input supply voltage and the output control will be lost) as long as the gating signals are applied to the thyristors T_1 and T_2. The load current waveform will appear as a full continuous sine wave and the load current waveform lags behind the output sine wave by the load power factor angle ϕ.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$Z = \sqrt{R^2 + \left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R}$ = Load circuit time constant.

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

Therefore $$A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 from the above equation into the expression for i_O, we obtain

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right] \quad ;$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase full wave ac voltage controller with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

To Calculate Extinction Angle β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin\left(\beta-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$ we can write

$$\left[\sin\left(\beta-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin\left(\beta-\phi\right)=\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = \left(\beta - \alpha\right)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a known value of β.

For $\delta < \pi$ radians, i.e., for $\left(\beta-\alpha\right) < \pi$ radians, for $\left(\pi+\alpha\right) > \beta$ the load current waveform appears as a discontinuous current waveform as shown in the figure. The output load current remains at zero during $\omega t = \beta$ to $\left(\pi+\alpha\right)$. This is referred to as discontinuous load current operation which occurs for $\beta < \left(\pi+\alpha\right)$.

When the trigger angle α is decreased and made equal to the load impedance angle ϕ i.e., when $\alpha = \phi$ we obtain from the expression for $\sin\left(\beta-\phi\right)$,

$$\sin\left(\beta-\phi\right)=0 \ \ ; \ \ \text{Therefore} \ \ \left(\beta-\phi\right)=\pi \ \text{radians.}$$

Extinction angle $\beta = \left(\pi+\phi\right) = \left(\pi+\alpha\right)$; for the case when $\alpha = \phi$

Conduction angle $\delta = \left(\beta-\alpha\right) = \pi$ radians $= 180^0$; for the case when $\alpha = \phi$

Each thyristor conducts for 180^0 (π radians) . T_1 conducts from $\omega t = \phi$ to $\left(\pi+\phi\right)$ and provides a positive load current. T_2 conducts from $\left(\pi+\phi\right)$ to $\left(2\pi+\phi\right)$ and provides a negative load current. Hence we obtain a continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle $\alpha \leq \phi$ and the control on the output is lost.

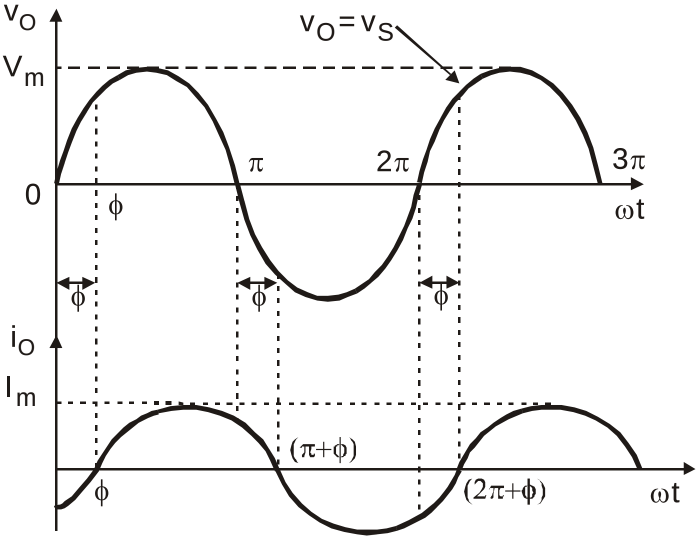


Fig.: Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \leq \phi$

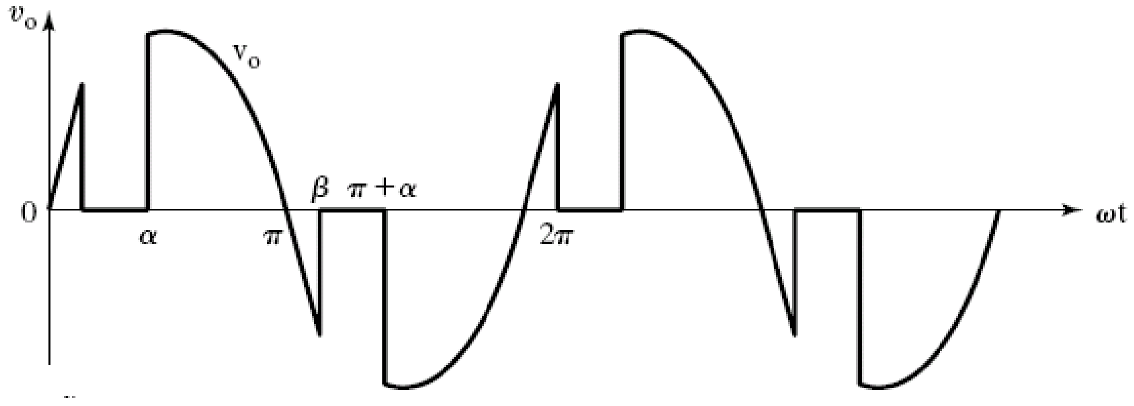
Thus we observe that for trigger angle $\alpha \leq \phi$, the load current tends to flow continuously and we have continuous load current operation, without any break in the load current waveform and we obtain output voltage waveform which is a continuous sinusoidal waveform identical to the input supply voltage waveform. We loose the control on the output voltage for $\alpha \leq \phi$ as the output voltage becomes equal to the input supply voltage and thus we obtain

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \quad ; \text{ for } \alpha \leq \phi$$

Hence,

RMS output voltage = RMS input supply voltage for $\alpha \leq \phi$

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$ OF A SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RL LOAD.



When $\alpha > \varnothing$, the load current and load voltage waveforms become discontinuous as shown in the figure above.

$$V_{O(RMS)} = \left[\frac{1}{\pi} \int_{\alpha}^{\beta} V_m{}^2 \sin^2 \omega t . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

Output $v_o = V_m \sin \omega t$, for $\omega t = \alpha$ to β, when T_1 is ON.

$$V_{O(RMS)} = \left[\frac{V_m{}^2}{\pi} \int_{\alpha}^{\beta} \frac{\left(1 - \cos 2\omega t\right)}{2} d\left(\omega t\right) \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m{}^2}{2\pi} \left\{ \int_{\alpha}^{\beta} d\left(\omega t\right) - \int_{\alpha}^{\beta} \cos 2\omega t . d\left(\omega t\right) \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m{}^2}{2\pi} \left\{ \left(\omega t\right) \Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{\beta} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m{}^2}{2\pi} \left\{ \left(\beta - \alpha\right) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ \left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left\{ \left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

The RMS output voltage across the load can be varied by changing the trigger angle α.

For a purely resistive load $L = 0$, therefore load power factor angle $\phi = 0$.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R} \right) = 0 \quad ;$$

Extinction angle $\beta = \pi$ radians $= 180^0$

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

- **RMS Output Voltage** $V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input supply voltage.

- $I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L}$ = RMS value of load current.

- $I_S = I_{O(RMS)}$ = RMS value of input supply current.

- **Output load power**

$$P_O = I^2_{O(RMS)} \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I^2_{O(RMS)} \times R_L}{V_S \times I_{O(RMS)}} = \frac{I_{O(RMS)} \times R_L}{V_S}$$

$$PF = \frac{V_{O(RMS)}}{V_S} = \sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

- **Average Thyristor Current,**

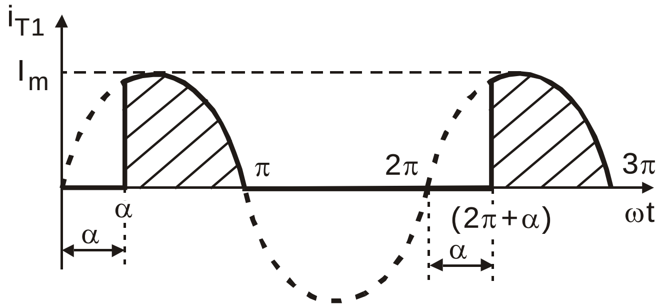


Fig.: Thyristor Current Waveform

$$I_{T(Avg)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T\, d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t . d(\omega t)$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t . d(\omega t) = \frac{I_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos\pi + \cos\alpha\right] = \frac{I_m}{2\pi}\left[1+\cos\alpha\right]$$

- **Maximum Average Thyristor Current, for** $\alpha = 0$,

$$I_{T(Avg)} = \frac{I_m}{\pi}$$

- **RMS Thyristor Current**

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}\right]}$$

- **Maximum RMS Thyristor Current, for** $\alpha = 0$,

$$I_{T(RMS)} = \frac{I_m}{2}$$

In the case of a single phase full wave ac voltage controller circuit using a Triac with resistive load, the average thyristor current $I_{T(Avg)} = 0$. Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH R-L LOAD

The Expression for the Output (Load) Current

The expression for the output (load) current which flows through the thyristor, during $\omega t = \alpha$ to β is given by

$$i_O = i_{T_1} = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}\left(\omega t-\alpha\right)}\right] \quad ; \quad \text{for } \alpha \le \omega t \le \beta$$

Where,

$V_m = \sqrt{2}V_S$ = Maximum or peak value of input ac supply voltage.

$Z = \sqrt{R^2+\left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (load power factor angle).

α = Thyristor trigger angle = Delay angle.

β = Extinction angle of thyristor, (value of ωt) at which the thyristor (load) current falls to zero.

β is calculated by solving the equation

$$\sin\left(\beta-\phi\right) = \sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}\left(\beta-\alpha\right)}$$

Thyristor Conduction Angle $\delta = \left(\beta - \alpha\right)$

Maximum thyristor conduction angle $\delta = \left(\beta - \alpha\right) = \pi$ radians = 180^0 for $\alpha \le \phi$.

RMS Output Voltage

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[\left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} i_{T_1} d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[\int_{\alpha}^{\beta} \sin\left(\omega t - \phi\right).d\left(\omega t\right) - \int_{\alpha}^{\beta} \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)} d\left(\omega t\right)\right]$$

Maximum value of $I_{T(Avg)}$ occur at $\alpha = 0$. The thyristors should be rated for maximum $I_{T(Avg)} = \left(\frac{I_m}{\pi}\right)$, where $I_m = \frac{V_m}{Z}$.

RMS Thyristor Current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta} i_{T_1}^2 d\left(\omega t\right)\right]}$$

Maximum value of $I_{T(RMS)}$ occurs at $\alpha = 0$. Thyristors should be rated for maximum $I_{T(RMS)} = \left(\frac{I_m}{2}\right)$

When a Triac is used in a single phase full wave ac voltage controller with RL type of load, then $I_{T(Avg)} = 0$ and maximum $I_{T(RMS)} = \frac{I_m}{\sqrt{2}}$

PROBLEMS

1. *A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has L = 10mH, R = 10Ω, the delay angle of thyristors T_1 and T_2 are equal, where $\alpha_1 = \alpha_2 = \frac{\pi}{3}$. Determine*
    a. *Conduction angle of the thyristor T_1.*
    b. *RMS output voltage.*
    c. *The input power factor.*
    Comment on the type of operation.

Given

$V_s = 230V$, $f = 50Hz$, $L = 10mH$, $R = 10\Omega$, $\alpha = 60^0$,

$\alpha = \alpha_1 = \alpha_2 = \frac{\pi}{3}$ radians, .

$$V_m = \sqrt{2} V_S = \sqrt{2} \times 230 = 325.2691193\ V$$

$$Z = \text{ Load Impedance} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (\omega L)^2}$$

$$\omega L = (2\pi fL) = (2\pi \times 50 \times 10 \times 10^{-3}) = \pi = 3.14159\Omega$$

$$Z = \sqrt{(10)^2 + (3.14159)^2} = \sqrt{109.8696} = 10.4818\Omega$$

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} \times 230}{10.4818} = 31.03179\ A$$

Load Impedance Angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{\pi}{10}\right) = \tan^{-1}(0.314159) = 17.44059^0$$

Trigger Angle $\alpha > \phi$. Hence the type of operation will be discontinuous load current operation, we get

$$\beta < (\pi + \alpha)$$

$$\beta < (180 + 60)\ ;\ \beta < 240^0$$

Therefore the range of β is from 180 degrees to 240 degrees. $(180^0 < \beta < 240^0)$

Extinction Angle β is calculated by using the equation

$$\sin\left(\beta-\phi\right)=\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

In the exponential term the value of α and β should be substituted in radians. Hence

$$\sin\left(\beta-\phi\right)=\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta_{Rad}-\alpha_{Rad})} \quad ; \quad \alpha_{Rad}=\left(\frac{\pi}{3}\right)$$

$$\left(\alpha-\phi\right)=\left(60-17.44059\right)=42.5594^0$$

$$\sin\left(\beta-17.44\right)^0=\sin\left(42.5594^0\right)e^{\frac{-10}{\pi}(\beta-\alpha)}$$

$$\sin\left(\beta-17.44\right)^0=0.676354e^{-3.183(\beta-\alpha)}$$

$$180^0\rightarrow\pi \text{ radians}, \quad \beta_{Rad}=\frac{\beta^0\times\pi}{180^0}$$

Assuming $\beta=190^0$**;**

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{190^0\times\pi}{180}=3.3161$$

L.H.S: $\sin\left(190-17.44\right)^0=\sin\left(172.56\right)=0.129487$

R.H.S: $0.676354\times e^{-3.183\left(3.3161-\frac{\pi}{3}\right)}=4.94\times10^{-4}$

Assuming $\beta=183^0$;

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{183^0\times\pi}{180}=3.19395$$

$$\left(\beta-\alpha\right)=\left(3.19395-\frac{\pi}{3}\right)=2.14675$$

L.H.S: $\sin\left(\beta-\phi\right)=\sin\left(183-17.44\right)=\sin165.56^0=0.24936$

R.H.S: $0.676354e^{-3.183(2.14675)}=7.2876\times10^{-4}$

Assuming $\beta\approx180^0$

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{180^0\times\pi}{180}=\pi$$

$$\left(\beta-\alpha\right)=\left(\pi-\frac{\pi}{3}\right)=\left(\frac{2\pi}{3}\right)$$

L.H.S: $\sin\left(\beta-\phi\right)=\sin\left(180-17.44\right)=0.2997$

R.H.S: $0.676354e^{-3.183\left(\pi-\frac{\pi}{3}\right)}=8.6092\times10^{-4}$

Assuming $\beta=196^{0}$

$$\beta_{Rad}=\frac{\beta^{0}\times\pi}{180^{0}}=\frac{196^{0}\times\pi}{180}=3.420845$$

L.H.S: $\sin\left(\beta-\phi\right)=\sin\left(196-17.44\right)=0.02513$

R.H.S: $0.676354e^{-3.183\left(3.420845-\frac{\pi}{3}\right)}=3.5394\times10^{-4}$

Assuming $\beta=197^{0}$

$$\beta_{Rad}=\frac{\beta^{0}\times\pi}{180^{0}}=\frac{197^{0}\times\pi}{180}=3.43829$$

L.H.S: $\sin\left(\beta-\phi\right)=\sin\left(197-17.44\right)=7.69=7.67937\times10^{-3}$

R.H.S: $0.676354e^{-3.183\left(3.43829-\frac{\pi}{3}\right)}=4.950386476\times10^{-4}$

Assuming $\beta=197.42^{0}$

$$\beta_{Rad}=\frac{\beta^{0}\times\pi}{180^{0}}=\frac{197.42\times\pi}{180}=3.4456$$

L.H.S: $\sin\left(\beta-\phi\right)=\sin\left(197.42-17.44\right)=3.4906\times10^{-4}$

R.H.S: $0.676354e^{-3.183\left(3.4456-\frac{\pi}{3}\right)}=3.2709\times10^{-4}$

Conduction Angle $\delta=\left(\beta-\alpha\right)=\left(197.42^{0}-60^{0}\right)=137.42^{0}$

RMS Output Voltage

$$V_{O(RMS)}=V_{S}\sqrt{\frac{1}{\pi}\left[\left(\beta-\alpha\right)+\frac{\sin2\alpha}{2}-\frac{\sin2\beta}{2}\right]}$$

$$V_{O(RMS)}=230\sqrt{\frac{1}{\pi}\left[\left(3.4456-\frac{\pi}{3}\right)+\frac{\sin2\left(60^{0}\right)}{2}-\frac{\sin2\left(197.42^{0}\right)}{2}\right]}$$

$$V_{O(RMS)}=230\sqrt{\frac{1}{\pi}\left[\left(2.39843\right)+0.4330-0.285640\right]}$$

$$V_{O(RMS)}=230\times0.9=207.0445\text{ V}$$

Input Power Factor

$$PF = \frac{P_O}{V_S \times I_S}$$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{207.0445}{10.4818} = 19.7527 \text{ A}$$

$$P_O = I_{O(RMS)}^2 \times R_L = (19.7527)^2 \times 10 = 3901.716 \text{ W}$$

$$V_S = 230V, \qquad I_S = I_{O(RMS)} = 19.7527$$

$$PF = \frac{P_O}{V_S \times I_S} = \frac{3901.716}{230 \times 19.7527} = 0.8588$$

2. *A single phase full wave controller has an input voltage of 120 V (RMS) and a load resistance of 6 ohm. The firing angle of thyristor is* $\pi/2$ *. Find*
   - a. *RMS output voltage*
   - b. *Power output*
   - c. *Input power factor*
   - d. *Average and RMS thyristor current.*

Solution

$$\alpha = \frac{\pi}{2} = 90^0, \quad V_S = 120 \text{ V}, \qquad R = 6\Omega$$

RMS Value of Output Voltage

$$V_O = V_S \left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 120\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 180}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 84.85 \text{ Volts}$$

RMS Output Current

$$I_O = \frac{V_O}{R} = \frac{84.85}{6} = 14.14 \text{ A}$$

Load Power

$$P_O = I_O^2 \times R$$

$$P_O = (14.14)^2 \times 6 = 1200 \text{ watts}$$

Input Current is same as Load Current

Therefore $I_S = I_O = 14.14$ Amps

Input Supply Volt-Amp $= V_S I_S = 120 \times 14.14 = 1696.8\ VA$

Therefore

$$\text{Input Power Factor} = \frac{\text{Load Power}}{\text{Input Volt-Amp}} = \frac{1200}{1696.8} = 0.707\left(lag\right)$$

Each Thyristor Conducts only for half a cycle

Average thyristor current $I_{T(Avg)}$

$$I_{T(Avg)} = \frac{1}{2\pi R}\int_{\alpha}^{\pi} V_m \sin\omega t.d\left(\omega t\right)$$

$$= \frac{V_m}{2\pi R}\left(1+\cos\alpha\right) \ ; \qquad \mathrm{V_m} = \sqrt{2}V_S$$

$$= \frac{\sqrt{2}\times 120}{2\pi\times 6}\left[1+\cos 90\right] = 4.5\ \mathrm{A}$$

RMS thyristor current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\frac{V_m^2\sin^2\omega t}{R^2}d\left(\omega t\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi R^2}\int_{\alpha}^{\pi}\frac{\left(1-\cos 2\omega t\right)}{2}d\left(\omega t\right)}$$

$$= \frac{V_m}{2R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}V_S}{2R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}\times 120}{2\times 6}\left[\frac{1}{\pi}\left(\pi-\frac{\pi}{2}+\frac{\sin 180}{2}\right)\right]^{\frac{1}{2}} = 10\ \text{Amps}$$

3. *A single phase half wave ac regulator using one SCR in anti-parallel with a diode feeds 1 kW, 230 V heater. Find load power for a firing angle of 45*^{0}.

Solution

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_S = 230 \text{ V} \;;\; P_O = 1KW = 1000W$$

At standard rms supply voltage of 230V, the heater dissipates 1KW of output power

Therefore

$$P_O = V_O \times I_O = \frac{V_O \times V_O}{R} = \frac{V_O^2}{R}$$

Resistance of heater

$$R = \frac{V_O^2}{P_O} = \frac{(230)^2}{1000} = 52.9\Omega$$

RMS value of output voltage

$$V_O = V_S \left[\frac{1}{2\pi} \left(2\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}} \qquad \text{; for firing angle } \alpha = 45^0$$

$$V_O = 230 \left[\frac{1}{2\pi} \left(2\pi - \frac{\pi}{4} + \frac{\sin 90}{2} \right) \right]^{\frac{1}{2}} = 224.7157 \text{ Volts}$$

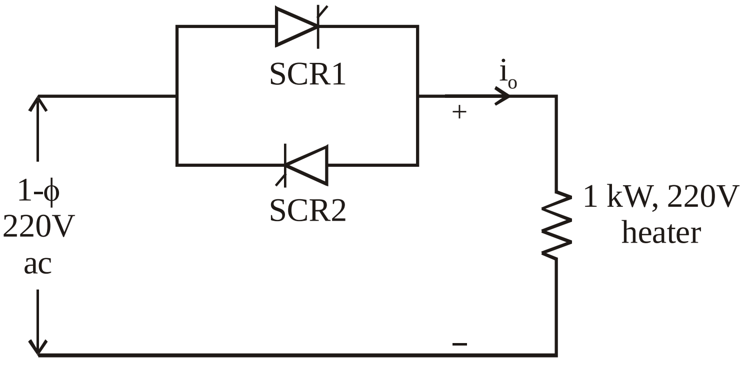
RMS value of output current

$$I_O = \frac{V_O}{R} = \frac{224.9}{52.9} = 4.2479 \text{ Amps}$$

Load Power

$$P_O = I_O^2 \times R = (4.25)^2 \times 52.9 = 954.56 \text{ Watts}$$

4. *Find the RMS and average current flowing through the heater shown in figure. The delay angle of both the SCRs is 45*^{0}.



Solution

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_S = 220 \text{ V}$$

Resistance of heater

$$R = \frac{V^2}{R} = \frac{(220)^2}{1000} = 48.4\Omega$$

Resistance value of output voltage

$$V_O = V_S \sqrt{\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]}$$

$$V_O = 220\sqrt{\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 90}{2}\right)\right]}$$

$$V_O = 220\sqrt{\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{1}{2}\right)\right]} = 209.769 \text{ Volts}$$

RMS current flowing through heater $= \frac{V_O}{R} = \frac{209.769}{48.4} = 4.334 \text{ Amps}$

Average current flowing through the heater $I_{Avg} = 0$

5. *A single phase voltage controller is employed for controlling the power flow from 220 V, 50 Hz source into a load circuit consisting of R = 4 Ω and ωL = 6 Ω. Calculate the following*
   a. *Control range of firing angle*
   b. *Maximum value of RMS load current*
   c. *Maximum power and power factor*
   d. *Maximum value of average and RMS thyristor current.*

Solution

For control of output power, minimum angle of firing angle α is equal to the load impedance angle θ

$$\alpha = \theta, \text{ load angle}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{6}{4}\right) = 56.3^0$$

Maximum possible value of α is 180^0

Therefore control range of firing angle is $56.3^0 < \alpha < 180^0$

Maximum value of RMS load current *occurs when* $\alpha = \theta = 56.3^0$. At this value of α the Maximum value of RMS load current

$$I_O = \frac{V_S}{Z} = \frac{220}{\sqrt{4^2 + 6^2}} = 30.5085 \text{ Amps}$$

Maximum Power $P_O = I_O^2 R = (30.5085)^2 \times 4 = 3723.077 \text{ W}$

Input Volt-Amp $= V_S I_O = 220 \times 30.5085 = 6711.87 \text{ W}$

Power Factor $= \frac{P_O}{Input\ VA} = \frac{3723.077}{6711.87} = 0.5547$

Average thyristor current *will be maximum when* $\alpha = \theta$ and conduction angle $\gamma = 180^0$.

Therefore maximum value of average thyristor current

$$I_{T(Avg)} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} \frac{V_m}{Z} \sin(\omega t - \theta)\, d(\omega t)$$

Note: $i_O = i_{T_1} = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right]$

At $\alpha = 0$,

$$i_{T_1} = i_O = \frac{V_m}{Z} \sin(\omega t - \theta)$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[-\cos(\omega t - \theta)\right]_{\alpha}^{\pi+\alpha}$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[-\cos(\pi + \alpha - \theta) + \cos(\alpha - \theta)\right]$$

But $\alpha = \theta$,

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[-\cos(\pi) + \cos(0)\right] = \frac{V_m}{2\pi Z}[2] = \frac{V_m}{\pi Z}$$

$$\therefore I_{T(Avg)} = \frac{V_m}{\pi Z} = \frac{\sqrt{2} \times 220}{\pi\sqrt{4^2 + 6^2}} = 13.7336 \text{ Amps}$$

Similarly, maximum RMS value occurs when $\alpha = 0$ and $\gamma = \pi$.

Therefore maximum value of RMS thyristor current

$$I_{TM} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} \left\{\frac{V_m}{Z} \sin(\omega t - \theta)\right\}^2 d(\omega t)}$$

$$I_{TM} = \sqrt{\frac{V_m^2}{2\pi Z^2}\int_{\alpha}^{\pi+\alpha}\left[\frac{1-\cos\left(2\omega t-2\theta\right)}{2}\right]d\left(\omega t\right)}$$

$$I_{TM} = \sqrt{\frac{V_m^2}{4\pi Z^2}\left[\omega t-\frac{\sin\left(2\omega t-2\theta\right)}{2}\right]_{\alpha}^{\pi+\alpha}}$$

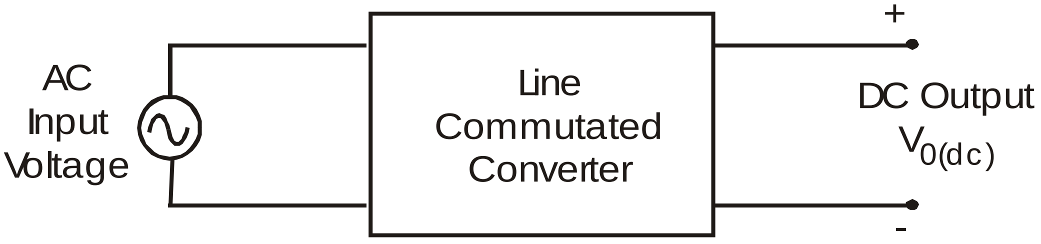
$$I_{TM} = \sqrt{\frac{V_m^2}{4\pi Z^2}\left[\pi+\alpha-\alpha-0\right]}$$

$$I_{TM} = \frac{V_m}{2Z} = \frac{\sqrt{2}\times 220}{2\sqrt{4^2+6^2}} = 21.57277 \text{ Amps}$$

CONTROLLED RECTIFIERS
(Line Commutated AC to DC converters)

INTRODUCTION TO CONTROLLED RECTIFIERS

Controlled rectifiers are line commutated ac to dc power converters which are used to convert a fixed voltage, fixed frequency ac power supply into variable dc output voltage.



Type of input: Fixed voltage, fixed frequency ac power supply.
Type of output: Variable dc output voltage

The input supply fed to a controlled rectifier is ac supply at a fixed rms voltage and at a fixed frequency. We can obtain variable dc output voltage by using controlled rectifiers. By employing phase controlled thyristors in the controlled rectifier circuits we can obtain variable dc output voltage and variable dc (average) output current by varying the trigger angle (phase angle) at which the thyristors are triggered. We obtain a uni-directional and pulsating load current waveform, which has a specific average value.

The thyristors are forward biased during the positive half cycle of input supply and can be turned ON by applying suitable gate trigger pulses at the thyristor gate leads. The thyristor current and the load current begin to flow once the thyristors are triggered (turned ON) say at $\omega t = \alpha$. The load current flows when the thyristors conduct from $\omega t = \alpha$ to β. The output voltage across the load follows the input supply voltage through the conducting thyristor. At $\omega t = \beta$, when the load current falls to zero, the thyristors turn off due to AC line (natural) commutation.

In some bridge controlled rectifier circuits the conducting thyristor turns off, when the other thyristor is (other group of thyristors are) turned ON.

The thyristor remains reverse biased during the negative half cycle of input supply. The type of commutation used in controlled rectifier circuits is referred to AC line commutation or Natural commutation or AC phase commutation.

When the input ac supply voltage reverses and becomes negative during the negative half cycle, the thyristor becomes reverse biased and hence turns off. There are several types of power converters which use ac line commutation. These are referred to as line commutated converters.

Different types of line commutated converters are

- Phase controlled rectifiers which are AC to DC converters.
- AC to AC converters
  - AC voltage controllers, which convert input ac voltage into variable ac output voltage at the same frequency.
  - Cyclo converters, which give low output frequencies.

All these power converters operate from ac power supply at a fixed rms input supply voltage and at a fixed input supply frequency. Hence they use ac line commutation for turning off the thyristors after they have been triggered ON by the gating signals.

DIFFERENCES BETWEEN DIODE RECTIFIERS AND PHASE CONTROLLED RECTIFIERS

The diode rectifiers are referred to as uncontrolled rectifiers which make use of power semiconductor diodes to carry the load current. The diode rectifiers give a fixed dc output voltage (fixed average output voltage) and each diode rectifying element conducts for one half cycle duration (T/2 seconds), that is the diode conduction angle = 180^0 or π radians.

A single phase half wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{V_m}{\pi}$ and single phase full wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{2V_m}{\pi}$, where V_m is maximum value of the available ac supply voltage.

Thus we note that we can not control (we can not vary) the dc output voltage or the average dc load current in a diode rectifier circuit.

In a phase controlled rectifier circuit we use a high current and a high power thyristor device (silicon controlled rectifier; SCR) for conversion of ac input power into dc output power.

Phase controlled rectifier circuits are used to provide a variable voltage output dc and a variable dc (average) load current.

We can control (we can vary) the average value (dc value) of the output load voltage (and hence the average dc load current) by varying the thyristor trigger angle.

We can control the thyristor conduction angle δ from 180^0 to 0^0 by varying the trigger angle α from 0^0 to 180^0, where thyristor conduction angle $\delta = (\pi - \alpha)$

APPLICATIONS OF PHASE CONTROLLED RECTIFIERS

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Reactor controls.
- Portable hand tool drives.
- Variable speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- Uninterruptible power supply systems (UPS).

Some years back ac to dc power conversion was achieved using motor generator sets, mercury arc rectifiers, and thyratorn tubes. The modern ac to dc power converters are designed using high power, high current thyristors and presently most of the ac-dc power converters are thyristorised power converters. The thyristor devices are phase controlled to obtain a variable dc output voltage across the output load terminals. The phase controlled thyristor converter uses ac line commutation (natural commutation) for commutating (turning off) the thyristors that have been turned ON.

The phase controlled converters are simple and less expensive and are widely used in industrial applications for industrial dc drives. These converters are classified as two quadrant converters if the output voltage can be made either positive or negative for a given polarity of output load current. There are also single quadrant ac-dc converters where the output voltage is only positive and cannot be made negative for a given polarity of output current. Of course single quadrant converters can also be designed to provide only negative dc output voltage.

The two quadrant converter operation can be achieved by using fully controlled bridge converter circuit and for single quadrant operation we use a half controlled bridge converter.

CLASSIFICATION OF PHASE CONTROLLED RECTIFIERS

The phase controlled rectifiers can be classified based on the type of input power supply as

- Single Phase Controlled Rectifiers which operate from single phase ac input power supply.
- Three Phase Controlled Rectifiers which operate from three phase ac input power supply.

DIFFERENT TYPES OF SINGLE PHASE CONTROLLED RECTIFIERS

*Single Phase Controlled Rectifier*s are further subdivided into different types

- *Half wave controlled rectifier* which uses a single thyristor device (which provides output control only in one half cycle of input ac supply, and it provides low dc output).

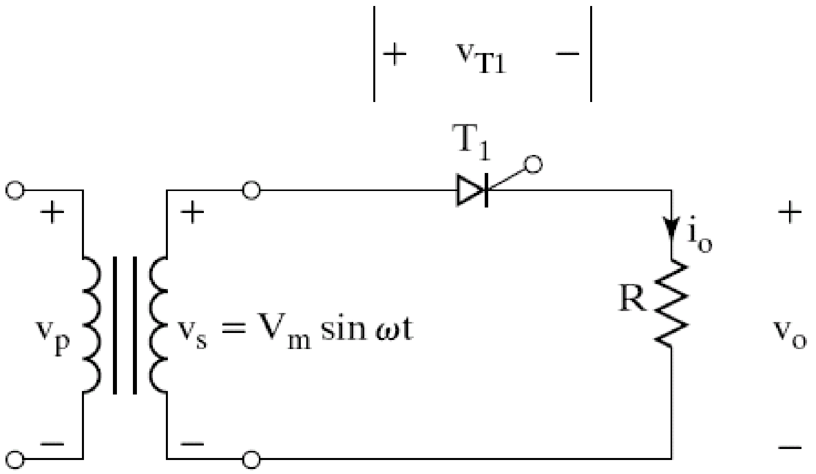
- *Full wave controlled rectifiers* (which provide higher dc output)
  - Full wave controlled rectifier using a center tapped transformer (which requires two thyristors).
  - Full wave bridge controlled rectifiers (which do not require a center tapped transformer)
    - *Single phase semi-converter* (half controlled bridge converter, using two SCR’s and two diodes, to provide single quadrant operation).
    - *Single phase full converter* (fully controlled bridge converter which requires four SCR’s, to provide two quadrant operation).

*Three Phase Controlled Rectifier*s are of different types

- Three phase half wave controlled rectifiers.
- Three phase full wave controlled rectiriers.
  - Semi converter (half controlled bridge converter).
  - Full converter (fully controlled bridge converter).

PRINCIPLE OF PHASE CONTROLLED RECTIFIER OPERATION

The basic principle of operation of a phase controlled rectifier circuit is explained with reference to a single phase half wave phase controlled rectifier circuit with a resistive load shown in the figure.



$R = R_L =$ Load Resistance

Fig.: Single Phase Half-Wave Thyristor Converter with a Resistive Load

A single phase half wave thyristor converter which is used for ac-dc power conversion is shown in the above figure. The input ac supply is obtained from a main supply transformer to provide the desired ac supply voltage to the thyristor converter depending on the output dc voltage required. v_P represents the primary input ac supply voltage. v_S represents the secondary ac supply voltage which is the output of the transformer secondary.

During the positive half cycle of input supply when the upper end of the transformer secondary is at a positive potential with respect to the lower end, the thyristor anode is positive with respect to its cathode and the thyristor is in a forward biased state. The thyristor is triggered at a delay angle of $\omega t = \alpha$, by applying a suitable gate trigger pulse to the gate lead of thyristor. When the thyristor is triggered at a delay angle of $\omega t = \alpha$, the thyristor conducts and assuming an ideal thyristor, the thyristor behaves as a closed switch and the input supply voltage appears across the load when the thyristor conducts from $\omega t = \alpha$ to π radians. Output voltage $v_O = v_S$, when the thyristor conducts from $\omega t = \alpha$ to π.

For a purely resistive load, the load current i_O (output current) that flows when the thyristor T_1 is on, is given by the expression

$$i_O = \frac{v_O}{R_L}, \text{ for } \alpha \le \omega t \le \pi$$

The output load current waveform is similar to the output load voltage waveform during the thyristor conduction time from α to π. The output current and the output voltage waveform are in phase for a resistive load. The load current increases as the input supply voltage increases and the maximum load current flows at $\omega t = \frac{\pi}{2}$, when the input supply voltage is at its maximum value.

The maximum value (peak value) of the load current is calculated as

$$i_{O(\max)} = I_m = \frac{V_m}{R_L}.$$

Note that when the thyristor conducts (T_1 is on) during $\omega t = \alpha$ to π, the thyristor current i_{T1}, the load current i_O through R_L and the source current i_S flowing through the transformer secondary winding are all one and the same.

Hence we can write

$$i_S = i_{T1} = i_O = \frac{v_O}{R} = \frac{V_m \sin \omega t}{R} \text{ ; for } \alpha \le \omega t \le \pi$$

I_m is the maximum (peak) value of the load current that flows through the transformer secondary winding, through T_1 and through the load resistor R_L at the instant $\omega t = \frac{\pi}{2}$, when the input supply voltage reaches its maximum value.

When the input supply voltage decreases the load current decreases. When the supply voltage falls to zero at $\omega t = \pi$, the thyristor and the load current also falls to zero at $\omega t = \pi$. Thus the thyristor naturally turns off when the current flowing through it falls to zero at $\omega t = \pi$.

During the negative half cycle of input supply when the supply voltage reverses and becomes negative during $\omega t = \pi$ to 2π radians, the anode of thyristor is at a negative potential with respect to its cathode and as a result the thyristor is reverse biased and hence it remains cut-off (in the reverse blocking mode). The thyristor cannot conduct during its reverse biased state between $\omega t = \pi$ to 2π. An ideal thyristor under reverse biased condition behaves as an open switch and hence the load current and load voltage are zero during $\omega t = \pi$ to 2π. The maximum or peak reverse voltage that appears across the thyristor anode and cathode terminals is V_m.

The trigger angle α (delay angle or the phase angle α) is measured from the beginning of each positive half cycle to the time instant when the gate trigger pulse is applied. The thyristor conduction angle is from α to π, hence the conduction angle $\delta = (\pi - \alpha)$. The maximum conduction angle is π radians (180^0) when the trigger angle $\alpha = 0$.

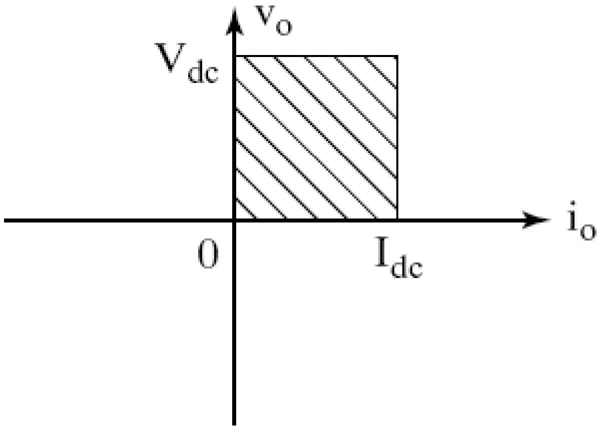


Fig: Quadrant Diagram

The waveforms shows the input ac supply voltage across the secondary winding of the transformer which is represented as v_S, the output voltage across the load, the output (load) current, and the thyristor voltage waveform that appears across the anode and cathode terminals.

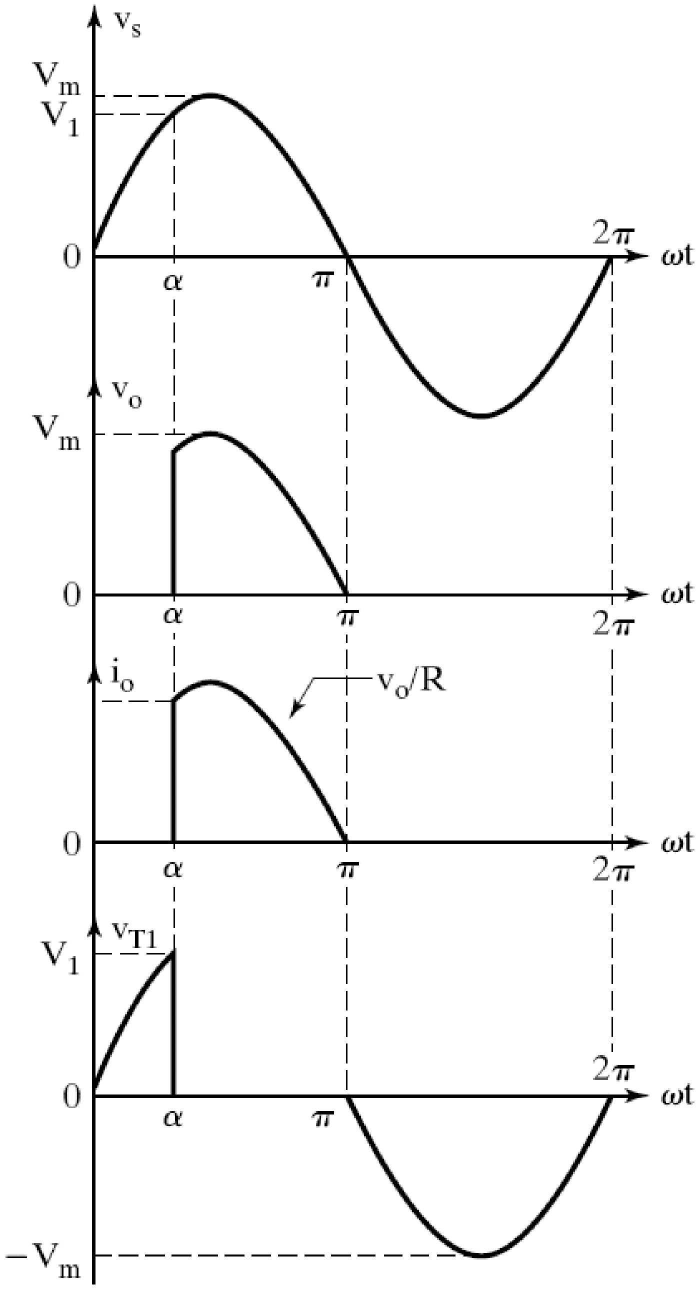


Fig: Waveforms of single phase half-wave controlled rectifier with resistive load

EQUATIONS

$v_s = V_m \sin \omega t =$ the ac supply voltage across the transformer secondary.

$V_m =$ max. (peak) value of input ac supply voltage across transformer secondary.

$V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input ac supply voltage across transformer secondary.

$v_O = v_L =$ the output voltage across the load ; $i_O = i_L =$ output (load) current.

When the thyristor is triggered at $\omega t = \alpha$ (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

$v_O = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to π , when the thyristor is on.

$i_O = i_L = \frac{v_O}{R}$ = Load current for $\omega t = \alpha$ to π , when the thyristor is on.

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE ACROSS THE LOAD

If V_m is the peak input supply voltage, the average output voltage V_{dc} can be found from

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_O . d\left(\omega t\right)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin \omega t . d\left(\omega t\right)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \pi + \cos \alpha\right] \qquad ; \quad \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos \alpha\right] \quad ; \quad V_m = \sqrt{2} V_S$$

The maximum average (dc) output voltage is obtained when $\alpha = 0$ and the maximum dc output voltage $V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$.

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of $180^0 \left(\pi \text{ radians}\right)$.

We can plot the control characteristic, which is a plot of dc output voltage versus the trigger angle α by using the equation for $V_{O(dc)}$.

CONTROL CHARACTERISTIC OF SINGLE PHASE HALF WAVE PHASE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The average dc output voltage is given by the expression

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	**%**	$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}	
30^0	0.933 V_{dm}	93.3 % V_{dm}	
60^0	0.75 V_{dm}	75 % V_{dm}	
90^0	0.5 V_{dm}	50 % V_{dm}	
120^0	0.25 V_{dm}	25 % V_{dm}	
150^0	0.06698 V_{dm}	6.69 % V_{dm}	
180^0	0	0	

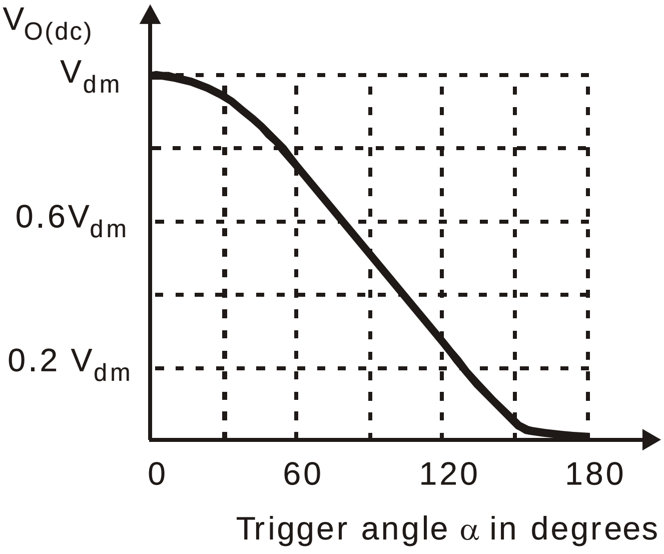


Fig.: Control characteristic

Normalizing the dc output voltage with respect to V_{dm}, the normalized output voltage

$$V_{dcn} = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE OF A SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The rms output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_O^2 .d\left(\omega t\right)\right]$$

Output voltage $v_O = V_m \sin\omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

By substituting $\sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \frac{\left(1-\cos 2\omega t\right)}{2}.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\int_\alpha^{\pi} \left(1-\cos 2\omega t\right).d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\left\{\int_\alpha^{\pi} d\left(\omega t\right) - \int_\alpha^{\pi} \cos 2\omega t.d\left(\omega t\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\left(\pi-\alpha\right) - \frac{\left(\sin 2\pi - \sin 2\alpha\right)}{2}\right)\right]^{\frac{1}{2}} \ ; \ \sin 2\pi = 0$$

Hence we get,

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

PERFORMANCE PARAMETERS OF PHASE CONTROLLED RECTIFIERS

Output dc power (average or dc output power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \;\; ; \text{ i.e., } P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ average or dc value of output (load) voltage.

$I_{O(dc)} = I_{dc} =$ average or dc value of output (load) current.

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \;\; ; \qquad \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The output voltage can be considered as being composed of two components

- The dc component $V_{O(dc)}$ = DC or average value of output voltage.
- The ac component or the ripple component $V_{ac} = V_{r(rms)} =$ RMS value of all the ac ripple components.

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

Therefore

$$V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output } (\text{load}) \text{ voltage}}{\text{DC output } (\text{load}) \text{ voltage}}$$

The Ripple Factor (RF) which is a measure of the ac ripple content in the output voltage waveform. The output voltage ripple factor defined for the output voltage waveform is given by

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

Therefore

$$r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor defined for the output (load) current waveform is given by

$$r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

Some times the peak to peak output ripple voltage is also considered to express the peak to peak output ripple voltage as

$$V_{r(pp)} = \text{ peak to peak ac ripple output voltage}$$

The peak to peak ac ripple load current is the difference between the maximum and the minimum values of the output load current.

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

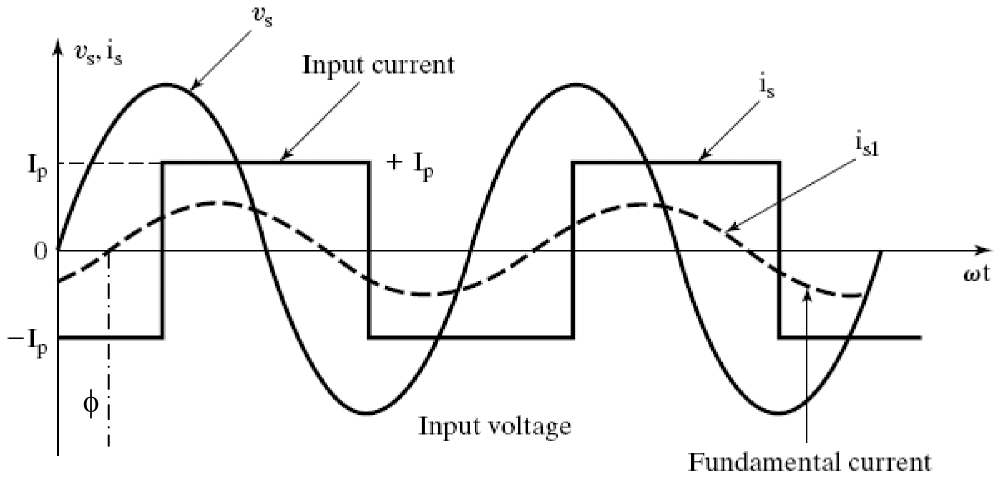
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$V_S =$ RMS value of transformer secondary output voltage (RMS supply voltage at the secondary)

$I_S =$ RMS value of transformer secondary current (RMS line or supply current).



$v_S =$ Supply voltage at the transformer secondary side.

$i_S =$ Input supply current (transformer secondary winding current).

$i_{S1} =$ Fundamental component of the input supply current.

$I_P =$ Peak value of the input supply current.

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of input supply current and the input supply voltage.

$\phi =$ Displacement angle (phase angle)

For an RL load $\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) *or* ***Fundamental Power Factor***

$$DF = Cos\phi$$

Harmonic Factor (HF) or Total Harmonic Distortion Factor (THD)

The harmonic factor is a measure of the distortion in the output waveform and is also referred to as the total harmonic distortion (THD)

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2}\right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the input supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

$FF = 1$; which means that $V_{O(RMS)} = V_{O(dc)}$.

Efficiency $\eta = 100\%$; which means that $P_{O(dc)} = P_{O(ac)}$.

$V_{ac} = V_{r(rms)} = 0$; so that $RF = r_v = 0$; Ripple factor = 0 (ripple free converter).

$TUF = 1$; which means that $P_{O(dc)} = V_S \times I_S$

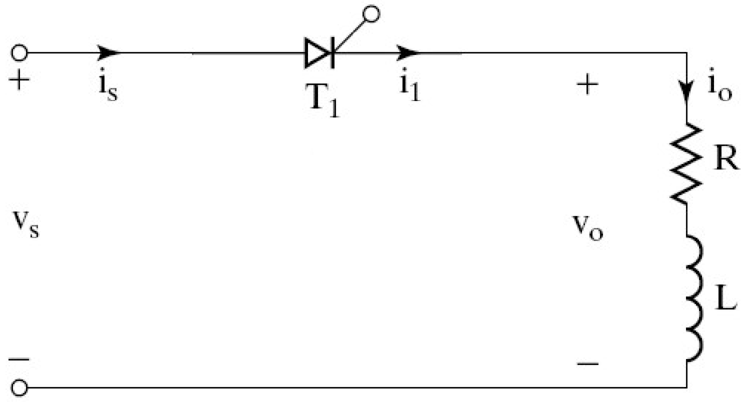
$HF = THD = 0$; which means that $I_S = I_{S1}$

$PF = DPF = 1$; which means that $\phi = 0$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH AN RL LOAD

In this section we will discuss the operation and performance of a single phase half wave controlled rectifier with RL load. In practice most of the loads are of RL type. For example if we consider a single phase controlled rectifier controlling the speed of a dc motor, the load which is the dc motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

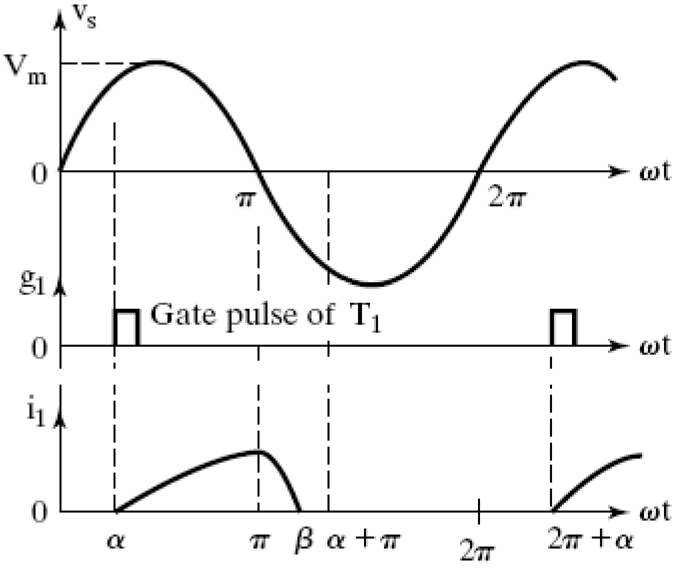
A single phase half wave controlled rectifier circuit with an RL load using a thyristor T_1 (T_1 is an SCR) is shown in the figure below.



The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative. A phase shift appears between the load voltage and the load current waveforms, due to the load inductance.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β . The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ . The waveforms of the input supply voltage, the gate trigger pulse of T_1, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.



$$i_1 = i_O = i_S$$

Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

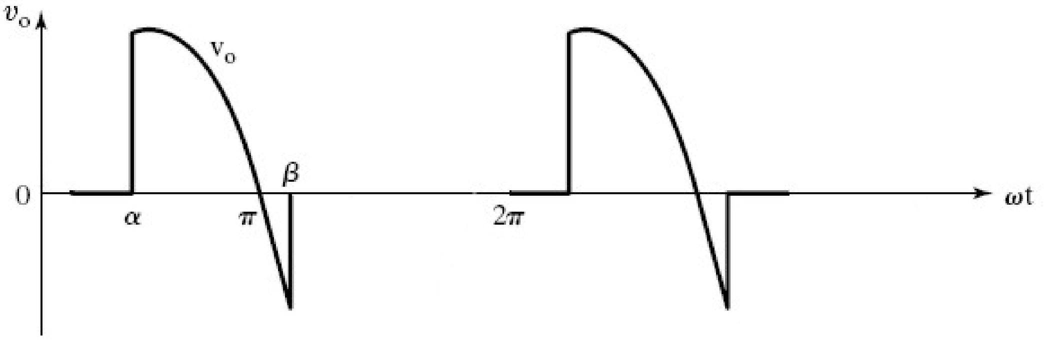


Fig.: Output (load) voltage waveform of a single phase half wave controlled rectifier with RL load

From β to 2π, the thyristor remains cut-off as it is reverse biased and behaves as an open switch. The thyristor current and the load current are zero and the output voltage also remains at zero during the non conduction time interval between β to 2π. In the next cycle the thyristor is triggered again at a phase angle of $\left(2\pi+\alpha\right)$, and the same operation repeats.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t=\alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t=\alpha$. The load current which flows through the thyristor T_1 during $\omega t=\alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right)+Ri_O=V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O=\frac{V_m}{Z}\sin\left(\omega t-\phi\right)+A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m=\sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$Z=\sqrt{R^2+\left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R} =$ Load circuit time constant.

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

Therefore $\quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 from the above equation into the expression for i_O, we obtain

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right] \quad ;$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t-\phi\right)+e^{\frac{-R}{\omega L}(\omega t-\alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha-\phi\right)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

TO CALCULATE EXTINCTION ANGLE β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin\left(\beta-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$, we can write

$$\left[\sin\left(\beta-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin\left(\beta-\phi\right) = \sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = \left(\beta-\alpha\right)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a specific value of β.

Conduction angle $\delta = \left(\beta-\alpha\right)$; for a purely resistive load or for an RL load when the load inductance L is negligible the extinction angle $\beta = \pi$ and the conduction angle $\delta = \left(\pi-\alpha\right)$

Equations

$v_s = V_m \sin\omega t =$ Input supply voltage

$v_O = v_L = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β,

when the thyristor T_1 conducts (T_1 is on).

Expression for the load current (thyristor current): for $\omega t = \alpha$ to β

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t-\phi\right)-\sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

Extinction angle β can be calculated using the equation

$$\sin\left(\beta-\phi\right) = \sin\left(\alpha-\phi\right)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

TO DERIVE AN EXPRESSION FOR AVERAGE (DC) LOAD VOLTAGE

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d\left(\omega t\right)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d\left(\omega t\right)+\int_{\alpha}^{\beta} v_O.d\left(\omega t\right)+\int_{\beta}^{2\pi} v_O.d\left(\omega t\right)\right] ;$$

$v_O = 0$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π ;

$$\therefore \quad V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d\left(\omega t\right)\right]; v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d\left(\omega t\right)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{\beta}\right] = \frac{V_m}{2\pi}\left(\cos\alpha-\cos\beta\right)$$

$$\therefore \quad V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos\alpha-\cos\beta\right)$$

Note: During the period $\omega t = \pi$ to β, we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE

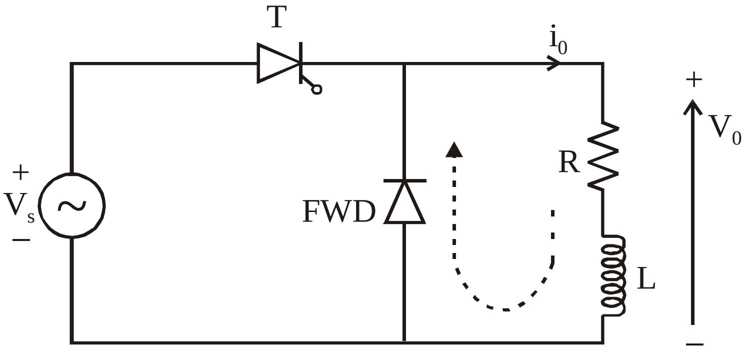
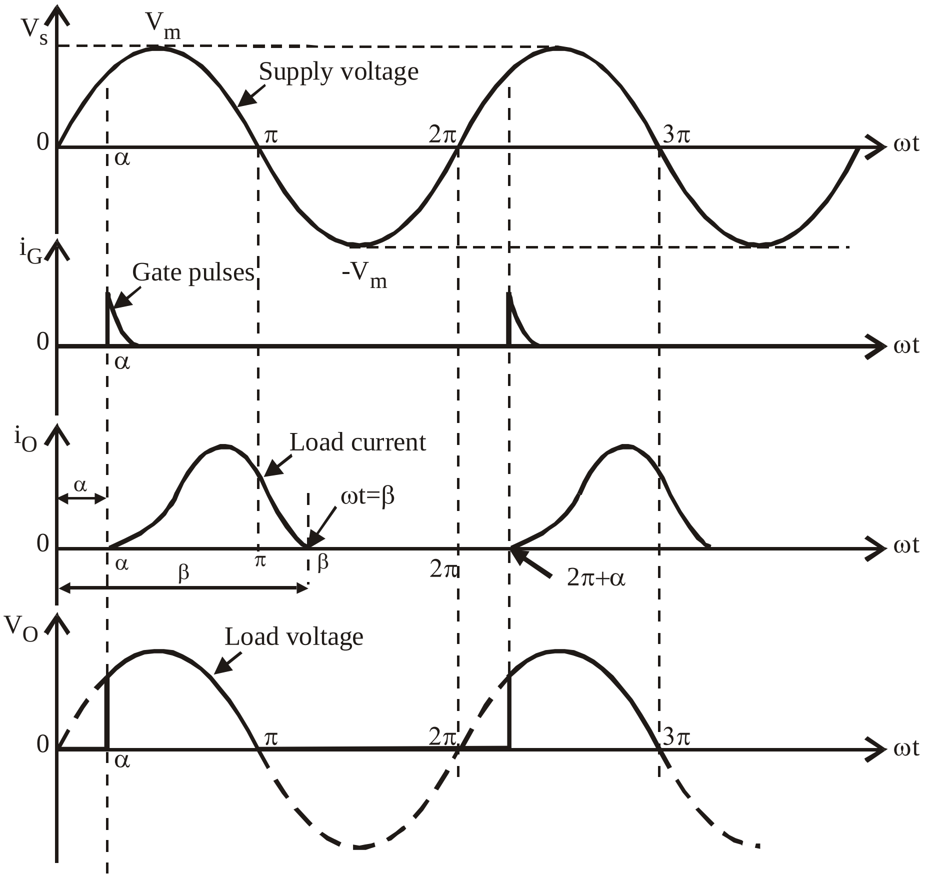


Fig. : Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode (FWD)

With a RL load it was observed that the average output voltage reduces. This disadvantage can be overcome by connecting a diode across the load as shown in figure. The diode is called as a *Free Wheeling Diode (FWD)*. The waveforms are shown below.



At $\omega t = \pi$, the source voltage v_S falls to zero and as v_S becomes negative, the free wheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to β, the load current flows through FWD (free wheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.

Also during this free wheeling time period the load is shorted by the conducting FWD and the load voltage is almost zero, if the forward voltage drop across the conducting FWD is neglected. Thus there is no negative region in the load voltage wave form. This improves the average output voltage.

The average output voltage $V_{dc} = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$, which is the same as that of a purely resistive load. The output voltage across the load appears similar to the output voltage of a purely resistive load.

The following points are to be noted.

- If the inductance value is not very large, the energy stored in the inductance is able to maintain the load current only upto $\omega t = \beta$, where $\pi < \beta < 2\pi$, well before the next gate pulse and the load current tends to become discontinuous.
- During the conduction period α to π, the load current is carried by the SCR and during the free wheeling period π to β, the load current is carried by the free wheeling diode.
- The value of β depends on the value of R and L and the forward resistance of the FWD. Generally $\pi < \beta < 2\pi$.

If the value of the inductance is very large, the load current does not decrease to zero during the free wheeling time interval and the load current waveform appears as shown in the figure.

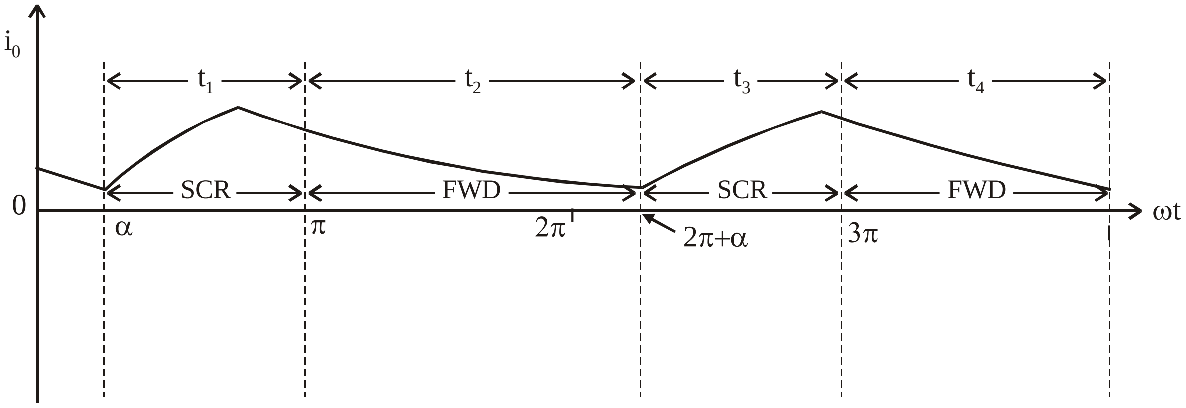


Fig. : Waveform of Load Current in Single Phase Half Wave Controlled Rectifier with a Large Inductance and FWD

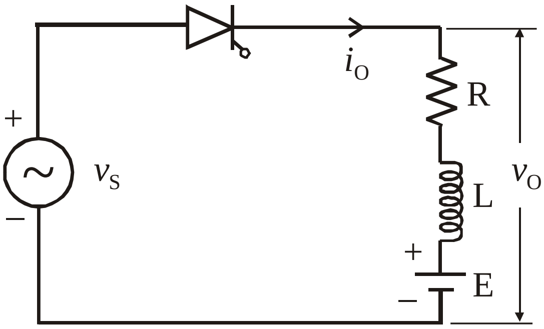
During the periods $t_1, t_3, ...$ the SCR carries the load current and during the periods $t_2, t_4, ...$ the FWD carries the load current.

It is to be noted that

- The load current becomes continuous and the load current does not fall to zero for large value of load inductance.
- The ripple in the load current waveform (the amount of variation in the output load current) decreases.

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH A GENERAL LOAD

A general load consists of R, L and a DC source 'E' in the load circuit



In the half wave controlled rectifier circuit shown in the figure, the load circuit consists of a dc source 'E' in addition to resistance and inductance. When the thyristor is in the cut-off state, the current in the circuit is zero and the cathode will be at a voltage equal to the dc voltage in the load circuit i.e. the cathode potential will be equal to 'E'. The thyristor will be forward biased for anode supply voltage greater than the load dc voltage.

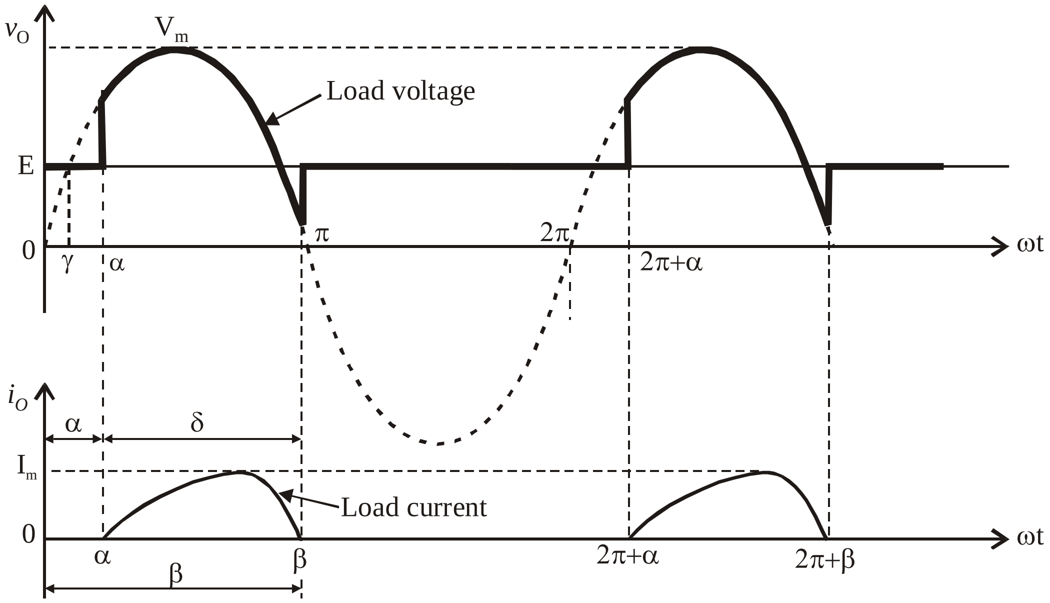
When the supply voltage is less than the dc voltage 'E' in the circuit the thyristor is reverse biased and hence the thyristor cannot conduct for supply voltage less than the load circuit dc voltage.

The value of ωt at which the supply voltage increases and becomes equal to the load circuit dc voltage can be calculated by using the equation $V_m \sin \omega t = E$. If we assume the value of ωt is equal to γ then we can write $V_m \sin \gamma = E$. Therefore γ is calculated as $\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$.

For trigger angle $\alpha < \gamma$, the thyristor conducts only from $\omega t = \gamma$ to β.

For trigger angle $\alpha > \gamma$, the thyristor conducts from $\omega t = \alpha$ to β.

The waveforms appear as shown in the figure



Equations

$$v_S = V_m \sin\omega t = \text{ Input supply voltage .}$$

$$v_O = V_m \sin\omega t = \text{ Output load voltage for } \omega t = \alpha \text{ to } \beta$$

$$v_O = E \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of α, the equation for the circuit can be written as

$$V_m \sin\omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \text{E} \ ; \ \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + \left(\omega L\right)^2} = \text{Load Impedance}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant}$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$$

To find the value of the constant ‘A’ apply the initial condition at $\omega t = \alpha$, load current $i_O = 0$. Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant ‘A’ as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant ‘A’ in the expression for the load current, we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

The Extinction angle β can be calculated from the final condition that the output current $i_O = 0$ at $\omega t = \beta$. By using the above expression we get,

$$i_O = 0 = \frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

To derive an expression for the average or dc load voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_0^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta)\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta)\right] + \frac{E}{2\pi}(2\pi - \beta + \alpha)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi}\right]E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

DISADVANTAGES OF SINGLE PHASE HALF WAVE CONTROLLED RECTIFIERS

Single phase half wave controlled rectifier gives

- Low dc output voltage.
- Low dc output power and lower efficiency.
- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest.

The above disadvantages of a single phase half wave controlled rectifier can be over come by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

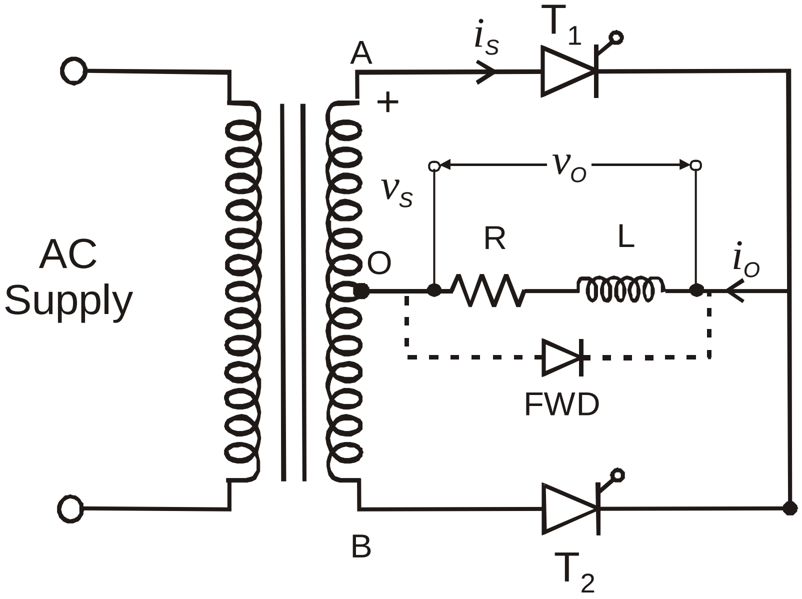
SINGLE PHASE FULL WAVE CONTROLLED RECTIFIERS

Single phase full wave controlled rectifier circuit combines two half wave controlled rectifiers in one single circuit so as to provide two pulse output across the load. Both the half cycles of the input supply are utilized and converted into a uni-directional output current through the load so as to produce a two pulse output waveform. Hence a full wave controlled rectifier circuit is also referred to as a two pulse converter.

Single phase full wave controlled rectifiers are of various types

- Single phase full wave controlled rectifier using a center tapped transformer (two pulse converter with mid point configuration).
- Single phase full wave bridge controlled rectifier
    - Half controlled bridge converter (semi converter).
    - Fully controlled bridge converter (full converter).

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER USING A CENTER TAPPED TRANSFORMER



v_S = Supply Voltage across the upper half of the transformer secondary winding

$$v_S = v_{AO} = V_m \sin \omega t$$

$v_{BO} = -v_{AO} = -V_m \sin \omega t =$ supply voltage across the lower half of the transformer secondary winding.

This type of full wave controlled rectifier requires a center tapped transformer and two thyristors T_1 and T_2. The input supply is fed through the mains supply transformer, the primary side of the transformer is connected to the ac line voltage which is available (normally the primary supply voltage is 230V RMS ac supply voltage at 50Hz supply frequency in India). The secondary side of the transformer has three lines and the center point of the transformer (center line) is used as the reference point to measure the input and output voltages.

The upper half of the secondary winding and the thyristor T_1 along with the load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor T_2 with the common load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

There are two types of operations possible.

- Discontinuous load current operation, which occurs for a purely resistive load or an RL load with low inductance value.
- Continuous load current operation which occurs for an RL type of load with large load inductance.

Discontinuous Load Current Operation (for low value of load inductance)

Generally the load current is discontinuous when the load is purely resistive or when the RL load has a low value of inductance.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point ‘O’ the thyristor T_1 is forward biased and it is triggered at a delay angle of α. The load current flows through the thyristor T_1, through the load and through the upper part of the secondary winding, during the period α to β, when the thyristor T_1 conducts.

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to β. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \beta$, where $\beta > \pi$ for RL type of load and the thyristor T_1 naturally turns off at $\omega t = \beta$.

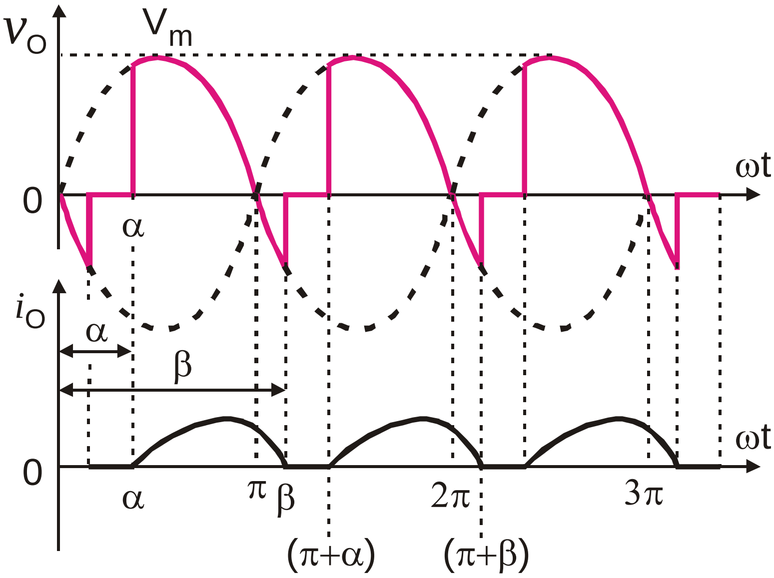


Fig.: Waveform for Discontinuous Load Current Operation without FWD

During the negative half cycle of the input supply the voltage at the supply line 'A' becomes negative whereas the voltage at line 'B' (at the lower side of the secondary winding) becomes positive with respect to the center point 'O'. The thyristor T_2 is forward biased during the negative half cycle and it is triggered at a delay angle of $(\pi + \alpha)$. The current flows through the thyristor T_2, through the load, and through the lower part of the secondary winding when T_2 conducts during the negative half cycle the load is connected to the lower half of the secondary winding when T_2 conducts.

For purely resistive loads when L = 0, the extinction angle $\beta = \pi$. The load current falls to zero at $\omega t = \beta = \pi$, when the input supply voltage falls to zero at $\omega t = \pi$. The load current and the load voltage waveforms are in phase and there is no phase shift between the load voltage and the load current waveform in the case of a purely resistive load.

For low values of load inductance the load current would be discontinuous and the extinction angle $\beta > \pi$ but $\beta < (\pi + \alpha)$.

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor T_1 conducts from α to $(\pi + \alpha)$, until the next thyristor T_2 is triggered. When T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biased and hence T_1 turns off.

TO DERIVE AN EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD (WITHOUT FREE WHEELING DIODE (FWD))

The average or dc output voltage of a full-wave controlled rectifier can be calculated by finding the average value of the output voltage waveform over one output cycle (i.e., π radians) and note that the output pulse repetition time is $\frac{T}{2}$ seconds where T represents the input supply time period and $T = \frac{1}{f}$; where f = input supply frequency.

Assuming the load inductance to be small so that $\beta > \pi$, $\beta < (\pi + \alpha)$ we obtain discontinuous load current operation. The load current flows through T_1 form $\omega t = \alpha$ to β, where α is the trigger angle of thyristor T_1 and β is the extinction angle where the load current through T_1 falls to zero at $\omega t = \beta$. Therefore the average or dc output voltage can be obtained by using the expression

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi} \int_{\omega t = \alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

Therefore $V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$**, for discontinuous load current operation,** $\pi < \beta < (\pi + \alpha)$**.**

When the load inductance is small and negligible that is $L \approx 0$, the extinction angle $\beta = \pi$ radians. Hence the average or dc output voltage for resistive load is obtained as

$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \pi) \quad ; \ \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - (-1))$$

CYCLOCONVERTERS

In industrial applications, two forms of electrical energy are used: direct current (dc) and alternating current (ac). Usually constant voltage constant frequency single-phase or three-phase ac is readily available. However, for different applications, different forms, magnitudes and/or frequencies are required. There are four different conversions between dc and ac power sources. These conversions are done by circuits called power converters. The converters are classified as:

1-rectifiers: from single-phase or three-phase ac to variable voltage dc

2-choppers: from dc to variable voltage dc

3-inverters: from dc to variable magnitude and variable frequency, single-phase or three-phase ac

4-cycloconverters: from single-phase or three-phase ac to variable magnitude and variable frequency, single-phase or three-phase ac

The first three classes are explained in other articles. This article explains what cycloconverters are, their types, how they operate and their applications.

Traditionally, ac-ac conversion using semiconductor switches is done in two different ways: 1- in two stages (ac-dc and then dc-ac) as in dc link converters or 2- in one stage (ac-ac) cycloconverters (Fig. 1). Cycloconverters are used in high power applications driving induction and synchronous motors. They are usually phase-controlled and they traditionally use thyristors due to their ease of phase commutation.

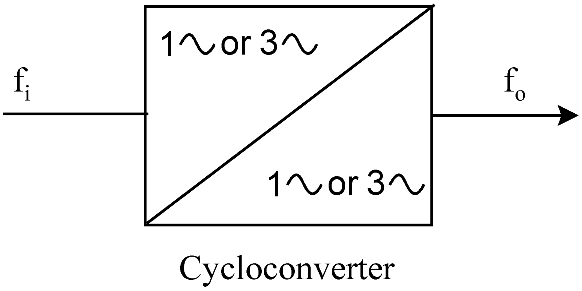


Fig.1 Block diagram of a cycloconverter

There are other newer forms of cycloconversion such as ac-ac matrix converters and high frequency ac-ac (hfac-ac) converters and these use self-controlled switches. These converters, however, are not popular yet.

Some applications of cycloconverters are:

- Cement mill drives
- Ship propulsion drives
- Rolling mill drives
- Scherbius drives
- Ore grinding mills
- Mine winders

1.Operation Principles:

The following sections will describe the operation principles of the cycloconverter starting from the simplest one, single-phase to single-phase (1ϕ-1ϕ) cycloconverter.

1.1. Single-phase to Single-phase (1ϕ-1ϕ) Cycloconverter:

To understand the operation principles of cycloconverters, the single-phase to single-phase cycloconverter (Fig. 2) should be studied first. This converter consists of back-to-back connection of two full-wave rectifier circuits. Fig 3 shows the operating waveforms for this converter with a resistive load.

The input voltage, v_s is an ac voltage at a frequency, f_i as shown in Fig. 3a. For easy understanding assume that all the thyristors are fired at $\alpha=0°$ firing angle, i.e. thyristors act like diodes. Note that the firing angles are named as α_P for the positive converter and α_N for the negative converter.

Consider the operation of the cycloconverter to get one-fourth of the input frequency at the output. For the first two cycles of $v_{s,}$ the positive converter operates supplying current to the load. It rectifies the input voltage; therefore, the load sees 4 positive half cycles as seen in Fig. 3b. In the next two cycles, the negative converter operates supplying current to the load in the reverse direction. The current waveforms are not shown in the figures because the resistive load current will have the same waveform as the voltage but only scaled by the resistance. Note that when one of the converters operates the other one is disabled, so that there is no current circulating between the two rectifiers.

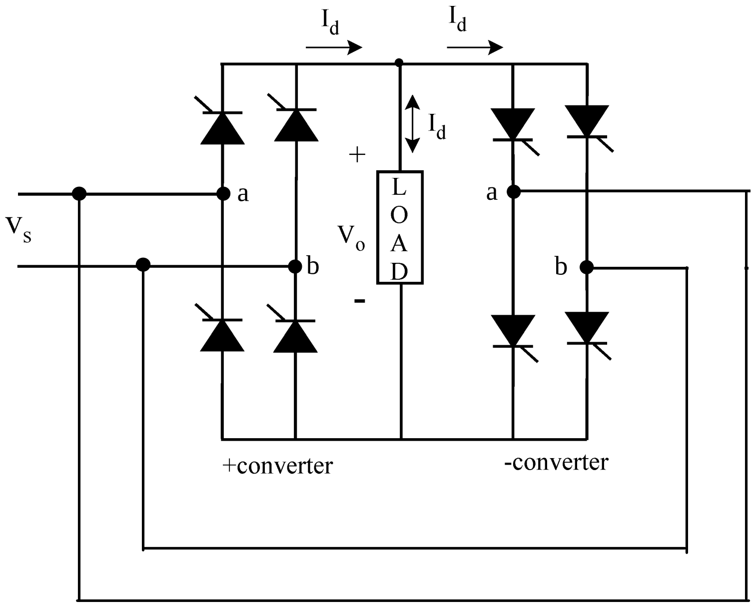


Fig. 2 Single-phase to single-phase cycloconverter

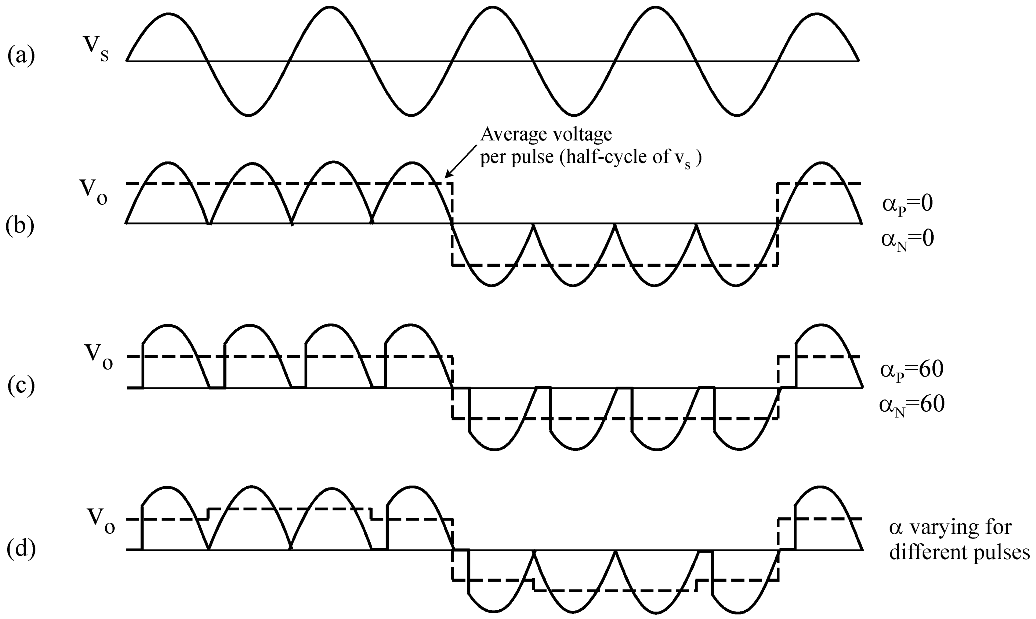


Fig. 3 Single-phase to single-phase cycloconverter waveforms
a) input voltage
b) output voltage for zero firing angle
c) output voltage with firing angle π/3 rad.
d) output voltage with varying firing angle

The frequency of the output voltage, v_o in Fig. 3b is 4 times less than that of v_s, the input voltage, i.e. $f_o/f_i=1/4$. Thus, this is a step-down cycloconverter. On the other hand, cycloconverters that have $f_o/f_i>1$ frequency relation are called step-up cycloconverters. Note that step-down cycloconverters are more widely used than the step-up ones.

The frequency of v_o can be changed by varying the number of cycles the positive and the negative converters work. It can only change as integer multiples of f_i in 1ϕ-1ϕ cycloconverters.

With the above operation, the 1ϕ-1ϕ cycloconverter can only supply a certain voltage at a certain firing angle α. The dc output of each rectifier is:

$$V_d = \frac{2\sqrt{2}}{p} V \cos a \qquad (1)$$

where V is the input rms voltage.

The dc value per half cycle is shown as dotted in Fig. 3d.

Then the peak of the fundamental output voltage is

$$v_{o_1}(t) = \frac{4}{p}\frac{2\sqrt{2}}{p} V \cos a \qquad (2)$$

Equation 2 implies that the fundamental output voltage depends on α. For α=0°, $V_{0_1} = V_{do} \times 1 = V_{do}$ where $V_{do} = \frac{4}{p}\frac{2\sqrt{2}}{p} V$. If α is increased to π/3 as in Fig. 3d, then $V_{o_1} = V_{do} \times 0.5$. Thus varying α, the fundamental output voltage can be controlled.

Constant α operation gives a crude output waveform with rich harmonic content. The dotted lines in Fig. 3b and c show a square wave. If the square wave can be modified to look more like a sine wave, the harmonics would be reduced. For this reason α is modulated as shown in Fig. 3d. Now, the six-stepped dotted line is more like a sinewave with fewer harmonics. The more pulses there are with different α's, the less are the harmonics.

1.2. Three-Phase to Single-Phase (3ϕ-1ϕ) Cycloconverter:

There are two kinds of three-phase to single-phase (3ϕ-1ϕ) cycloconverters: 3ϕ-1ϕ half-wave cycloconverter (Fig. 4) and 3ϕ-1ϕ bridge cycloconverter (Fig. 5). Like the 1ϕ-1ϕ case, the 3ϕ-1ϕ cycloconverter applies rectified voltage to the load. Both positive and negative converters can generate voltages at either polarity, but the positive converter can only supply positive current and the negative converter can only supply negative current. Thus, the cycloconverter can operate in four quadrants: (+v, +i) and (-v, -i) rectification modes and (+v, -i) and (-v, +i) inversion modes. The modulation of the output voltage and the fundamental output voltage are shown in Fig. 6. Note that α is sinusoidally modulated over the cycle to generate a harmonically optimum output voltage.

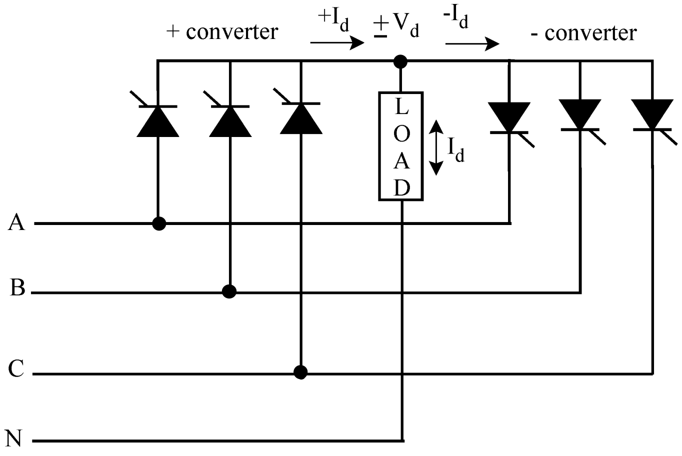


Fig. 4 3ϕ-1ϕ half-wave cycloconverter

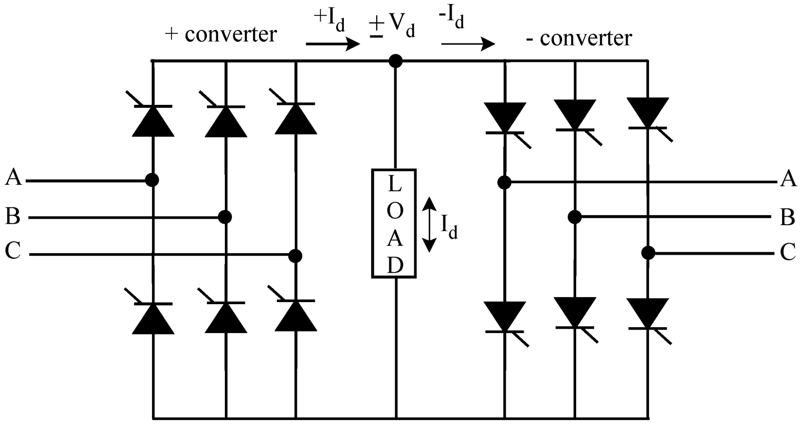


Fig. 5 3ϕ-1ϕ bridge cycloconverter

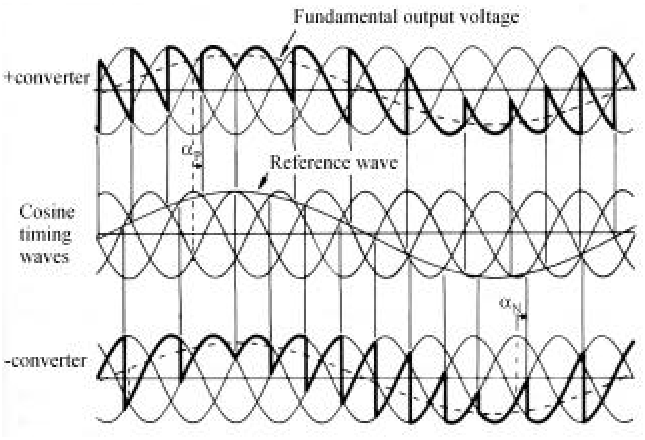


Fig. 6 3ϕ-1ϕ half-wave cycloconverter waveforms
a) + converter output voltage
b) cosine timing waves
c) – converter output voltage

The polarity of the current determines if the positive or negative converter should be supplying power to the load. Conventionally, the firing angle for the positive converter is named α_P, and that of the negative converter is named α_N. When the polarity of the current changes, the converter previously supplying the current is disabled and the other one is enabled. The load always requires the fundamental voltage to be continuous. Therefore, during the current polarity reversal, the average voltage supplied by both of the converters should be equal. Otherwise, switching from one converter to the other one would cause an undesirable voltage jump. To prevent this problem, the converters are forced to produce the same average voltage at all times. Thus, the following condition for the firing angles should be met.

$$a_P + a_N = p \qquad (3)$$

The fundamental output voltage in Fig. 6 can be given as:

$$v_{o_1}(t) = \sqrt{2}V_o \sin w_o t \qquad (4)$$

where V_o is the rms value of the fundamental voltage

At a time t_o the output fundamental voltage is

$$v_{o_1}(t_o) = \sqrt{2}V_o \sin w_o t_o \qquad (5)$$

The positive converter can supply this voltage if α_P satisfies the following condition.

$$v_{o_1}(t_o) = \sqrt{2}V_o \sin w_o t_o = V_{do} \cos a_P \qquad (6)$$

where $V_{do} = \sqrt{2}V_o \frac{p}{p}\sin\frac{p}{p}$ (p=3 for half wave converter and 6 for bridge converter)

From the α condition (3)

$$v_{o_1} = V_{do}\cos a_P = -V_{do}\sin a_N \qquad (7)$$

The firing angles at any instant can be found from (6) and (7).

The operation of the 3ϕ-1ϕ bridge cycloconverter is similar to the above 3ϕ-1ϕ half-wave cycloconverter. Note that the pulse number for this case is 6.

1.3 Three-Phase to Three-Phase (3ϕ-3ϕ) Cycloconverter:

If the outputs of 3 3ϕ-1ϕ converters of the same kind are connected in wye or delta and if the output voltages are 2π/3 radians phase shifted from each other, the resulting converter is a three-phase to three-phase (3ϕ-3ϕ) cycloconverter. The resulting cycloconverters are shown in Figs. 7 and 8 with wye connections. If the three converters connected are half-wave converters, then the new converter is called a 3ϕ-3ϕ half-wave cycloconverter. If instead, bridge converters are used, then the result is a 3ϕ-3ϕ bridge cycloconverter. 3ϕ-3ϕ half-wave cycloconverter is also called a 3-pulse cycloconverter or an 18-thyristor cycloconverter. On the other hand, the 3ϕ-3ϕ bridge cycloconverter is also called a 6-pulse cycloconverter or a 36-thyristor cycloconverter. The operation of each phase is explained in the previous section.

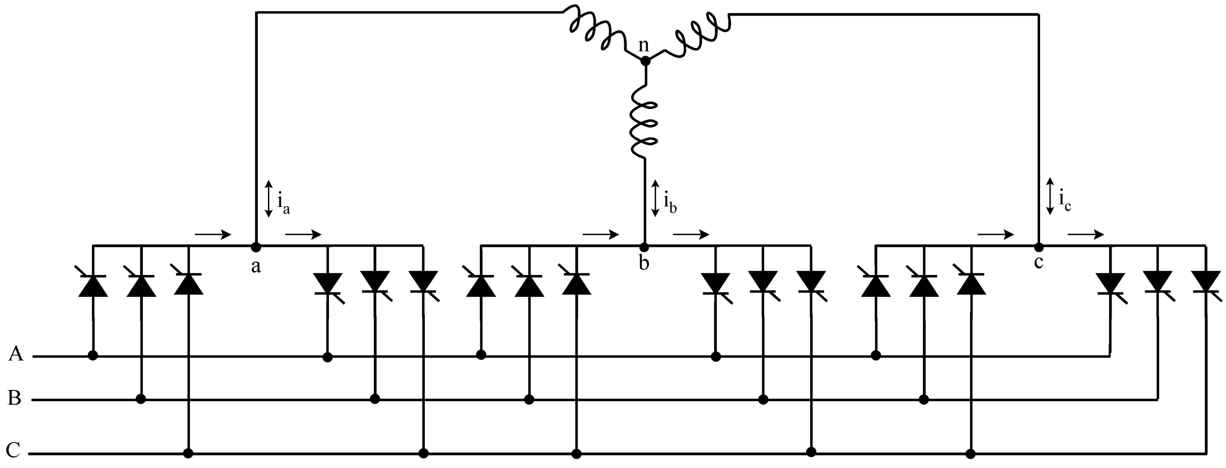


Fig. 7 3ϕ-3ϕ half-wave cycloconverter

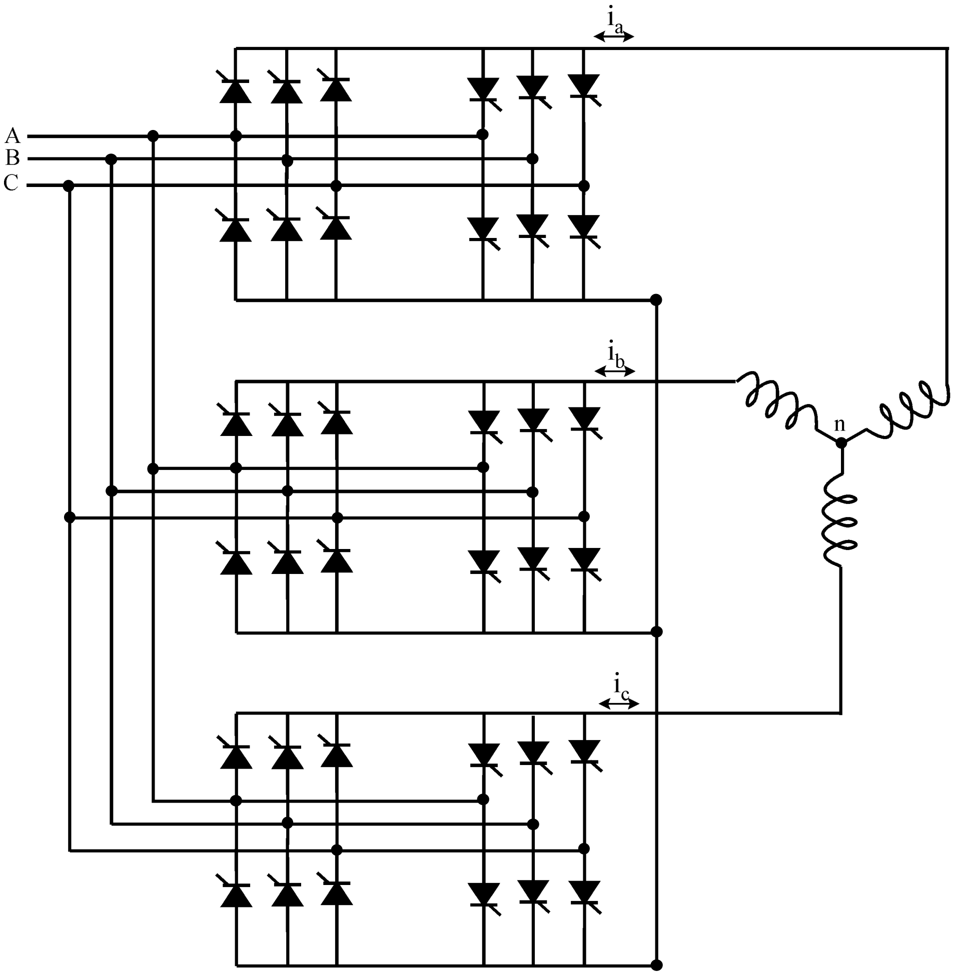


Fig. 8 3ϕ-3ϕ bridge cycloconverter

The three-phase cycloconverters are mainly used in ac machine drive systems running three-phase synchronous and induction machines. They are more advantageous when used with a synchronous machine due to their output power factor characteristics. A cycloconverter can supply lagging, leading, or unity power factor loads while its input is always lagging. A synchronous machine can draw any power factor current from the converter. This characteristic operation matches the cycloconverter to the synchronous machine. On the other hand, induction machines can only draw lagging current, so the cycloconverter does not have an edge compared to the other converters in this aspect for running an induction machine. However, cycloconverters are used in Scherbius drives for speed control purposes driving wound rotor induction motors.

Cycloconverters produce harmonic rich output voltages, which will be discussed in the following sections. When cycloconverters are used to run an ac machine, the leakage inductance of the machine filters most of the higher frequency harmonics and reduces the magnitudes of the lower order harmonics.

2. Blocked Mode and Circulating Current Mode:

The operation of the cycloconverters is explained above in ideal terms. When the load current is positive, the positive converter supplies the required voltage and the negative converter is disabled. On the other hand, when the load current is negative, then the negative converter supplies the required voltage and the positive converter is blocked. This operation is called the blocked mode operation, and the cycloconverters using this approach are called blocking mode cycloconverters.

However, if by any chance both of the converters are enabled, then the supply is short-circuited. To avoid this short circuit, an intergroup reactor (IGR) can be connected between the converters as shown in Fig. 9. Instead of blocking the converters during current reversal, if they are both enabled, then a circulating current is produced. This current is called the circulating current. It is unidirectional because the thyristors allow the current to flow in only one direction. Some cycloconverters allow this circulating current at all times. These are called circulating current cycloconverters.

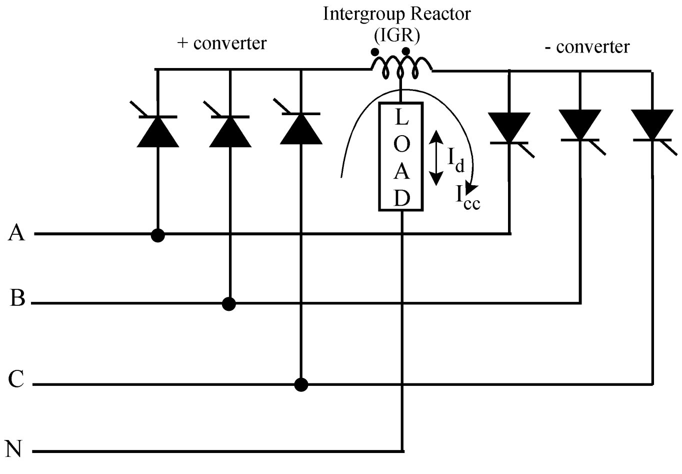


Fig. 9 Circulating current and IGR

2.1 Blocking Mode Cycloconverters:

The operation of these cycloconverters was explained briefly before. They do not let circulating current flow, and therefore they do not need a bulky IGR. When the current goes to zero, both positive and negative converters are blocked. The converters stay off for a short delay time to assure that the load current ceases. Then, depending on the polarity, one of the converters is enabled. With each zero crossing of the current, the converter, which was disabled before the zero crossing, is enabled. A toggle flip-flop, which toggles when the current goes to zero, can be used for this purpose. The operation waveforms for a three-pulse blocking mode cycloconverter are given in Fig. 10.

The blocking mode operation has some advantages and disadvantages over the circulating mode operation. During the delay time, the current stays at zero distorting the voltage and current waveforms. This distortion means complex harmonics patterns compared to the circulating mode cycloconverters. In addition to this, the current reversal problem brings more control complexity. However, no bulky IGRs are used, so the size and cost is less than that of the circulating current case. Another advantage is that only one converter is in conduction at all times rather than two. This means less losses and higher efficiency.

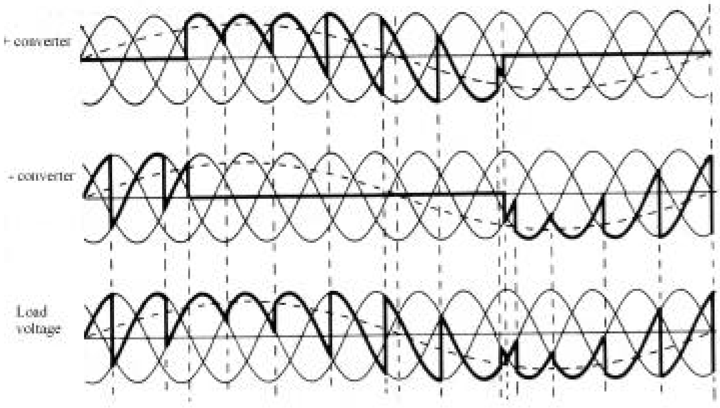


Fig. 10 Blocking mode operation waveforms
a) + converter output voltage
b) – converter output voltage
c) load voltage

2.2 Circulating Current Cycloconverters:

In this case, both of the converters operate at all times producing the same fundamental output voltage. The firing angles of the converters satisfy the firing angle condition (Eq. 3), thus when one converter is in rectification mode the other one is in inversion mode and vice versa. If both of the converters are producing pure sine waves, then there would not be any circulating current because the instantaneous potential difference between the outputs of the converters would be zero. In reality, an IGR is connected between the outputs of two phase controlled converters (in either rectification or inversion mode). The voltage waveform across the IGR can be seen in Fig. 11d. This is the difference of the instantaneous output voltages produced by the two converters. Note that it is zero when both of the converters produce the same instantaneous voltage. The center tap voltage of IGR is the voltage applied to the load and it is the mean of the voltages applied to the ends of IGR, thus the load voltage ripple is reduced.

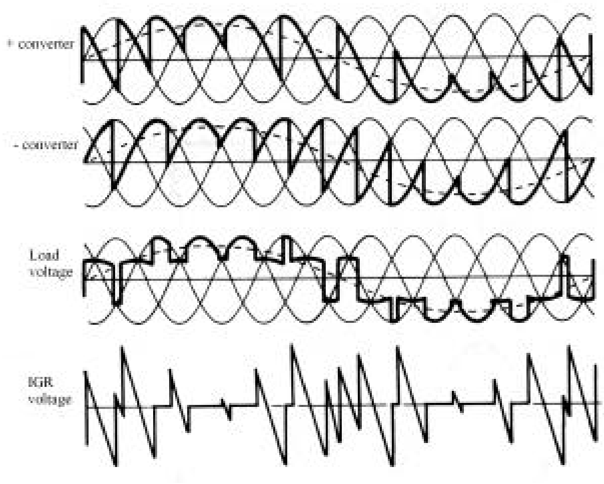


Fig. 11 Circulating mode operation waveforms
a) + converter output voltage
b) – converter output voltage
c) load voltage
d) IGR voltage

The circulating current cycloconverter applies a smoother load voltage with less harmonics compared to the blocking mode case. Moreover, the control is simple because there is no current reversal problem. However, the bulky IGR is a big disadvantage for this converter. In addition to this, the number of devices conducting at any time is twice that of the blocking mode converter. Due to these disadvantages, this cycloconverter is not attractive.

The blocked mode cycloconverter converter and the circulating current cycloconverter can be combined to give a hybrid system, which has the advantages of both. The resulting cycloconverter looks like a circulating mode cycloconverter circuit, but depending on the polarity of the output current only one converter is enabled and the other one is disabled as with the blocking mode cycloconverters. When the load current decreases below a threshold, both of the converters are enabled. Thus, the current has a smooth reversal. When the current increases above a threshold in the other direction, the outgoing converter is disabled. This hybrid cycloconverter operates in the blocking mode most of the time so a smaller IGR can be used. The efficiency is slightly higher than that of the circulating current cycloconverter but much less than the blocking mode cycloconverter. Moreover, the distortion caused by the blocking mode operation disappears due to the circulating current operation around zero current. Moreover, the control of the converter is still less complex than that of the blocking mode cycloconverter.

3. Output and Input Harmonics:

The cycloconverter output voltage waveforms have complex harmonics. Higher order harmonics are usually filtered by the machine inductance, therefore the machine current has less harmonics. The remaining harmonics cause harmonic losses and torque pulsations. Note that in a cycloconverter, unlike other converters, there are no inductors or capacitors, i.e. no storage devices. For this reason, the instantaneous input power and the output power are equal.

There are several factors effecting the harmonic content of the waveforms. Blocking mode operation produces more complex harmonics than circulating mode of operation due to the zero current distortion. In addition to this, the pulse number effects the harmonic content. A greater number of pulses has less harmonic content. Therefore, a 6-pulse (bridge) cycloconverter produces less harmonics than a 3-pulse (half-wave) cycloconverter. Moreover, if the output frequency gets closer to the input frequency, the harmonics increase. Finally, low power factor and discontinuous conduction, both contribute to harmonics.

For a typical p-pulse converter, the order of the input harmonics is "$pn \pm 1$" and that of the output harmonics is "pn", where p is the pulse number and n is an integer. Thus for a 3-pulse converter the input harmonics are at frequencies $2f_i$, $4f_i$ for n=1, $5f_i$, $7f_i$ for n=2, and so on. The output harmonics, on the other hand, are at frequencies $3f_i$, $6f_i$, …

The firing angle, α, in cycloconverter operation is sinusoidally modulated. The modulation frequency is the same as the output frequency and sideband harmonics are induced at the output. Therefore, the output waveform is expected to have harmonics at frequencies related to both the input and output frequencies.

For blocking mode operation, the output harmonics are found at "$pnf_i \pm Nf_o$", where N is an integer and $pn \pm N$=odd condition is satisfied. Then the output harmonics for a 3-pulse cycloconverter in blocking mode will be found at frequencies
n=1 $3f_i$, $3f_i \pm 2f_o$, $3f_i \pm 4f_o$, $3f_i \pm 6f_o$, $3f_i \pm 8f_o$, $3f_i \pm 10f_o$ …
n=2 $6f_i$, $6f_i \pm 1f_o$, $6f_i \pm 3f_o$, $6f_i \pm 5f_o$, $6f_i \pm 7f_o$, $6f_i \pm 9f_o$ …
n=3 $9f_i$, $9f_i \pm 2f_o$, $9f_i \pm 4f_o$, $9f_i \pm 6f_o$, $9f_i \pm 8f_o$, $9f_i \pm 10f_o$, …
n=4, 5,…
Some of the above harmonics might coincide to frequencies below f_i. These are called subharmonics. They are highly unwanted harmonics because the machine inductance cannot filter these.

For the circulating mode operation, the harmonics are at the same frequencies as the blocking mode, but N is limited to (n+1). Thus, the output harmonics for a 3-pulse cycloconverter in circulating mode will be found at frequencies
n=1 $3f_i$, $3f_i \pm 2f_o$, $3f_i \pm 4f_o$
n=2 $6f_i \pm 1f_o$, $6f_i \pm 3f_o$, $6f_i \pm 5f_o$, $6f_i \pm 7f_o$
n=3 $9f_i$, $9f_i \pm 2f_o$, $9f_i \pm 4f_o$, $9f_i \pm 6f_o$, $9f_i \pm 8f_o$, $9f_i \pm 10f_o$
n=4, 5,…

With N limited in the circulating mode, there are fewer subharmonics expected. According to calculations done in [1], subharmonics in this mode exist for $f_o/f_i>0.6$. For the blocking mode, [1] states that the subharmonics exist for $f_o/f_i>0.2$.

The output voltage of a cycloconverter has many complex harmonics, but the output current is smoother due to heavy machine filtering. The input voltages of a cycloconverter are sinusoidal voltages. As stated before the instantaneous output and input powers of a cycloconverter are balanced because it does not have any storage devices. To maintain this balance on the input side with sinusoidal voltages, the input current is expected to have complex harmonic patterns. Thus as expected, the input current harmonics are at frequencies "$(pn\pm1)f_i\pm Mf_o$" where M is an integer and $(pn+1)\pm M$=odd condition is satisfied. Thus, a 3-pulse cycloconverter has input current harmonics at the following frequencies:

n=0 f_i, $f_i\pm6f_o$, $f_i\pm12f_o$, …
n=1 $2f_i\pm3f_o$, $2f_i\pm9f_o$, $2f_i\pm15f_o$ …
$4f_i\pm3f_o$, $4f_i\pm9f_o$, $4f_i\pm15f_o$,…
n=2, 3,…

4. Newer Types of Cycloconverters:

4.1 Matrix Converter:

The matrix converter is a fairly new converter topology, which was first proposed in the beginning of the 1980s. A matrix converter consists of a matrix of 9 switches connecting the three input phases to the three output phases directly as shown in Fig. 12. Any input phase can be connected to any output phase at any time depending on the control. However, no two switches from the same phase should be on at the same time, otherwise this will cause a short circuit of the input phases. These converters are usually controlled by PWM to produce three-phase variable voltages at variable frequency.

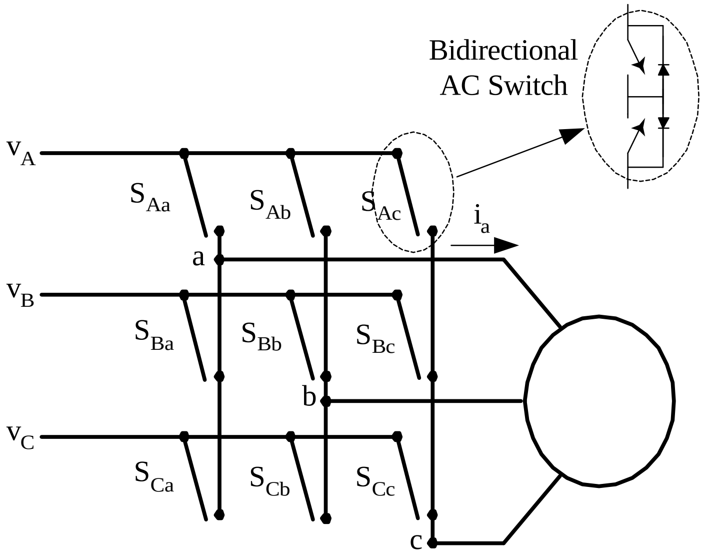


Fig. 12 Matrix converter

This direct frequency changer is not commonly used because of the high device count, i.e. 18 switches compared to 12 of a dc link rectifier-inverter system. However, the devices used are smaller because of their shorter ON time compared to the latter.

4.2 Single-Phase to Three-Phase (1ϕ-3ϕ) Cycloconverters:

Recently, with the decrease in the size and the price of power electronics switches, single-phase to three-phase cycloconverters (1ϕ-3ϕ) started drawing more research interest. Usually, an H-bridge inverter produces a high frequency single-phase voltage waveform, which is fed to the cycloconverter either through a high frequency transformer or not. If a transformer is used, it isolates the inverter from the cycloconverter. In addition to this, additional taps from the transformer can be used to power other converters producing a high frequency ac link. The single-phase high frequency ac (hfac) voltage can be either sinusoidal or trapezoidal. There might be zero voltage intervals for control purposes or zero voltage commutation. Fig. 13 shows the circuit diagram of a typical hfac link converter. These converters are not commercially available yet. They are in the research state.

Among several kinds, only two of them will be addressed here:

4.2.1 Integral Pulse Modulated (1ϕ-3ϕ) Cycloconverters [4]:

The input to these cycloconverters is single-phase high frequency sinusoidal or square waveforms with or without zero voltage gaps. Every half-cycle of the input signal, the control for each phase decides if it needs a positive pulse or a negative pulse using integral pulse modulation. For integral pulse modulation, the command signal and the output phase voltage are integrated and the latter result is subtracted from the former. For a positive difference, a negative pulse is required, and vice versa for the negative difference. For the positive (negative) input half-cycle, if a positive pulse is required, the upper (lower) switch is turned on; otherwise, the lower (upper) switch is turned on.

Therefore, the three-phase output voltage consists of positive and negative half-cycle pulses of the input voltage. Note that this converter can only work at output frequencies which are multiples of the input frequency.

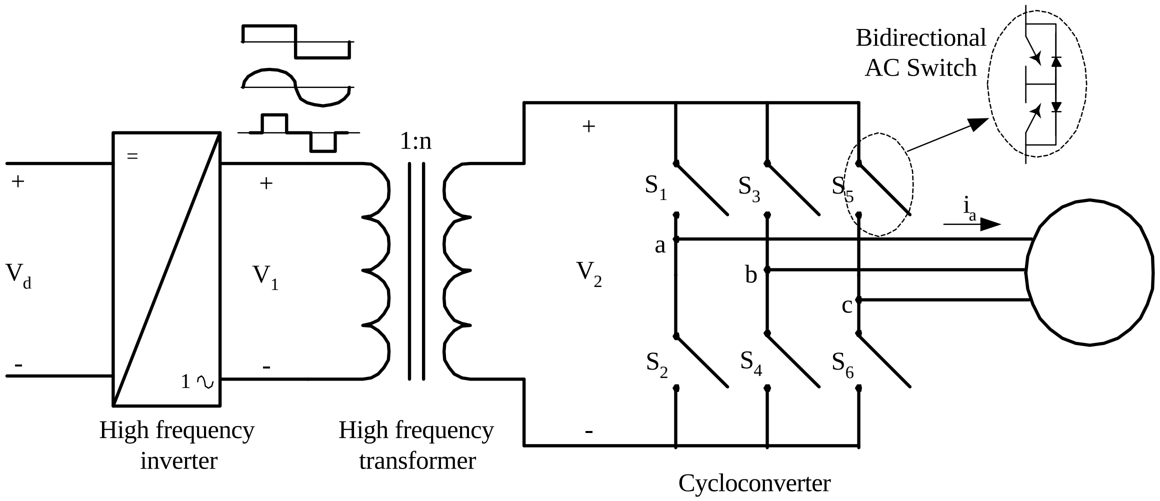


Fig. 13 High frequency ac link converter (1ϕ hf inverter + (1ϕ-3ϕ) Cycloconverter)

4.2.2 Phase-Controlled (1ϕ-3ϕ) Cycloconverter [5]:

This cycloconverter converts the single-phase high frequency sinusoidal or square wave voltage into three-phase voltages using the previously explained phase control principles. The voltage command is compared to a sawtooth waveform to find the firing instant of the switches. Depending on the polarity of the current and the input voltage, the next switch to be turned on is determined. Compared to the previous one, this converter has more complex control but it can work at any frequency.

5. Summary:

Cycloconverters are widely used in industry for ac-to-ac conversion. With recent device advances, newer forms of cycloconversion are being developed. These newer forms are drawing more research interest.

In this article, the most commonly known cycloconverter schemes are introduced, and their operation principles are discussed. For more detailed information, the following references can be used.

MODULE 6

INTRODUCTION TO DRIVE SYSTEMS

Drives: Systems employed for motion control are called drives. Motion control is required in industrial as well as domestic applications like transportation system, rolling mills, paper mills, textile mills, machine tools, fans, pumps, robots, washing machines etc. Motion control may be translational, rotational or combination of both. Generally, a drive system is basically has a mechanical load, a transmission system and a prime mover. The prime mover may be I.C. engine, steam engine, turbine or electric motors. However, electric motors are predominantly used employed as prime mover due to certain advantages.

Advantages of Electric Drives:

- Flexible control characteristics.
- Starting and braking is easy and simple
- Provides a wide range of torques over a wide range of speeds (both ac and dc motor)
- Availability of wide range of electric power
- Works to almost any type of environmental conditions
- No exhaust gases emitted
- Capable of operating in all 4 quadrants of torque –speed plane
- Can be started and accelerated at very short time

Choice of Electrical Drives:

The choice of an electrical drive depends on a number of factors. Some important factors are:

- Steady state operation requirements: (nature of speed-torque characteristics, speed regulation, speed range, efficiency, duty cycle, quadrants of operation, speed fluctuations, rating etc)
- Transient operation requirement(values of acceleration and deceleration, starting, braking, speed reversing)
- Requirement of sources:(types of source, its capacity, magnitude of voltage, power factor, harmonics etc)
- Capital and running cost, maintenance needs, life periods
- Space and weight restrictions
- Environment and location
- Reliability

Basic Elements of the Electric Drive Systems:

Functional Block of Electric Drive System

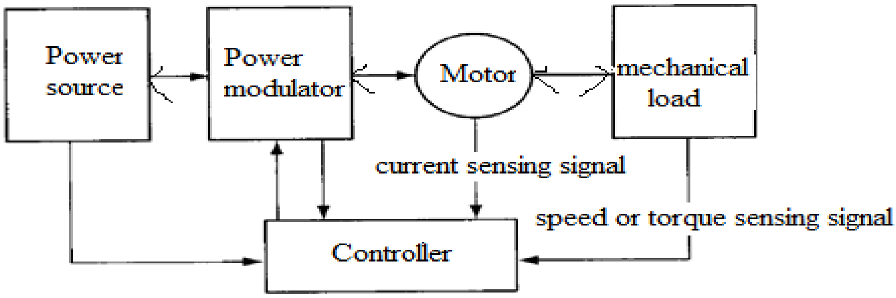


Fig 1.1

A modern electric drive system has five main functional blocks as shown above a mechanical load, a motor, a power modulator, a power source and a controller.

Power source: The power source provides the energy to the drive system. It may be dc or ac (single-phase or three-phase)

Power Converter: The converter interfaces the motor with the power source and provides the motor with adjustable voltage, current and frequency. During transient period such as starting, braking and speed reversal, it restricts source and motor current within permissible limits Also the converter converts the electric waveform into required signal that requires the motor.

Types of modulator:

- Controlled Rectifier(ac to dc)
- Inverter (dc to ac
- AC Voltage Regulator (ac to ac
- DC Chopper (dc to dc)
- Cyclo-converter (ac to ac) (Frequency converter)

Controller:

A well designed controller has several functions. The basic function is to monitor system variables, compare them with desire values, and then adjust the converter output until the system achives a desired performance. This feature is used in speed and position control.

Electric motor: i) The basic criterion in selecting an electric motor for a given drive application is it meets power level and performance required by the load during steady state and dynamic operation.

ii) Environmental factors: In industry such as in food processing, chemical industries and aviation where the environment must be clean and free from arc. Induction motors are used instead of DC motor.

Mechanical Load:

The mechanical load usually called as machinery such as flow rates in pump, fans, robots, machine tools, trains and drills are coupled with motor shaft.

Classification of Load torque: Various load torques are broadly classified into two categories.

A) Active Load Torque

B) Passive Load Torque

Load torques which have the potential to drive the motor under equilibrium conditions are called active load torques. Such load torques usually retain their sign when the direction of the drive rotation is changed. Torque due to the force of gravity, hoists, lifts or elevators and locomotive trains also torques due to tension, compression, and torsion undergone by an elastic body come under this category.

Components of the Load Torque (T_L) :

The load torque T_L can be further divided into the following components:

1.Friction torque T_F: The friction will be present at the motor shaft and also in the various parts of the load.

2.Windage torque T_W: When a motor runs, the wind generates a torque opposing the motion. This is known as the windage torque.

3.Torque required to do the useful mechanical work, T_M: The nature of this torque depends on the type of load. It may be constant and independent of speed, it may be some function of speed, it may be time invariant or time variant, and its nature may also vary with the change in the load's mode of operation.

The friction torque 'T_F' can be resolved into three components as shown in figure 1.2b.

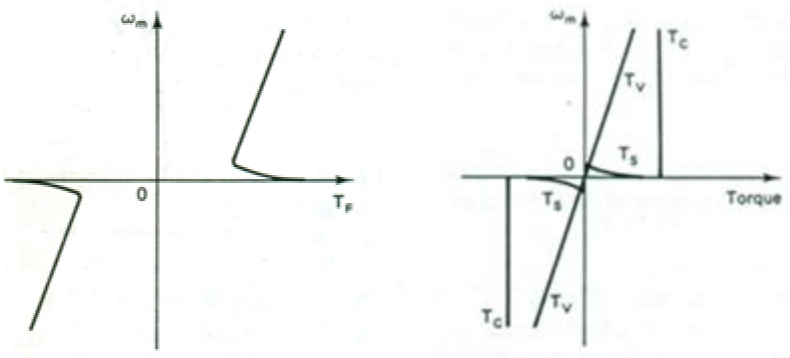


Fig 1.2 (a) Fig 1.2 (b)

Friction torque and its components.

$$T_F = T_V + T_C + T_S \qquad (1.1)$$

The first component Tv which varies linearly with speed is called viscous friction and is given by the following equation:

$$T_V = Bw_m \qquad (1.2)$$

where B is the viscous friction coefficient.

The windage torque T_w, which is proportional to speed squared, is given by the following equation:

$$T_w = Cw_m^2 \qquad (1.3)$$

where C is a constant.

So, $T_F = T_V$ is taken in account.

Now, the load torque can be represented by $T_L = T_M + T_V + T_w$

$$T_L = T_M + Bw_m + Cw_m^2 \qquad (1.4)$$

In many applications $T_w = Cw_m^2$ is very small compared to Bw_m and negligible compared to T_M . To simplify the analysis, the term T_w is neglected.

$$T_L = T_M + Bw_w \qquad (1.5)$$

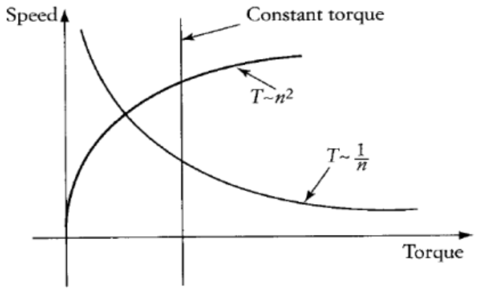
Torque-Speed Characteristics of Mechanical Load:

Fundamental Torque Equations

Mechanical load exhibit wide variations of speed-torque characteristics. Load torques are generally speed dependent and can be represented by an empirical formula such as

$$T = CT_r \left(\frac{n}{n_r} \right)^k \qquad (1.6)$$

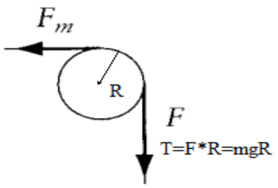
Where C is a proportionally constant,T$_r$ is the load torque at the rated speed n$_r$, n is the operating speed , and k is an exponential coefficient representing the torque dependency speed. Figure shows the typical mechanical loads.



Typical speed-torque characteristics of mechanical loads

1.**Torque independent of speed.**

The characteristics of this type of mechanical load are represented by setting k is equal to zero and C equals to 1.While torque is independent of speed. Such examples are hoists,the pumping of water etc.



2. **Torque linearly dependent on speed.** The torque is linearly proportional to speed k=1,and the mechanical power is proportional to the square of the speed. An example would be a motor driving a dc generator connected to a fixed resistance load with constant field. It can be shown as

$$T = \frac{P}{w} \quad \text{where P is the power generated by generator} \qquad (1.7)$$

But P=VI and $T = \frac{P}{w} = \frac{k^2 w^2}{Rw} = \frac{k^2 w}{R}$ (1.8)

$$\Rightarrow T \propto w \quad (1.9)$$

3. Torque proportional to the square of speed.

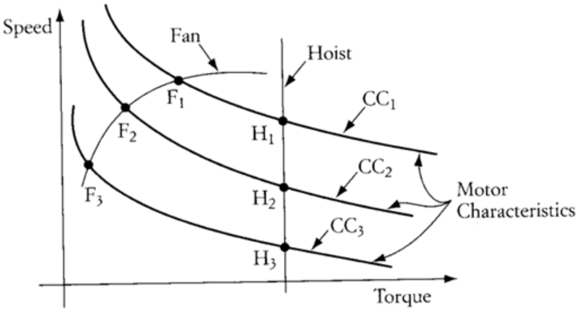
The torque-speed characteristic is parabolic, k=2. Such examples of loads are fans, centrifugal pumps, and propellers. $T \propto w^2$ (1.10)

4. Torque inversely proportional to speed.

In this case, k=-1.Examples are milling, boring machines, road vehicle and traction etc. $T \propto \frac{1}{w}$ (1.11)

Combined torque-speed characteristics of Motor-Load system:

Speed-torque characteristics of motor and mechanical load

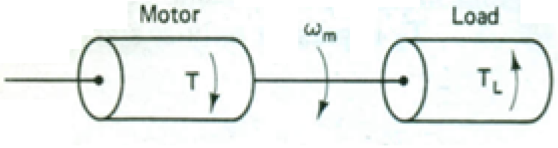


Dynamic of Motor-Load System:

Fundamental Torque Equations

The dynamic relations applicable to all types of motors and loads. The dynamic or transient condition. These condition appears during starting, braking and speed reversal of the drive.

A motor generally drives a load (machine) through some transmission system. While the motor always rotates, the load may rotate or may undergo a translational motion. It is convenient, however, to represent the motor load system by an equivalent rotational system, as shown in figure .



The following notations is adapted:

J = polar moment of inertia of the motor-load system referred to the motor shaft,Kg-m^2

w_m= instantaneous angular velocity of the motor shaft, rad/ sec

T_m = developed torque of the motor, N-m

T_L = the load (resisting) torque, referred to the motor shaft, N-m

Any motor-load system can be described by the following fundamental torque equation during dynamic condition:

$$T_m = T_L \pm \frac{d}{dt}(Jw_m) \tag{1.12}$$

$$\Rightarrow T_m = T_L \pm J\frac{dw_m}{dt} \pm w_m\frac{dJ}{dt} \tag{1.13}$$

This equation is applicable for variable inertia drives such as mines winder, industrial robots etc.

And $T_m = T_L \pm J\frac{dw_m}{dt}$ (1.14)

This equation is for constant inertia i.e. $\frac{dJ}{dt} = 0$

negative sign for deceleration and positive sign for acceleration.

Acceleration or deceleration depends on whether T_m is greater or less than T_L .During acceleration, motor should supply not only load torque T_L but also an additional torque component called inertia torque $J\frac{dw_m}{dt}$ to overcome the drive inertia. During deceleration, dynamic torque $J\frac{dw_m}{dt}$ has negative sign. Therefore, it assists the motor torque T_m and maintains drive motion by extracting from stored kinetic energy.

The fundamental torque equation balance between the various torques in the drive may be considered while investing the dynamic behavior is

$$T_m = T_L + J\frac{dw_m}{dt} \tag{1.15}$$

Where, $T_L = T_M + Bw_w$ (1.16)

It is seen from the above equation that, when

i) $T_m > T_L$ *i.e.* $\frac{dw_m}{dt} > 0$ (1.17)

the drive will be accelerating, in particular, picking up speed to reach rated speed

ii) $T_m < T_L$ *i.e.* $\frac{dw_m}{dt} < 0$ (1.18)

the drive will be decelerating and particularly, coming to rest

iii) $T_m = T_L$ *i.e.* $\frac{dw_m}{dt} = 0$ (1.19)

the motor will continue to run the same speed, if it were running or will continue to be at rest, if it were not running.

Stability:

Steady State Stability

A) Transient state Stability or Dynamic Stability

Criteria for Steady State Stability:

Let us assume that the motor-load speed-torque curve is at in equilibrium i.e. at steady state.

The disturbances changes the equilibrium states. The disturbances are two types. Such as

1.Due to slow change of inertia of rotating masses or that of inductances changes the equilibrium states slowly. So, effect of the inertia and the inductances are neglected for dynamics.

2. Due to large and sudden changes of inertia and inductances there is a sudden changes of equilibrium states. So, the inertia and inductances are taken for dynamic study.

Study of stability under conditions given above for the first type of disturbance relate to the field of steady state stability while for the second type of disturbance pertain to the field of dynamic or transient stability.

Let the equilibrium values of the torques and speed be denoted by T_m, T_L and w_m

Then at equilibrium, when deviation is not occurred

$$T_m = T_L \tag{1.20}$$

Let a small deviation in load torque is done, so that all equilibrium changes by $\Delta T_m, \Delta T_L$ and Δw_m

then, the dynamics $T_m = T_L + J\,\frac{dw_m}{dt}$ becomes $T_m + \Delta T_m = T_L + \Delta T_L + J\,\frac{d(w_m + \Delta w_m)}{dt}$ (1.21)

$$\Rightarrow \Delta T_m = \Delta T_L + J\,\frac{d\Delta w_m}{dt} \tag{1.22}$$

If we assume that these increments are so small that may be expressed as linear functions of the change in speed, then

$$\Delta T_m = \frac{dT_m}{dw_w}\Delta w_m \tag{1.23}$$

$$\Delta T_L = \frac{dT_L}{dw_m}\Delta w_m \tag{1.24}$$

Where $\frac{dT_m}{dw_m}$ and $\frac{dT_L}{dw_m}$ indicates derivatives at the point of equilibrium.Sybstituting these relations in early equation and rearranging, we have

$$J\,\frac{d\Delta w_m}{dt} + \left(\frac{dT_L}{dw_m} - \frac{dT_m}{dw_m}\right)\Delta w_m = 0$$

$$\Delta w_m = (\Delta w_m)_0\, e^{-\frac{t}{\tau}} \tag{1.25}$$

$$\tau = \frac{J}{\left(\frac{dT_L}{dw_m} - \frac{dT_m}{dw_m}\right)}$$ called mechanical time constant.

For the system to be stable when the exponent of the equation be negative. This exponent will be negative when

$$\frac{dT_L}{dw_m} - \frac{dT_m}{dw_m} > 0 \tag{1.26}$$

or $$\frac{dT_L}{dw_m} > \frac{dT_m}{dw_m} \tag{1.27}$$

Transient state stability:

Concept of Transient Stability

Thus the equation of motion in terms of power, can be written as

$$P_m = P_{dyn} + P_L \tag{1.28}$$

Where P_m, P_{dyn} and P_L denote the motor power, dynamic power and the load power at the shaft respectively. The dynamic power is determined from the angular acceleration. Let the angular position of the shaft at any instant is taken as the δ between a point and reference which is rotating at synchronous speed. With sudden application of load, since the rotor slows down, the angular acceleration will be negative and hence the dynamic power will be given by

$$P_{dyn} = -P_j \frac{d^2\delta}{dt^2} \tag{1.29}$$

Where, $P_j = J * w * \frac{2}{Poles}$ (1.30)

The electromagnetic power P_m has two components (i) damping power which linearly varies with $\frac{d\delta}{dt}$ from synchronous speed and (ii) Synchronous power which is a function of load angle δ. Thus,

$$P_j \frac{d^2\delta}{dt^2} + P_d \frac{d\delta}{dt} + P(\delta) = P_L \tag{1.31}$$

Where, P_d is the damping power. Neglecting damping and assuming cylindrical rotor, then the above equation will be

$$P_j \frac{d^2\delta}{dt^2} + P_m \sin\delta = P_L \tag{1.32}$$

Where, $P_m = \frac{VE}{X_s}$

Now, $$\frac{d^2\delta}{dt^2} = \frac{P_L - P_m \sin\delta}{P_j} \tag{1.33}$$

Mul;tiplying both sides by $\frac{d\delta}{dt}$, we have

$$\frac{d^2\delta}{dt^2}\left(\frac{d\delta}{dt}\right) = \left(\frac{P_L - P_m \sin\delta}{P_j}\right)\frac{d\delta}{dt}$$

$$\frac{d\delta}{dt} = \sqrt{\int_{\delta_0}^{\delta} \frac{2(P_L - P_m \sin\delta)}{P_j} d\delta} \tag{1.34}$$

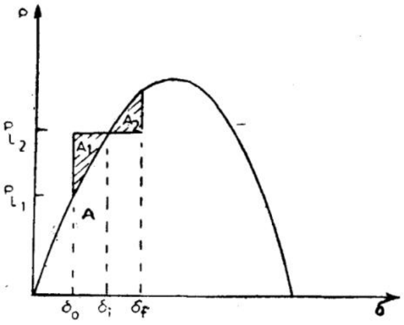
Where δ_0 is the load angle before the disturbance, i.e., at time t=0. So, for the machine to be stable at the synchronous speed $\frac{d\delta}{dt} = 0$. Hence,

$$\sqrt{\int_{\delta_0}^{\delta} \frac{2(P_L - P_m \sin\delta)}{P_j} d\delta} = 0 \tag{1.35}$$

$$\sqrt{\int_{\delta_0}^{\delta} (P_L - P_m \sin\delta) d\delta} \ . \tag{1.36}$$

With the motor initial load P_{L1}, the operating point is at A corresponding to point δ_0. As the load is suddenly increased to P_{L2}, the power angle swings to δ_f at which the speed is again synchronous. When the system is stable $\int_{\delta_0}^{\delta_i} (P_{L2} - P_m \sin\delta) d\delta + \int_{\delta_i}^{\delta_f} (P_{L2} - P_m \sin\delta) d\delta = 0$ (1.37)

where δ_i is the power angle corresponding to new load P_{L2}. So, from the equation it is to be written as



$$\int_{\delta_0}^{\delta_i} (P_{L2} - P_m \sin\delta) d\delta = \int_{\delta_i}^{\delta_f} (P_m \sin\delta - P_{L2}) d\delta \tag{1.38}$$

Or Area $A_1 = A_2$. This method of determining the transient stability of a drive system is called equal area criterion of stability.

Conclusion :

i) If area $A_1 > A_2$, the drive is stable

ii) If area $A_1 = A_2$, the drive is just stable

iii) If $A_1 < A_2$, the motor loses synchronism

Rating of Motor:

From the classes of duty the motor rating is selected. A motor can be selected for a given class of duty based on its thermal rating with due consideration to pull out torque i.e. the overload must be within the pull out torque. The various classes of duties are

- Continuous duty

- Intermittent duty
- Short time duty

Continuous duty: Two classes of continuous duty are there. a) continuous duty at constant load b) continuous duty with variable load

i) continuous duty at constant load: It denotes the motor operation at a constant load torque for a long duration enough to attain the steady state temperature. Ex. Paper mill drives, centrifugal pumps and fans etc. Frequent starting is not required. The rating of the machine is decided by input power.

If efficiency of the load and transmission is η ,the input power to the load is

$$P = \frac{wT}{\eta} \quad \text{(for rotational body)} \tag{1.39}$$

T= load torque

$$P = \frac{F * V}{0.201 * \eta} \quad \text{(for linear motion)} \tag{1.40}$$

F= force exerted by load in kg
V= velocity of motion in m/sec

$$P = \frac{F * V}{2 * 0.102 * \eta} \quad \text{(for elevator)} \tag{1.41}$$

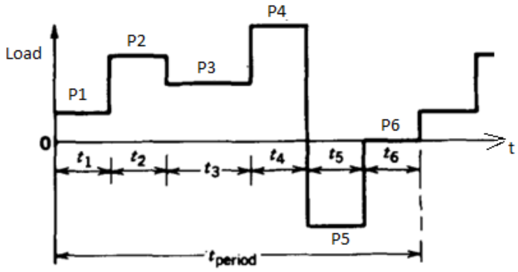
$$P = \frac{HQ\rho}{0.102 * \eta} \quad \text{(for pump)} \tag{1.42}$$

H= gross head comprising suction in mt
Q= quantity of delivery of pump in mt^3/sec
ρ = density of liquid in kg/mt^3

$$P = \frac{HQ}{0.102 * \eta} \quad \text{(for fan)} \tag{1.43}$$

Q=volume of air in mt^3/sec
H= pressure of air in kg/mt^2

ii) continuous duty with variable load: Load has several steps in one cycle. The motor rating is neither selected for highest load nor lowest load rather the rating is selected on average losses for the load cycle. Ex. Metal cutting lathes, conveyors etc.



For variable motor mechanical load, the current to the motor is variable. To find the rating of the motor the equivalent current I_{eq} method is used for finding the motor rating.

Let each step of the load the power loss is composed of constant loss which is independent of load called core and variable loss called copper loss. The variable load consists of motor current $I_{1,}I_2….I_6$ for one cycle. Thus

$$P_c + I_{eq}^2 R = \frac{(P_c + I_1^2 R)t_1 + (P_c + I_2^2 R)t_2 + ... + (P_c + I_6^2 R)t_6}{t_1 + t_2 + ... + t_6}$$

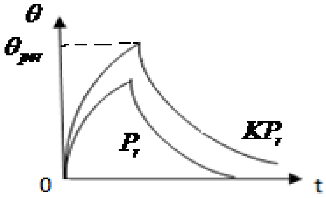
$$I_{eq} = \sqrt{\frac{I_1^2 t_1 + I_2^2 t_2 + ...I_6^2 t_6}{t_1 + t_2 + t_3 + ... + t_6}} \qquad (1.44)$$

After I_{eq} is determined, a motor with next higher current rating (= I_{rated}) from commercially available ratings is selected.

DC motor: For the design of dc motor, the maximum allowable current is 2 times the rated current.

Induction and Synchronous motor: Here the maximum breakdown torque to rated is 2 to 2.25

Short Time Duty: In short time duty, the time of operation is less than the heating time constant and the motor is allowed to cool down to the ambient temperature before it is required to start again. If a motor rating of power P_r is subjected to short time duty load of P_r , then the temperature rise will be far below than the permissible temperature θ_{per} . Therefore, the motor can be overloaded by a factor K>1 such that, the maximum temperature rise just reaches the permissible value θ_{per}.Now, for the load KP_r ,the time of operation is t_r .



$$\theta_{per} = \theta_{ss}(1 - e^{-\frac{t_r}{\tau}}) \qquad (1.45)$$

$$\frac{\theta_{ss}}{\theta_{per}} = \left(\frac{1}{1 - e^{-\frac{t_r}{\tau}}} \right)$$

For motor power P_r the loss is P_{1r}

For power KP_r, the loss is P_{1s}, then $\frac{\theta_{ss}}{\theta_{per}} = \frac{P_{1s}}{P_{1r}} = \left(\frac{1}{1 - e^{-\frac{t_r}{\tau}}} \right)$

But, $P_{1r} = P_c + P_{cu}$ and let $\alpha = \frac{P_c}{P_{cu}}$

then, $P_{1s} = P_c + P_{cu}\left(\frac{KP_r}{P_r} \right)^2 = P_c + K^2 P_{cu} = P_{cu}(\alpha + K^2)$ (1.46)

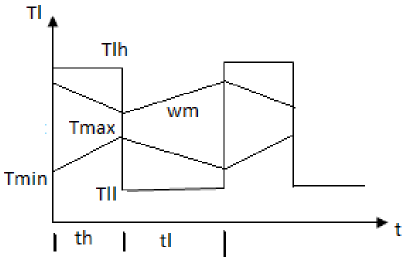
From the equations, the overloading factor $K=\sqrt{\frac{1+\alpha}{1-e^{-\frac{t_r}{\tau}}}}-\alpha$ (1.47)

Intermittent periodic duty: Here, the temperature neither reaches the steady state value during on nor reaches to ambient temperature during off period. So, the over-loading can be applied to the motor to bring to the steady state temperature for which the motor rating can be selected. The overloading factor can be found as

$$K=\sqrt{(\alpha+1)\frac{1-e^{-\{\left(\frac{t_r}{\tau_r}\right)+\left(\frac{t_s}{\tau_s}\right)}}{1-e^{-\frac{t_r}{\tau_r}}}}-\alpha \qquad (1.48)$$

Load Equalization:

In application such as electric hammer, pressing job, steel rolling mills etc, load fluctuates widely within short intervals of time. In such drives, to meet the required load the motor rating has to be high or the motor would draw the pulse current from the supply. Such pulse current from the supply gives voltage fluctuations which affects to the other load connected to it and affects to the stability of the source. The above problem can be met by using a flywheel connected to the motor shaft for non-reversible drives.This is called load equalization. The moment of inertia and the mechanical time constant can be found out from the load equalization problem.



$$J=\frac{T_r}{w_{m0}-w_{mr}}\left(\frac{t_l}{\ln\left(\frac{T_{\max}-T_{ll}}{T_{\min}-T_{ll}}\right)}\right) \quad \text{or} \quad J=\frac{T_r}{w_{m0}-w_{mr}}\left(\frac{t_h}{\ln\left(\frac{T_{lh}-T_{\min}}{T_{lh}-T_{\max}}\right)}\right) \qquad (1.49)$$

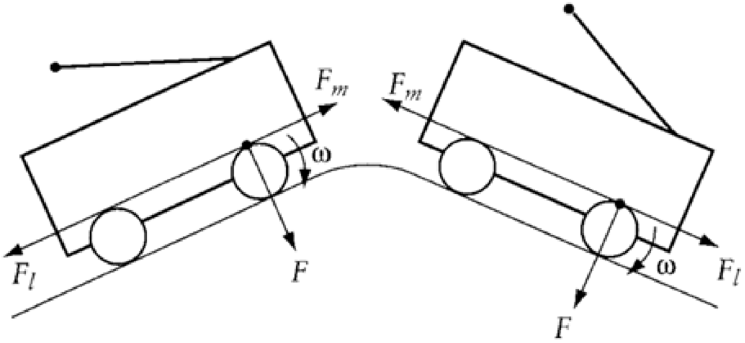
The mechanical time constant is

$$\tau_m=\left(\frac{t_h}{\ln\left(\frac{T_{lh}-T_{\min}}{T_{lh}-T_{\max}}\right)}\right) \qquad (1.50)$$

The symbols used have their respective meaning.

Bidirectional Electrical Drives (1st and 2nd quadrant)

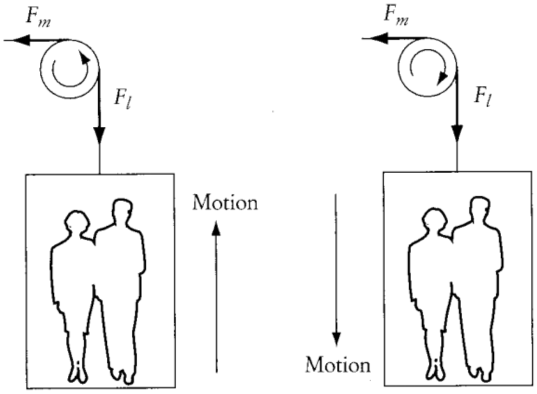
From the action-reaction theory of Newton's Law, when an electric motor driving a mechanical load in a steady state operation, a force exerted by either part (motor or load) of drive system, is opposed by a force equal in magnitude and opposite in direction from the other. This can be understood by taking a bidirectional drives with **unidirectional speed and bidirectional load torque.**



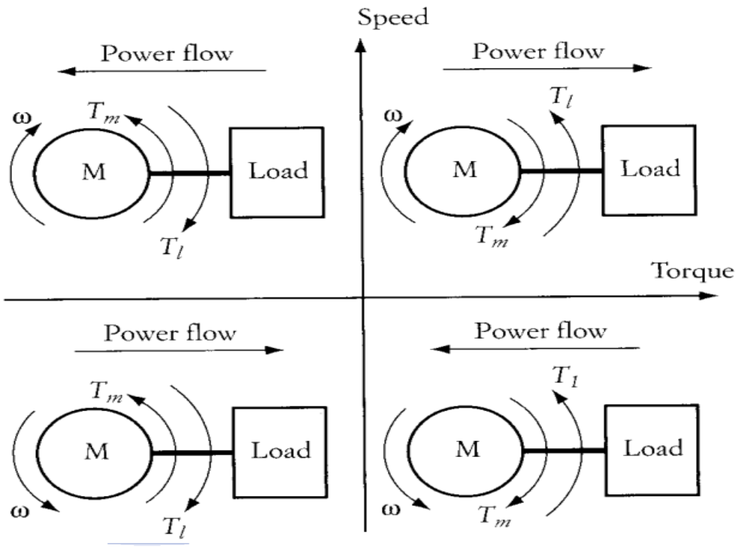
Bi-directional speed drive (1st and 4th quadrant)

In the figure shown below an elevator is moving passengers in both directions (up and down).For simplicity, let us assume that the elevator does not have a counterweight. In the upward directions,the motor sees the load force F_l which is a function of the weight of the passengers plus elevator cabin,cable etc. Since the weight and F_l are unidirectional, the motor force F_m is also unidirectional. The speed of the motor in this operation is bidirectional.

Bidirectional speed



Four-Quadrant Drives



The Following conventions are to be followed.

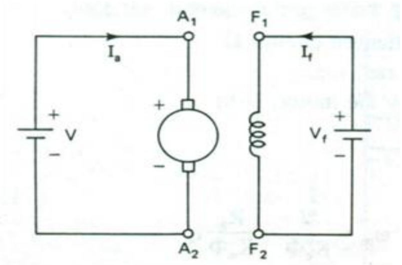
1. When the torque of an electric machine is in the same direction as system speed, the machine consumes electric power from the source and deliver the mechanical power to the laod. The machine is then operating as a motor.
2. If the speed and the torque of the machine are in opposite directions, the machine is consuming mechanical power from the load and delivering electric power to the source. In this case, the machine is acting as generator.

Characteristics of Motor:

Three types of electric motors generally used for drive purposes. DC, Induction and Synchronous motor.

DC Drives:

Separately Excited Dc motor:



Separately excited

The basic equations for DC motor are

$$E = K_e \phi w_m \quad (1.51)$$

$$V = E + I_a R_a \quad (1.52)$$

$$T = K_e \phi I_a \quad (1.53)$$

Where, E = back emf in volt; ϕ = flux per pole in weber; V = supply voltage in volt; I_a = Armature current in Amp; R_a = Armature resistance in ohm; w_m = speed of armature in rad/sec; T = torque developed in motor in N-mt

From the above set of steady state equations the steady state torque speed relation can be found out as

$$w_m = \frac{V}{K_e\phi} - \frac{I_a R_a}{K_e\phi} \qquad (1.54)$$

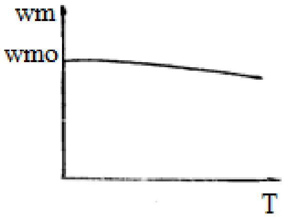
$$w_m = \frac{V}{K_e\phi} - \frac{R_a T}{(K_e\phi)^2} \qquad (1.55)$$

This equation can be applied to all series, shunt, compound and separately excited dc motors.
In the case of separately excited motors, if the field voltage is maintained constant,and assuming the flux as constant, then $K_e\phi = K$ (constant) (1.56)

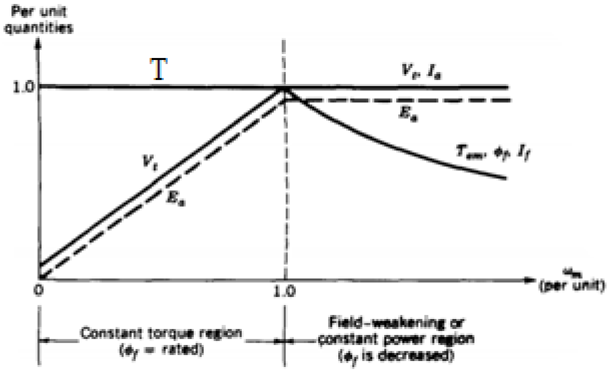
The speed equation is written for the separately as well as shunt motor is

$$w_m = \frac{V}{K} - \frac{R_a T}{K^2} \qquad (1.57)$$

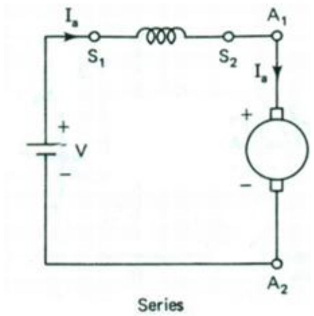
The speed increases from the zero upto the base speed. This method is called the constant torque method. Beyond the rated voltage, and rated armature current the voltage can not be increased further due to insulation problem. So, to control the speed the flux control can be done. By decreasing the flux, speed can be increased above the base speed w_{m0}. This method is called constant power method where both voltage and armature current is kept constants. Further, in the below base speed region, the speed can be decreased from the no load speed w_{m0} by increasing the load. When the load increased, the speed decreased from its no load speed. This motor is used where the speed regulation is good.



Speed-Torque ch. when load varies



Dc Series Motor:



Series

From the basic equation the speed can be written as

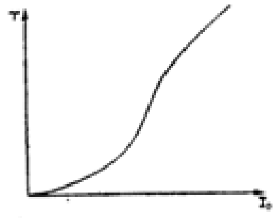
$$w_m = \frac{V}{K_e\phi} - \frac{R_a T}{(K_e\phi)^2}$$

In series motor, $T = K_e \phi I_a$, but $\phi \propto I_a$

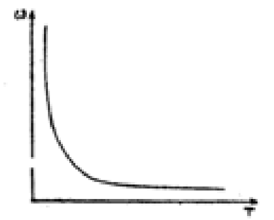
So, $$T = K_e K_f I_a^2 \tag{1.58}$$

$$w_m = \frac{V}{\sqrt{K_e K_f}} \frac{1}{\sqrt{T}} - \frac{R_a}{K_e K_f} \tag{1.59}$$

In the case of series motor, any increase in torque is accompanied by an increase in the armature current and therefore, an increase in flux. Because the flux increases with torque, the speed must drop to maintain a balance between the induced voltage and the supply voltage. The characteristic is therefore, highly drooping.



Torque vs. current of series motor.



Speed vs. torque of series motor.

Methods of speed control

From the speed-torque relation from the equation it is seen that, the speed can be controlled by any one of the following three methods.

1. Armature voltage control
2. Armature resistance control (Rheostatic control)
3. Field flux control

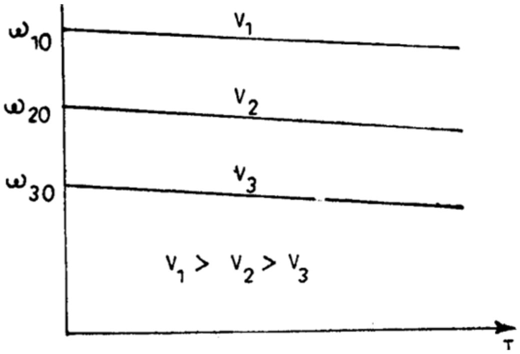
Armature voltage control method: (DC shunt motor)

The speeds corresponding to two different armature voltages are V_1 and V_2 of a dc shunt motor are given by

$$w_{m1} = \frac{V_1}{K_e \phi} - \frac{R_a T}{(K_e \phi)^2} = w_{10} - \Delta w \tag{1.60}$$

$$w_{m2} = \frac{V_2}{K_e \phi} - \frac{R_a T}{(K_e \phi)^2} = w_{20} - \Delta w \tag{1.61}$$

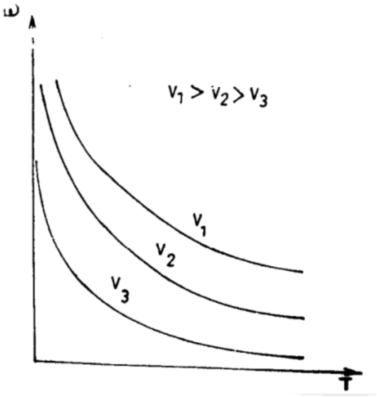
The no load speed is directly proportional to the supply voltages. Keeping the load torque as constant, the family of motor torque-speed characteristics can be drawn for a given load torque.



Speed-Torque Characteristics of dc shunt motor for different armature voltages

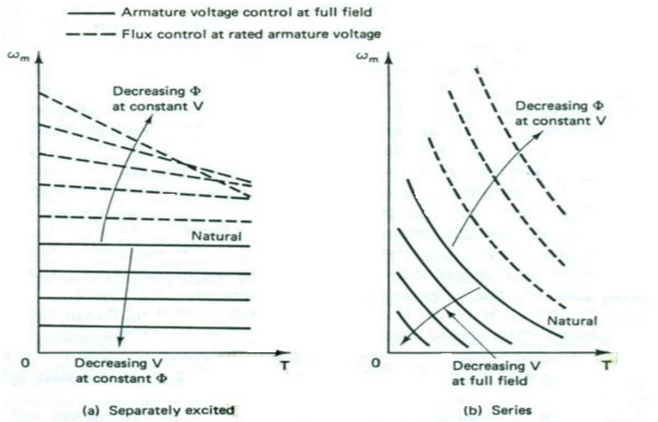
This method is only for below rated speed since the voltage magnitude should not be greater than the rated voltage. The variable voltages can be obtained by phase controlled rectifier and DC-DC Chopper concerter.

DC Series motor:



Field flux control method.

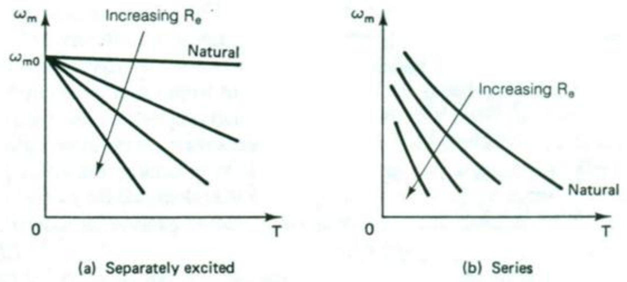
If the field of a separately or series excited motor running at a speed is weakened, its induced emf decreases. Because of low armature resistance, the current increases by an amount much larger than the decrease in the field flux. As a result, in spite of the weakened field, the torque is increased by a large amount, considerably exceeding the load torque. The surplus torque thus available causes the motor to accelerate and the back emf to rise. The motor will finally settle down to a new speed, higher than the previous one, at which the motor torque with the weakened field becomes equal to the load torque. Any attempt to weaken the field by a large amount will cause a dangerous inrush of current. Care should therefore be taken to weaken the field only slowly and gradually.



Speed-torque curves of separately excited and series motors.

Armature resistance control:

Speed torque characteristics of separately excited (or shunt) and series motors for various values of external resistance R_e in series with the armature are shown.



Speed-torque curves of dc motors with resistance control.

The main drawback of this method of speed control is its poor efficiency. Because of the poor efficiency, this method is seldom used with separately excited motors, except for getting speeds which are required for very short times.

Braking:

There are three methods of braking a dc motor

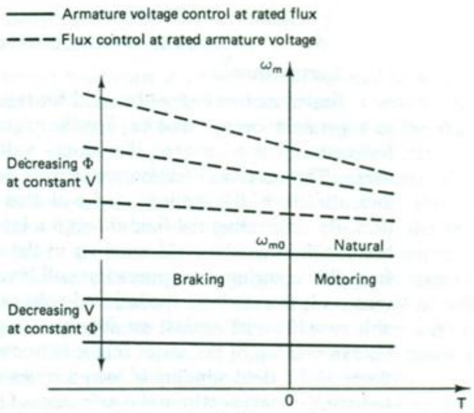
1. Regenerative braking.
2. Dynamic braking or rheostatic braking.
3. Plugging or reverse voltage braking

Regenerative Braking:

In regenerative braking, the energy generated is supplied to the source.

Separately Excited Motor:

The steady-state equivalent circuit of a separately excited motor and source is given in figure. If by some method the induced emf E is made greater than the source voltage V, the current will reverse. The machine will work as a generator and the source will act as a sink of energy, thus giving regenerative braking.



Regenerative braking speed-torque curves of a separately excited motor.

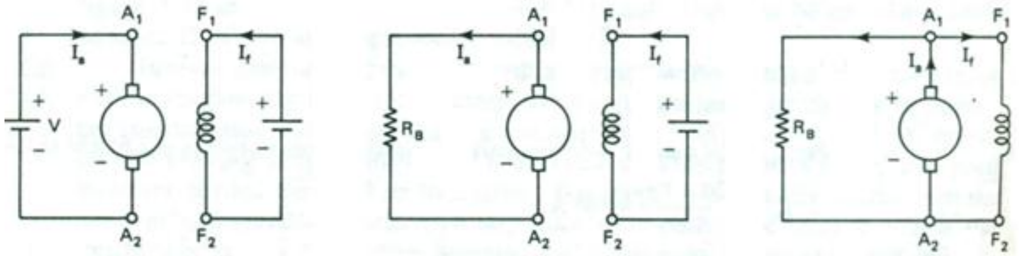
Series motor

Series motors cannot be used for regenerative braking in the same simple way as separately excited motors. For the regenerative braking to take place, the motor induced emf must exceed the supply voltage and the armature current should reverse. The reversal of armature current will reverse the current through the field, and, therefore, the induced emf will also reverse. The main advantage of regenerative braking is that the generated electrical energy is usefully employed instead of being wasted in rheostats as in the case of dynamic braking and plugging.

Dynamic Braking:

The dynamic braking of a dc motor is done by disconnecting it from the source and closing the armature circuit through a suitable resistance. The motor now works as a generator, producing the braking torque.

For the braking operation, the separately excited (or shunt) motor can be connected either as a separately excited generator (fig.b), where the flux remains constant, or it can be connected as a self-excited shunt generator, with the field winding in parallel with the armature (fig.c).



(a) Motoring

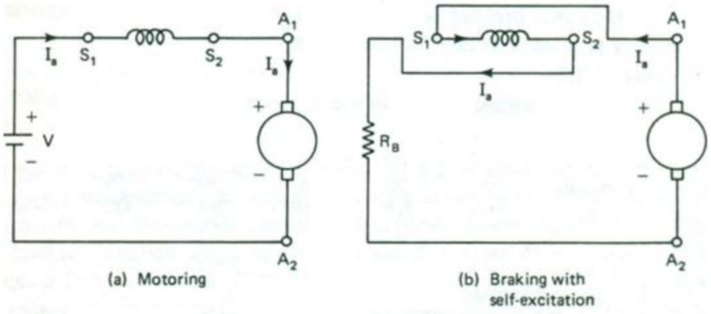
(b) Braking with separate excitation

(c) Braking with self-excitation

Dynamic braking of separately excited motor.

Series Motor:

For dynamic braking, the series motor is usually connected as a self-excited series generator. For the self-excitation, it is necessary that the current forced through the field winding by the induced emf aids the residual flux. This requirement is satisfied either by reversing the armature terminals or the field terminals.

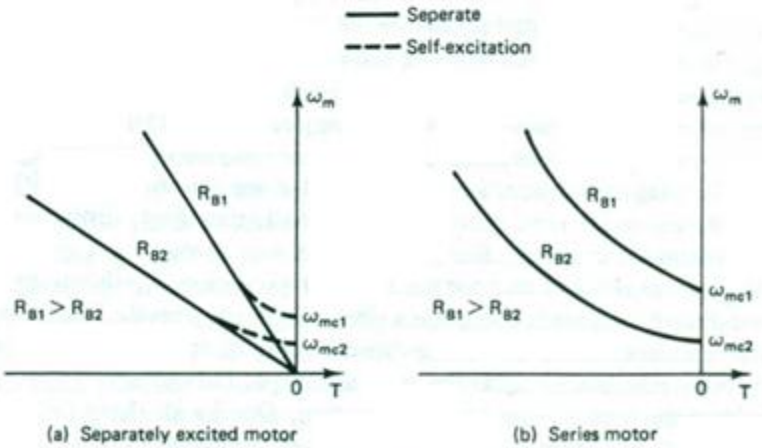


(a) Motoring

(b) Braking with self-excitation

Dynamic braking of series motor.

Speed-Torque Characteristics during dynamic braking:



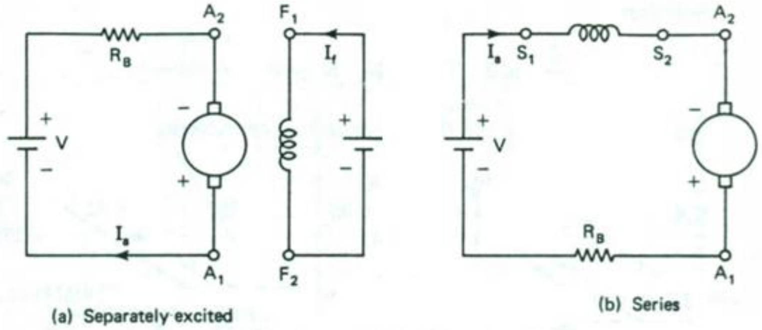
(a) Separately excited motor

(b) Series motor

Speed-torque curves of dc motors under dynamic braking.

Plugging:

If the armature terminals (or supply polarity) of a separately excited (or shunt) motor when running are reversed, the supply voltage and the induced voltage will act in the same direction and the motor current will reverse, producing braking torque. This type of braking is called plugging. In the case of a series motor, either the armature terminals or field terminal s should be reversed. Reversing of both gives only the normal motoring operation.



(a) Separately excited

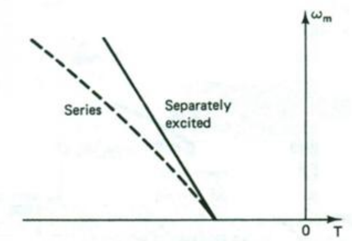
(b) Series

Plugging operation of dc motors.

Torque-speed characteristics:

When running at the rated speed, the induced voltage will be nearly equal to the supply voltage V. Therefore, at the initiation of braking, the total voltage in the armature circuit will be nearly 2 V. To limit the current within the safe value, a resistance equal to twice the starting resistance will be required.

Plugging is a highly inefficient method of braking. Not only is power supplied by the load, but also the power taken from the source is wasted in resistances.



Speed-torque curves of dc motors under plugging.

Starting:

Separately excited dc motor:

The maximum current that a de motor can safely carry during transients of short duration is limited by the maximum armature current that can be commutated without sparking. From the speed equation we see

$$w_m = w_0 - \Delta w$$

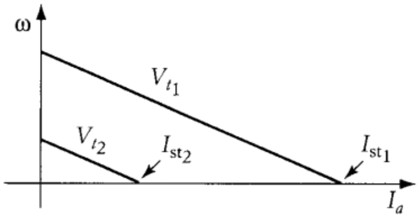
For large motors (greater than 10hp),the armature resistance R_a is very small. For these motors, the speed drop Δw is very small, and the machine is considered to be constant speed machines. The torque developed at starting T_{st} and starting current I_{st} can be calculated by keeping speed as zero during starting.

$$\frac{V}{K\phi} = \frac{R_a T_{st}}{(K\phi)^2} \qquad (1.62)$$

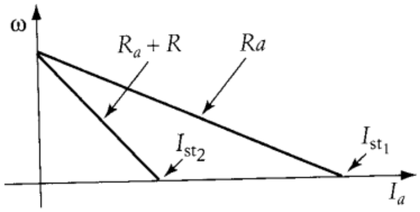
$$\Rightarrow T_{st} = (K\phi)\frac{V}{R_a} \qquad (1.63)$$

The starting current is $$I_{st} = \frac{V}{R_a} \qquad (1.64)$$

Effect of reducing source voltage during starting.



Effect of reducing external resistances.



Series Motor:

In series motor, the starting current is less due to presence of field resistances in series with armature resistance.

$$I_{st} = \frac{V}{R_a + R_f} \tag{1.65}$$

$$T_{st} = KC\left(\frac{V}{R_a + R_f}\right)^2 \quad \text{(series motor)} \tag{1.66}$$

$$T_{st} = KC\left(\frac{V}{R_{fsh}}\right)\left(\frac{V}{R_a}\right) \quad \text{(shunt motor)} \tag{1.67}$$

From the two equations it is seen that,

T_{st} is less in shunt motor and more in series motor.

I_{st} is more in shunt motor and less in series motor. So, series motor is widely used in traction drive.

Induction motor:

Advantages

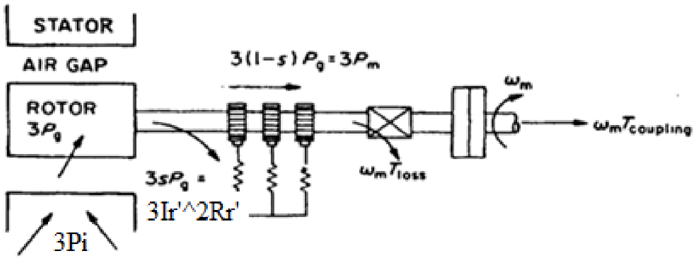
- Light in weight (cage type motor is usually used)
- Higher efficiency
- Low maintainance
- Robust and reliable
- Less cost than commutator type motor
- Ability to operate in dirty and explosive environment
- Advance feedback control technique such as field oriented control

Disadvantages

- Armature and field windings are highly coupled
- Non-linear modeling
- Multi-variable structure
- Controller such as power converter, inverter are relatively complex and expensive.

Steady-state performance of three phase induction motor:

The steady state performance can be studied from the power flow and equivalent circuit.



Power flow

Input power to the stator is $P_i = 3V_s I_s \cos\theta_s$ (1.68)

Input power = stator cu loss+ core loss+ air gap power

Stator cu loss $P_{scu} = 3(I_r^{'})^2 R_s$ (1.69)

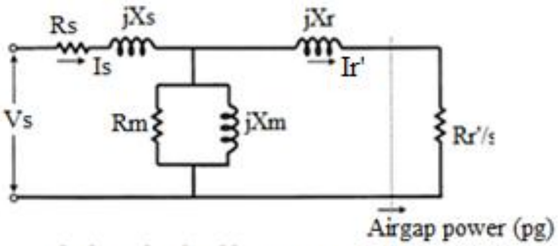
Core loss $P_c = 3\frac{V_s^2}{R_m}$ (1.70)

The air gap power per phase $P_g = (I_r^{'})^2 \frac{R_r^{'}}{s}$ (1.71)

Rotor circuit power per phase $(I_r^{'})^2 R_r^{'} = sP_g$ (1.72)

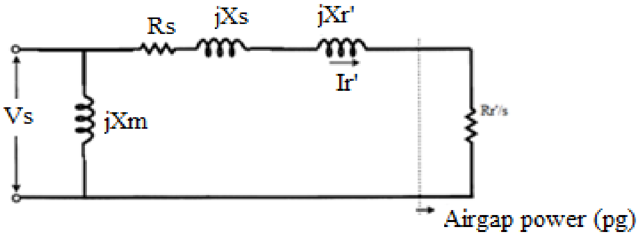
Mechanical power $P_m = (1-s)P_g = (1-s)\frac{(I_r^{'})^2 R_r^{'}}{s}$ (1.73)

Thus electromagnetic torque is $T_m = \frac{P_m}{w_m} = \frac{P_g(1-s)}{w_s(1-s)} = \frac{P_g}{w_s} = \frac{1}{w_s}\frac{(I_r^{'})^2 R_r^{'}}{s}$ (1.74)



Equivalent circuit with reapect to the stator

Per phase equivalent circuit of induction motor



Approximate equivalent circuit

per phase equivalent circuit of induction motor

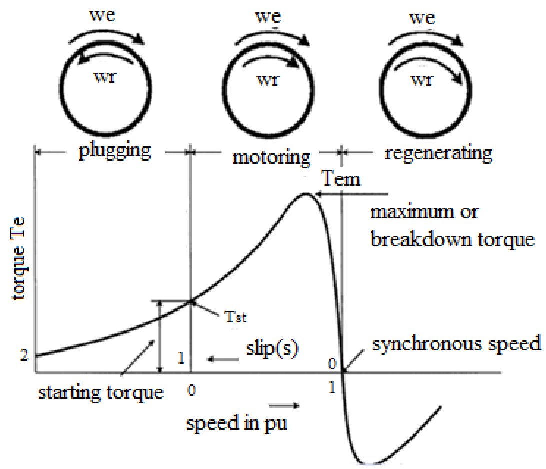
But from equivalent circuit $I_r^{'} = \frac{V_s}{\sqrt{\left(X_s + X_r^{'}\right)^2 + \left(R_s + \frac{R_r^{'}}{s}\right)^2}}$

Hence $$T_m = \frac{1}{w_s}\frac{V_s^2}{\left(R_s + \frac{R_r^{'}}{s}\right)^2 + \left(X_s + X_r^{'}\right)^2} * \frac{R_r^{'}}{s}$$ (1.75)

Steady-state torque-speed Characteristics:

When the slip is very small, $T_m \propto s$

When the slip is very large $T_m \propto \frac{1}{s}$



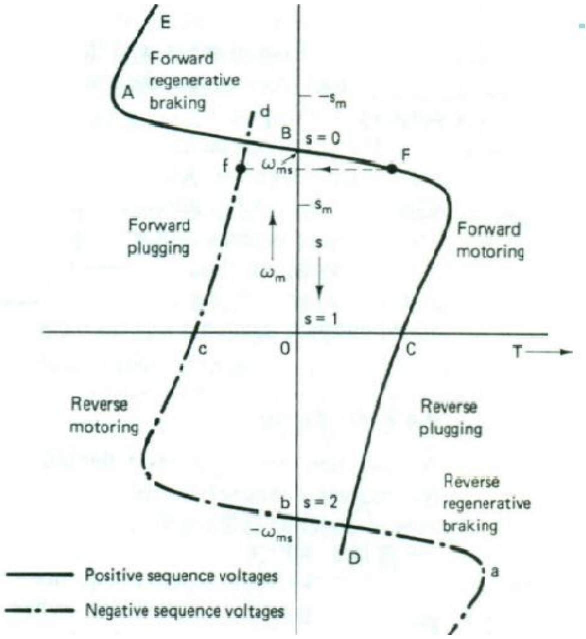
Three zones are there i) motoring zone (0<s<1) ii) regenerating zone (s<0) iii) plugging zone (1<s<2)

In the normal motoring zone Tm=0 at s=0. When slip increases, speed decreases but torque approaches maximum value. In the breakdown zone called quasi region, the stator drop is small and flux remains constant.

Features

- At s=0,Tm=0. Because there is no induced current and zero relative speed
- Tem is the maximum at sm where $R_r' = sX_r'$
- Tst is starting totque when s=1
- The motor is stable between (0 to sm)

Four-quadrant operation of Induction motor :



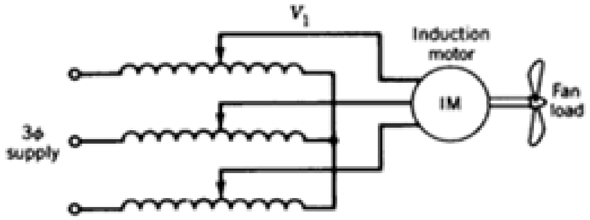
Speed-Control

There are five methods for speed control for modifying speed-torque characteristics. **i)** Stator voltage control **ii)** Stator Frequency control **iii)** Slip power recovery control ((Kramer drive)) or Rotor emf injection method **iv)** Rotor resistance control

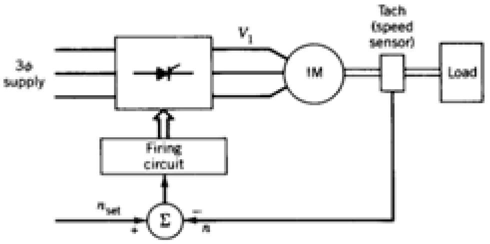
Last two methods are only for slip ring induction motor.

Supply voltage control Method

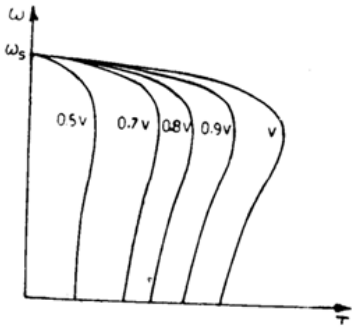
$$T_m \propto V_s^2$$



Auto-transformer voltage controller



Closed loop operation



Speed-torque characteristics with voltage control

The curves indicates that, the slip at maximum torque is independent of terminal voltages. The range of speeds within which steady state operation (for constant torque loads) may takes place for same for all voltages i.e. between the maximum torque and synchronous speed. With in that region there will be a small speed drop with decrease in voltage.This method is suitable for fan, pump and centrifugal drives.

Drawbacks:

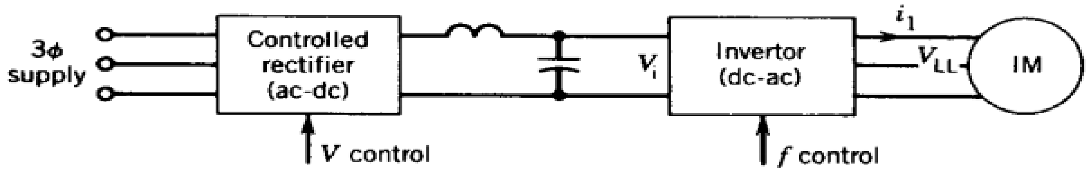
- Gives poor energy efficiency at low speed
- This method is only suitable for below base speed

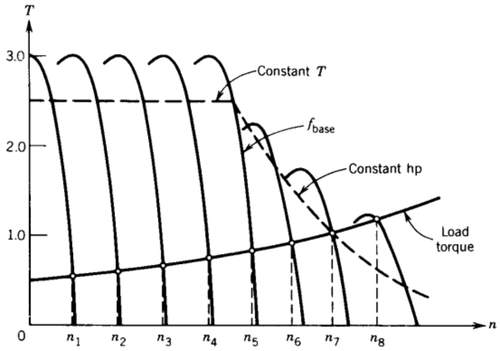
Stator Frequency control

By controlling the stator frequency ‘f’, synchronous speed which in turn determines the rotor speed of the motor. When the frequency is varied, then the magnetizing current I_m is also affected, which is given by

$$I_m = \frac{V_s}{X_m} = \frac{V_s}{2\pi f I_m} \qquad (1.76)$$

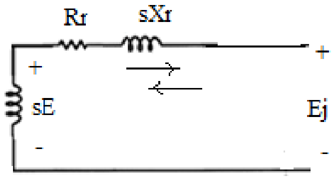
But, the magnetizing current must be constant for constant breakdown torque (maximum torque). Therefore, for constant breakdown torque, $\frac{V_s}{f}$ ratio should be maintained constant. When the operating frequency is increased beyond the breakdown torque, then the torque gets reduced but the starting torque is increased. For further decrease in supply frequency $\frac{V_s}{f}$ can not be maintained constant. At very low frequency the apparent increases that increased the voltage drop. Hence V_s decreased.





Slip power recovery control (rotor emf injection method):

In an induction motor, torque is equal to the power crossing the air gap divided by the synchronous mechanical speed. In early slip-ring induction motor drives, power was transferred through the motor to be dissipated in external resistances, connected to the slip-ring terminals of the rotor. This resulted in an inefficient drive over most of the speed range. More modern slip-ring drives use an inverter to recover the power called slip power from the rotor circuit, feeding it back to the supply system. One of the best recovery drive circuit is static Scherbius drive.



motor operation with injection voltage (Ej) in rotor

At running condition of slip with constant load, voltage and frequency the rotor current

$$I_r = \frac{sE}{\sqrt{\left(R_r^2 + (sX_r)^2\right)}}$$ For slip to be very small, $R_r^2 \geq (sX_r)^2$

$$I_r = \frac{sE}{R_r} \tag{1.77}$$

By giving additional voltage E_j at the rotor end, then $I_r = \frac{s_j E - E_j}{R_r}$, (1.78)

Since the load torque is constant, then

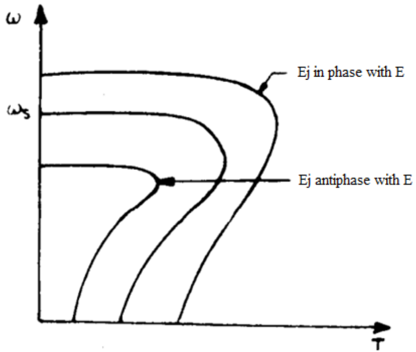
$$\frac{sE}{R_r} = \frac{s_j E - E_j}{R_r}$$

$$\Rightarrow s_j = s + \frac{E_j}{E} \tag{1.79}$$

It is seen the slip increases when the injected emf is in phase opposition to the induced emf. Now, as the slip increases, the induced emf increases and hence the current till the developed torque is equal to the load torque. In this way the injected emf controls the speed. Similarly when the injected emf is in same phase then the slip decreases.

$$s_j = s - \frac{E_j}{E} \tag{1.80}$$

This slip in term of slip power in rotor circuit can be recovered and send back to ac supply by Scherbius drive and efficient thus increase.



Speed- torque curves with injection of rotor voltage

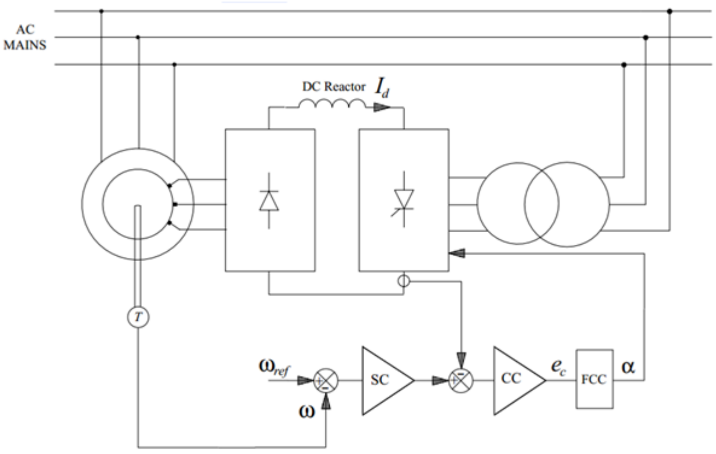
Scherbius method for slip power recovery

In this scheme, the rotor terminals are connected to a three-phase diode bridge that rectifies the rotor voltage. This rotor output is then inverted into mains frequency ac by a fully controlled thyristor converter operating off the same mains as the motor stator. The dc link current, smoothed by a reactor, may be regulated by controlling the firing angle of the converter in order to maintain the developed torque at the level required by the load. The current controller (CC) and speed controller (SC) are also indicated. The current controller output determines the converter firing angle α from the firing control circuit (FCC).

From the equivalent circuit and ignoring the stator impedance, the RMS voltage per phase in the rotor circuit is given by

$$V_R = \frac{V_s}{n}\frac{w_r}{w_s} = \frac{V_s}{n}\frac{sw_s}{w_s} = \frac{sV_s}{n} \tag{1.81}$$

Where w_r and w_r are the angular frequencies of rotor and stator voltages respectively. And ‘n’ is the ratio of the equivalent stator to rotor turns. The dc-link voltage at the rectifier terminals of the rotor, v_d is given by $v_d = \frac{3\sqrt{6}}{\pi} V_R$ (1.82)



Static Scherbius schme of slip power control

Assuming that the transformer interposed between the inverter output is and the ac supply has the same turn ratio ‘n’ as the effective stator-to-rotor turns of the motor.

$$v_d = -\frac{3\sqrt{6}}{\pi}\frac{V_s}{n}\cos\alpha \tag{1.83}$$

The negative sign arises because the thyristor converter develops negative dc voltage in the inverter mode of operation. The dc-link inductor is mainly to ensure continuous current through the converter so that the expression (1.83) holds for all conditions of operation. Combining the preceding three equations gives

$$sw_s = -w_s\cos\alpha \qquad \text{so that} \qquad s = -n\cos\alpha \tag{1.84}$$

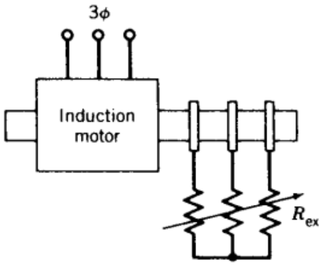
And the rotor speed is

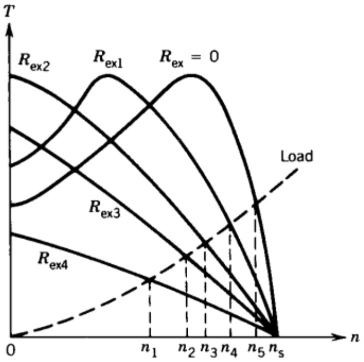
$$w_0 = \frac{1}{P}(1-s)w_s = \frac{1}{P}w_s(1+n\cos\alpha) \quad \text{rad/sec} \tag{1.85}$$

Thus, the motor speed can be controlled by adjusting the firing angle α. By varying α between 180^0 and 90^0, the speed of the motor can be varied from zero to full speed, respectively.

Rotor resistance control

The introduction or rotor resistance in slip ring induction motor will modifies the speed-torque curves. The operating points from zero to synchronous speed can be obtained in this method.

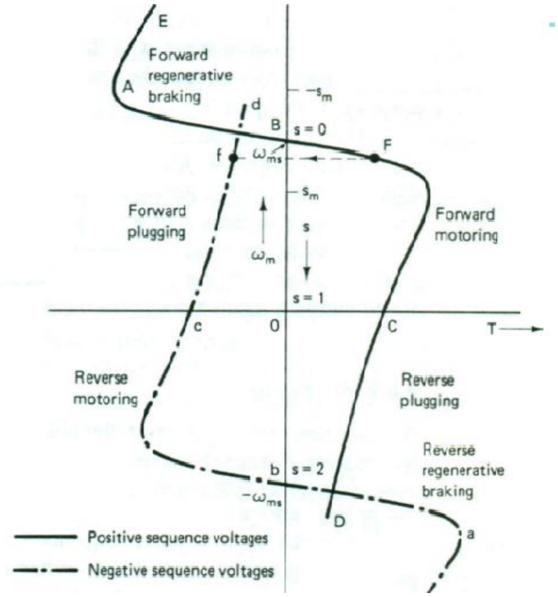




Braking:

Regenerative braking

The speed-torque curves obtained by the reversal of the phase sequence of the motor terminal voltages are also shown by dotted lines. With a positive sequence voltage across the motor terminals, the operation above synchronous speed gives the regenerative braking operation (portion BAE).Similarly, with a negative sequence voltage across the motor terminals, regenerative braking is obtained for speeds above the synchronous speed in the reverse direction (portion bae). In regenerative braking, the motor works as an induction generator, converting mechanical energy supplied by the load to electrical energy, which is fed to the source. Thus the generated energy is usefully employed.



Plugging

An induction motor operates in the plugging mode for slips greater than l. For positive sequence voltages, a slip greater than 1 is obtained when the rotor moves in the reverse direction (portion CD). Since the motor is running in the reverse direction, a positive torque provides the braking operation. With negative sequence voltages, plugging takes place on portion cd, shown by the chain-dotted line. When running in the forward direction, the motor can be braked by changing the phase sequence of the motor terminal voltages by simply interchanging the connections of any two motor terminals. This will transfer the operation from point F to f and braking will commence. The motor torque is not zero at zero speed. When braked for stopping, the motor should be disconnected from the supply at or near zero speed. An additional device will be required for detecting zero speed and disconnecting the motor from the supply. Therefore, plugging is not suit-able for stopping. It is, however, quite suitable for reversing the motor. From the forward motoring (portion BC), the reverse plugging operation (portion CD) is obtained when an active load drives the motor in the reverse direction, as in crane and hoist applications. When operating this way, plugging is some-times called counter-torque braking.

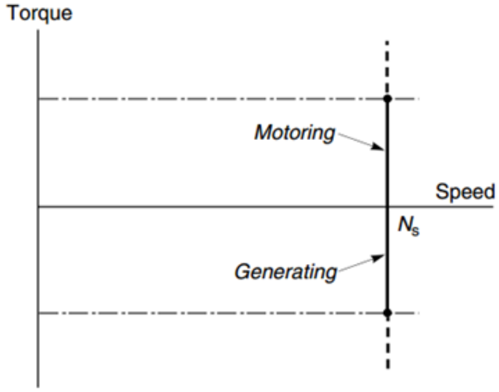
Dynamic braking

In dc dynamic braking, the motor is disconnected from the ac supply and connected to a dc supply. The flow of direct current through the stator windings sets up a stationary magnetic field. The relative speed between the stationary stator field and the moving rotor is now negative. Consequently, 3-phase voltages of reverse polarity and phase sequence (compared to the motoring in the same direction) are induced in the rotor. The resultant three-phase rotor currents produce a rotating field, moving at the rotor speed in the direction opposite to that of rotor, thus giving a stationary rotor field. Since both stator and rotor fields are stationary and rotor current flows in the reverse direction, a steady braking torque is produced at all speeds. It, however, becomes zero at standstill due to zero rotor currents.

Synchronous motor

In the synchronous motor, the stator windings are exactly the same as in the induction motor, so when connected to the 3-phase supply, a rotating magnetic field is produced. But instead of having a cylindrical rotor with a cage winding, the synchronous motor has a rotor with either a d.c. excited winding (supplied via slip rings), or permanent magnets, designed to cause the rotor to ‘lock-on’ or ‘synchronise with’ the rotating magnetic field produced by the stator. Once the rotor is synchronised, it will run at exactly the same speed as the rotating field despite load variation, so under constant-frequency operation the speed will remain constant as long as the supply frequency is stable.

With the synchronous machine we again find that, the maximum (pull-out) torque which can be developed before the rotor is forced out of synchronism with the rotating field. This ‘pull-out’ torque will typically be 1.5 times the continuous rated torque, but for all torques below pull-out the steady running speed will be absolutely constant. The torque–speed curve is therefore simply a vertical line at the synchronous speed.. We can see from Figure that the vertical line extends into quadrant 2, which indicates that if we try to force the speed above the synchronous speed the machine will act as a generator.

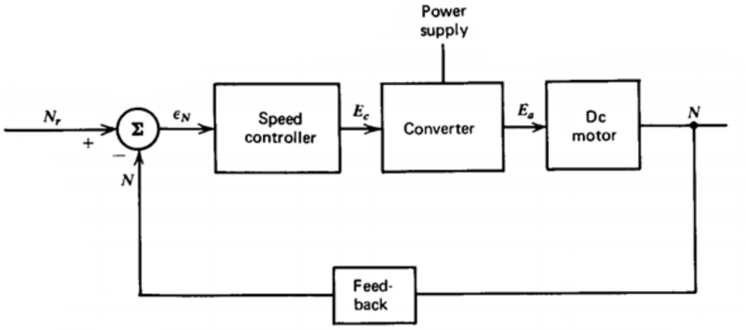


Steady state torque-speed characteristic for
Synchronous motor at constant frequency

MODULE-2

CONTROL OF DRIVE SYSTEMS

Dc motors are widely used in many speed-control drives. Open-loop operation of dc motors may be satisfactory in many applications. When the load increases the speed of the motor drops and the new operating point of speed is obtained after the transient. For getting constant speed i.e. the initial operating point the open loop does not work. So, closed-loop control system is required. The basic block diagram of closed-loop control system is shown.



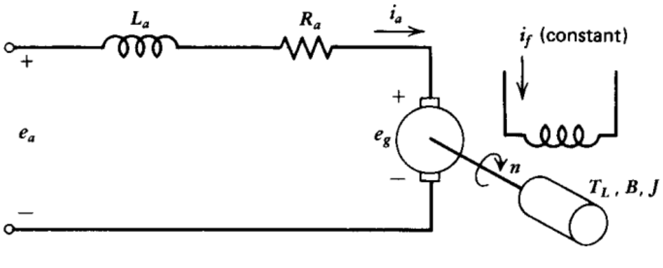
Basic block diagram of a closed-loop speed-control system.

If the motor speed decreases due to application of additional load torque, the speed error ε_N increases, which increases the control signal E_c. This in turn changes the firing angle of the converter, and thus increases the motor torque to restore the speed of the drive system. The system passes through a transient period until the developed torque matches the applied torque. A closed-loop system improves the dynamic response specially during acceleration, deceleration and disturbances such as loading in drive system. The response of a closed –loop system can be studied by using transfer function techniques.

Separately Excited DC motor Drives

Armature voltage control is inherently a closed loop control system in dc motor drives.However, the output speed signal can not be measured and the speed error is not found properly. This closed loop is further extended by using a feed back tachogenerator with speed controller and converter for modern control drives.

Motor Transfer function without tachogenerator and converter. (Armature voltage speed control)



The basic set of equations are

$$e_a = i_a R_a + L_a \frac{di_a}{dt} + e_g \qquad (2.1)$$

Where $e_g = K_a n$ (2.2)

The torque balance equation is $T_m = T_L + Bn + J\dfrac{dn}{dt}$ (2.3)

Also $T_m = K_a i_a$ (2.4)

In Laplace domain all time domain equations are brought into frequency domain

$$E_a(s) = I_a(s)R_a + L_a s I_a(s) + E_g(s) \tag{2.5}$$

$$E_g(s) = K_a N(s) \tag{2.6}$$

$$T_m(s) = T_L(s) + BN(s) + JsN(s) \tag{2.7}$$

$$T_m(s) = K_a I_a(s) \tag{2.8}$$

From the eq.(2.5)

$$I_a(s) = \frac{E_a(s) - E_g(s)}{R_a + sL_a} = \frac{\left[E_a(s) - E_g(s)\right]*1/R_a}{1 + s\tau_a} \tag{2.9}$$

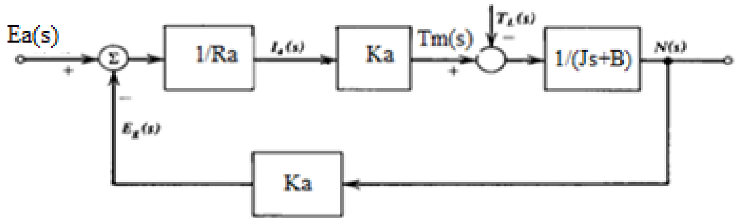
where $\tau_a = \dfrac{L_a}{R_a}$ electrical time constant

From eq.(2.7)

$$N(s) = \frac{T_m(s) - T_L(s)}{B + sJ} = \frac{\left[T_m(s) - T_L(s)\right]*1/B}{1 + s\tau_m} \tag{2.10}$$

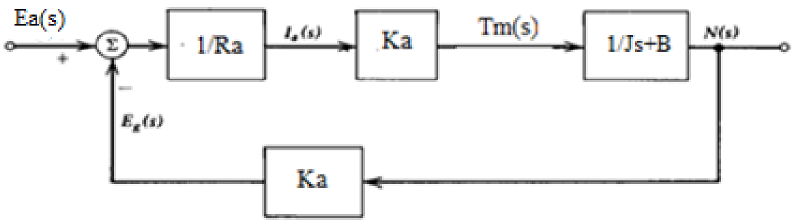
where $\tau_m = \dfrac{J}{B}$ mechanical time constant

The closed loop T.F.is



There are two inputs one electrical input voltage E_a and the other mechanical load torque T_L .So, considering,one input at a time neglecting others the total T.F. is coming as

$T_L = 0$ and neglecting L_a

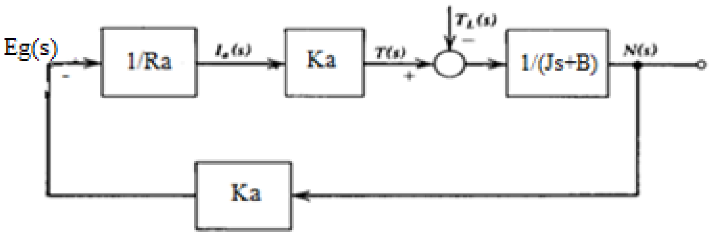


$$\frac{N(s)}{E_a(s)} = \frac{K_m}{1 + s\tau_m} \tag{2.11}$$

Where $K_m = \dfrac{(K_a / R_a)}{(B + K_a^2 / R_a)}$ motor gain constant and $\tau_m = \dfrac{J}{B + K_a^2 / R_a}$ mechanical time constant

K_a is called <u>electric friction</u> and $B + K_a^2 / R_a$ called <u>total friction</u>

Letting $E_a(s) = 0$



$$\frac{N(s)}{T_L(s)} = \frac{-K}{s\tau_m + 1} \tag{2.12}$$

where $K = \dfrac{1}{B + K_a^2 / R_a}$

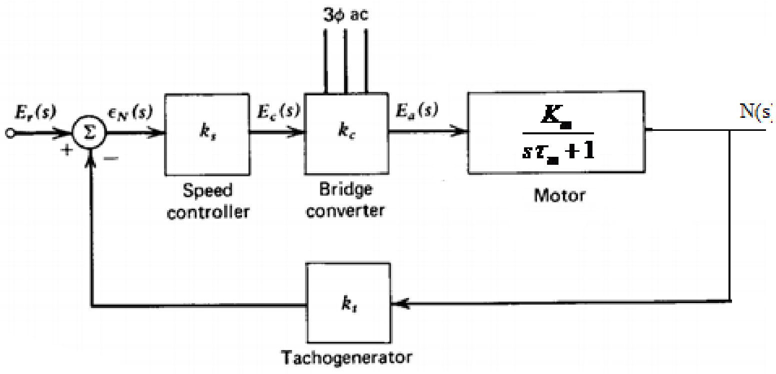
Combining eq.(2.11 and 2.12)

$$N(s) = \frac{K_m}{s\tau_m + 1} E_a(s) - \frac{K}{s\tau_m + 1} T_L(s) \tag{2.13}$$

Neglecting electrical time constant the armature voltage control is said to be <u>first order system</u>.

For the simolicity the T.F. can be represented by neglecting the torque now $\dfrac{N(s)}{E_a(s)} = \dfrac{K_m}{s\tau_m + 1}$

Motor Transfer function with tachogenerator and converter. (Armature voltage speed control)



closed-loop speed control

$$\frac{E_a(s)}{E_c(s)} = K_c = \frac{3\sqrt{2}V_{LL}}{\pi \hat{E}_c} \tag{2.14}$$

Where $\hat{E}_c$ corresponds to 0^0 firing angle and V_{LL} is the ac line to line rms value. The speed controller may be P or PI type can be taken.

Now, the closed loop T.F. can be formed sa

$$\frac{N(s)}{E_r(s)} = \frac{G(s)}{1+G(s)H(s)} \tag{2.15}$$

Where $G(s) = \dfrac{K_s K_c K_m}{1+s\tau_{m1}}$ (2.16)

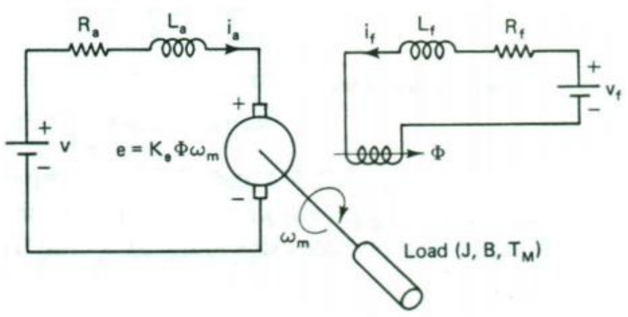
$$H(s) = K_t \tag{2.17}$$

From the eq (2.15), (2.16), (2.17) $\dfrac{N(s)}{E_r(s)} = \dfrac{K_1}{1+s\tau_1}$ (2.18)

Where $K_1 = \dfrac{K_s K_c K_m}{K_s K_c K_m K_t + 1}$ and $\tau_1 = \dfrac{\tau_{m1}}{K_s K_c K_m K_t + 1}$

Transfer function for field coontrol method:

Some de drives are operated with field control and with a constant current in thearrnature circuit. Usually, the arrnature current is maintained constant using a closed-loop system. Since the arrnature timeconstantis very small compared to the fieldtime constant, the response time of the closed-loop system controlling the arrnaturecurrent can be considered zero, and thus the change in the arrnature current due to the variation of field current and motor speed can be neglected.



The dynamics for the field control are

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \tag{2.19}$$

Assuming the armature current constant

$$T_m = K_a i_f \tag{2.20}$$

$$T_m = T_L + Bn + J\frac{dn}{dt} \tag{2.21}$$

$$\Rightarrow J\frac{dn}{dt} = T_m - T_L - Bn \Rightarrow J\frac{dn}{dt} = K_a i_f - T_L - Bn$$

Putting the Laplac transformation in those equations we get

$$V_f(s) = I_f(s)R_f + L_f s I_f(s) \Rightarrow I_f(s) = \frac{V_f(s)}{R_f(1+s\tau_f)} \tag{2.22}$$

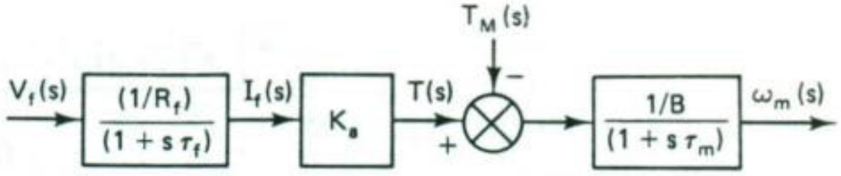
$$JsN(s) = K_a I_f(s) - T_L(s) - BN(s) \Rightarrow N(s) = \frac{K_a I_f(s) - T_L(s)}{Js+B}$$

Putting the value of $I_f(s)$ we have $N(s) = \dfrac{K_a V_f(s) - T_L(s)}{R_f(1+s\tau_f)(Js+B)}$

$$N(s) = \frac{V_f(s)(K_a / R_f B) - T_L(s)}{(1+s\tau_f)(1+s\tau_m)} \qquad (2.23)$$

For field control system which is suitable for above base speed ,the load torque is assumed to be small.so, $N(s) = \dfrac{V_f(s)(K_a / R_f B)}{(1+s\tau_f)(1+s\tau_m)} = \dfrac{N(s)}{V_f(s)} = \dfrac{K_m}{(1+s\tau_f)(1+s\tau_m)}$ (2.24)

Field control method is a <u>second order system</u>.



Block diagram of separately excited motor with field control.

Solid State Control

DC motor speed control by solid state can be done by two methods i) dc-dc Chopper control ii) Phasr rectifier control method

Chopper control of Dc motor drive: (separately excited)

This is one of the simplest power-electronic/machine circuits. With a battery, it is currently the most common electric road vehicle controller; the 'chopper' is also used for some d.c. rail traction applications. The principal difference between the thyristor-controlled rectifier and the chopper is that in the former the motor current always flows through the supply, whereas in the latter, the motor current only flows from the supply terminals for part of each cycle.

The chopper may use transistor,thyristor,MOSFET or IGBT as switches.

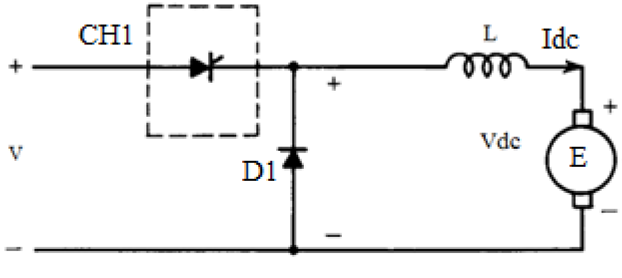
A single-switch chopper using a thyristor can supply positive voltage and current to a d.c. motor, and is therefore,restricted to quadrant 1 motoring operation. When regenerative and/or rapid speed reversal is called for, more complex circuitry is required,involving two or more power switches. When the motor voltage is less than the battery,the step down chopper is used and when the motor voltage is greater than the battery voltage, a‘step-up’ chopper using an additional inductance as an intermediate energy store is used.

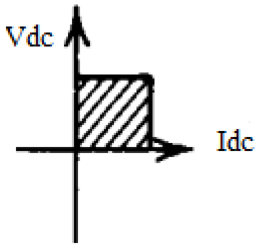
<u>Function:</u>

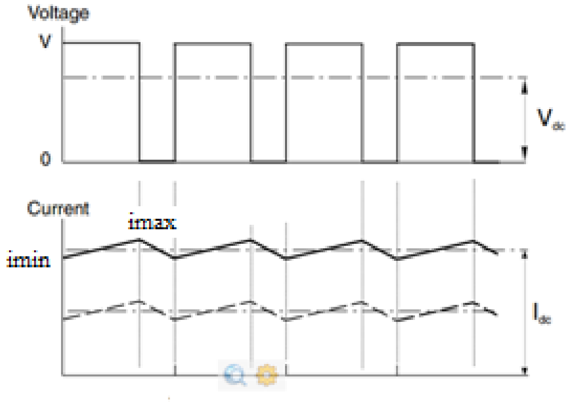
$V_{dc} = V, CH1$ on

$V_{dc} = 0, CH1$ off

D_1 on







The shape of the armature voltage waveform reminds us that when the transistor is switched on, the battery voltage V is applied directly to the armature, and during this period the path of the armature current is indicated by the dotted line in Figure . For the remainder of the cycle the transistor is turned 'off' and the current freewheels through thediode, as shown by the dotted line in Figure. When the current is freewheeling through the diode, the armature voltage is clamped at (almost) zero.

The speed of the motor is determined by the average armature volt-age, (Vdc), which in turn depends on the proportion of the total cycle time (T) for which the transistor is 'on'. If the on and off times are defined as $T_{on} = \delta T$ and $T_{on} = (1-\delta)T$ where $0 < \delta < 1$, then the average voltage is simply given by

$$V_{dc} = \frac{1}{T}\int_0^{Ton} V dt$$

$$V_{dc} = \delta V \qquad (2.25)$$

Where, $\delta = \frac{T_{on}}{T}$ duty ratio or time ratio and speed control is effected via the on time ratio, δ .

If we ignore resistance, the equation governing the current during the'on' period is

$$V = E + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} = \frac{1}{L}(V - E) \qquad (2.26)$$

During this 'on' period the battery is supplying power to the motor.So, the current rises.

During the 'off' period, the equation governing the current is

$$0 = E + L\frac{di}{dt} \qquad \text{or} \quad \frac{di}{dt} = -\frac{E}{L} \qquad (2.27)$$

So, the current fall. We note that the rise and fall of the current (i.e. the current ripple) is inversely proportional to the inductance, but is independent of the mean dc current, i.e. the ripple does not depend on the load. The current waveforms shown in Figure, the upper waveform corresponds to full load, i.e. the average current I_{dc}.produces the full rated torque of the motor. If now the load torque onthe motor shaft is reduced to half rated torque, and assuming that theresistance is negligible, the steady-state speed will remain the same but the new mean steady-state current will be halved, as shown by the lower dotted curve.

The average current I_{dc} further depends on the armature resistance that is

$$I_{dc} = \frac{\delta V - E}{R_a} \tag{2.28}$$

Now, the steady state sped of the motor is given by

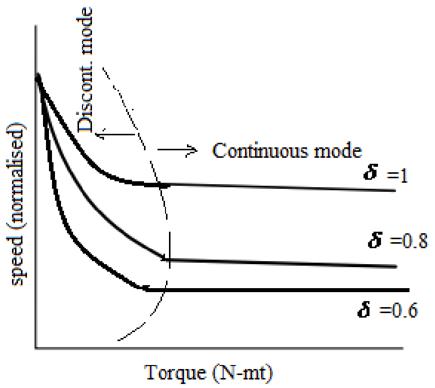
$$w_m = \frac{\delta V}{K} - \frac{R_a}{K^2} T_a \tag{2.29}$$

Torque–speed characteristics

When the armature current is continuous the speed falls only slightly with load, because the mean armature voltage remains constant. But when the armature current is discontinuous (which is most likely at high speeds and light load) the speed falls of rapidly when the load increases, because the mean armature voltage falls as the load increases. Discontinuous current can be avoided by adding an inductor in series with the armature, or by raising the chopping frequency, but when closed-loop speed control is employed, the undesirable effects of discontinuous current are masked by the control loop.

Separately excited DC motor:

The torque-speed characteristics are drooping in nature as like in phase control. However, the drop in speed is less for chopper control than for phase control because of the nature of the suplly voltage,which does not change with time.The region of discontinuous motor current operation can be reduced with chopper control by increasing the chopping frequency or indrocuing more inductance in motor circuit.



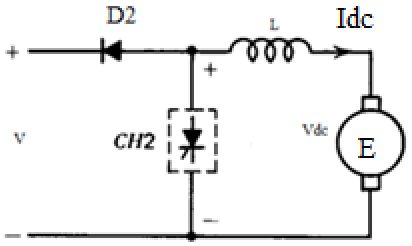
Regenerative braking (chopper drive)

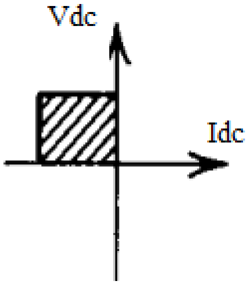
Function

$V_{dc} = 0, CH2$ on

$V_{dc} = V, CH2$ off D_2 on

The torque-speed curve is on second qudrant plane. So, regenerative braking takes place in second quadrant.





During turn on, the average voltage $V_{dc} = 0$ but, the current rises exponentially in motor circuit.During off, the average voltage $V_{dc} = \delta V$ and the current falls exponentially. The average current for this opration is

$$I_{dc} = \frac{E - \delta V}{R_a} \tag{2.30}$$

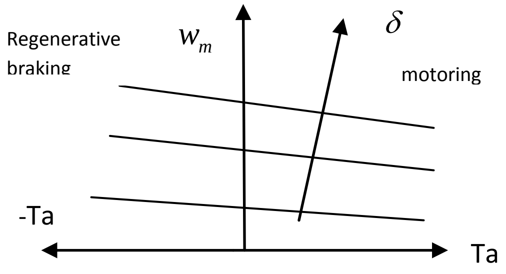
Since the torque is reversed, the toruqe is negative but the voltage polarity does not change.

$$T_b = -KI_{dc} \tag{2.31}$$

And speed is

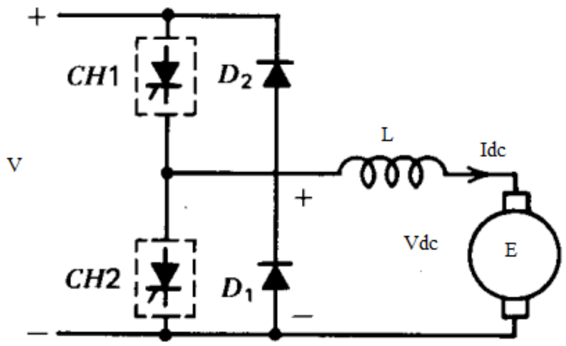
$$w_m = \frac{\delta V}{K} - \frac{R_a}{K^2} T_a \tag{2.32}$$

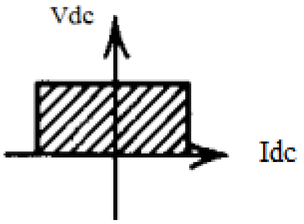
Torque-speed characteristics for motoring and regenerative braking



However**, both motoring and regenerative braking** can be brought into single chopper structure (two qudrant Type-A chopper or type-B chopper)

Type-A chopper





Function:

$V_{dc} = V, CH1$ or D_2 on

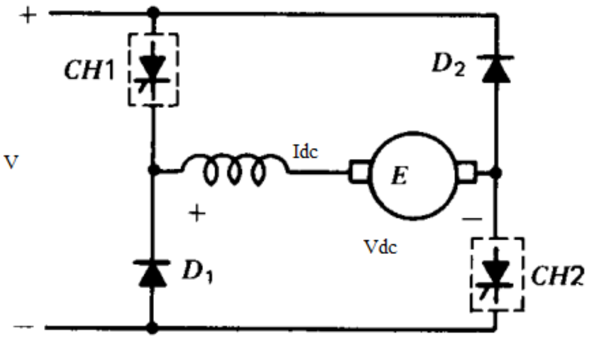
$= 0, CH2$ or D_1 on $I_{dc} = positive$

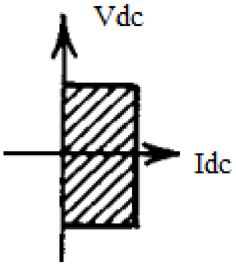
$CH1$ or D_1 on

$= negative$

$CH2$ or D_2 on

Type-B (two qudrant chopper)





Function

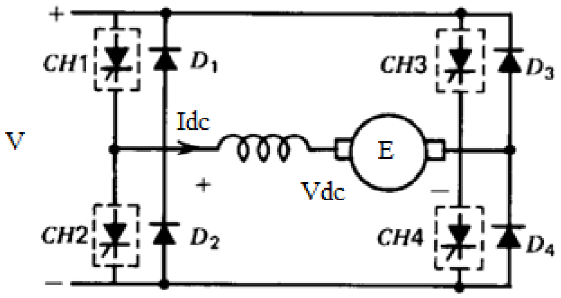
$V_{dc} = +V, CH1$ and $CH2$ on

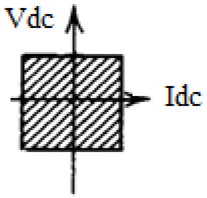
$V_{dc} = -V, CH1$ and $CH2$ off

D_1 and D_2 on

$V_{dc} = -V, CH1$

Fourth quadrant operation of Chopper fed Dc drive





Function

$V_{dc} = positive$

$I_{dc} = reversible$

$CH4$ on and $CH3$ off

$CH1$ and $CH2$ operated

$V_{dc} = negative$

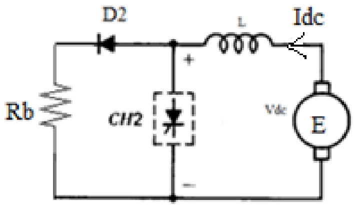
$I_{dc} = reversible$

$CH2$ on and $CH1$ off

$CH3$ and $CH4$ operated

Dynamic braking (chopper drive)

During dynamic braking,the supply is taken away and the braking resistance Rb is connected across the supply terminal. Now, before the supply is disconnected the switch (chopper) is made on.The stored kinetic energy is made to pass through the switch,and the motor parameters. Then the braking resistance is connected and the switch is made open. So, the energy is made to pass through the diode and braking resistance.The currennt is flown in reverse direction and the braking torque is developed.



The energy consumed by the braking resistance of chopper is $E_b = I_{dc}^2 R_b (T - T_{on})$ (2.33)

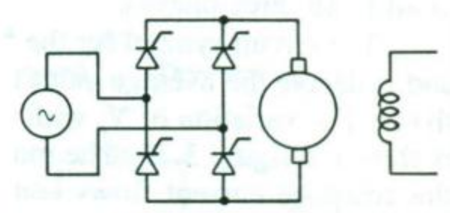
Average power consumed by $P = \frac{E_b}{T} = I_{dc}^2 R_b (1-\delta)$ (2.34)

Effective value of braking resistance is $R_b = \frac{P}{I_{dc}^2} = R_b (1-\delta)$ (2.35)

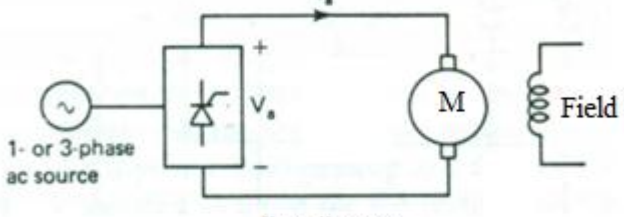
Phase controlled converter for speed control

There are a number of controlled rectifier circuits, some fed from a l-phase supply and others from a 3-phase supply. For the motor control, controlled rectifier circuits are classified as fully-controlled and half-controlled rectifiers. Single-phase controlled rectifiers are employed up to a rating of 10 kW and in some special cases up to 50 kW. For higher power ratings 3-phase controlled rectifiers are employed. As explained later in this chapter, the performance of a drive is improved when the rectifier pulse number is increased. Six-pulse operation is realized by employing the three-phase fully-controlled bridge rectifier. Twelve-pulse operation can also be obtained by connecting two six-pulse bridge controlled rectifiers.

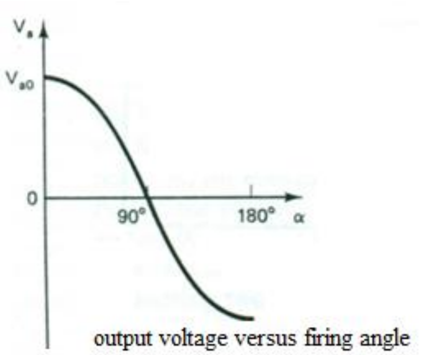
Single-phase fully controlled rectifier (Two quadrant operation)



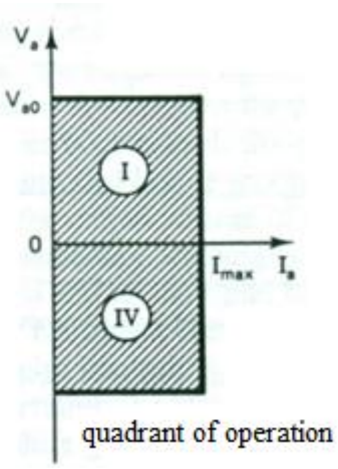
2-pulse bridge rectifier



line diagram



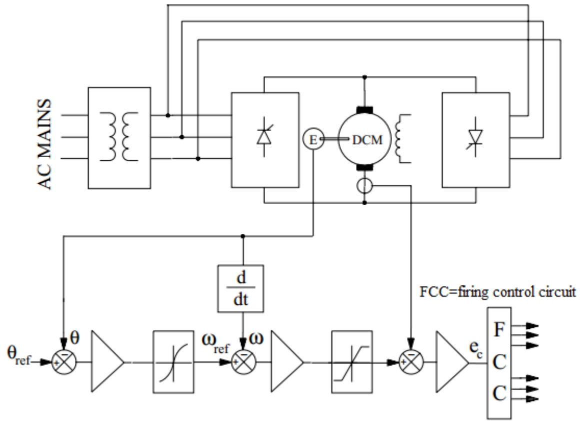
output voltage versus firing angle



quadrant of operation

Let the output average voltage from the controlled ac-to dc converter is V_a and average current is I_a respectively. The variation of V_a with the firing angle α, assuming continuous conduction, is shown in figure. The motor is said to operate in continuous conduction when the armature current flows continuously - that is, it does not become zero for a finite time interval. The output voltage can be controlled from a full-positive ($+V_{a0}$) to a full negative ($-V_{a0}$) by controlling the firing angle from 0^0 to 180^0.Since the output voltage can be controlled in either direction, the fully-controlled rectifiers are two-quadrant converters, providing operation in the first and fourth quadrants of the $V_a - I_a$ plane as shown. $I_{\max}$ is the rated rectifier current. With a negative output voltage, the rectifier works as a line-commutated inverter and the power flows from the load to the ac source.

Four quadrant operation (Closed-loop control)

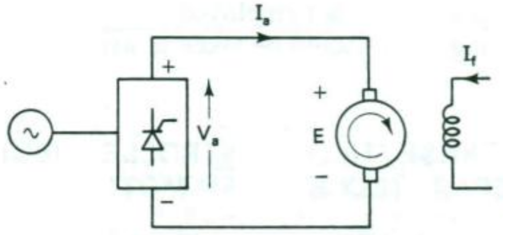


Bidirectional speed and position control system with a back-to-back (dual) thyristor converter.

A high-performance dc drive for a rolling mill drive may consist of such converter circuits connected for bi-directional operation of the drive. The interfacing of the firing control circuit to other motion-control loops, such as speed and position controllers, for the desired motion is also indicated. Two fully-controlled bridge ac–dc converter circuits are used back-to-back from the same ac supply. One is for forward and the other is for reverse driving of the motor. Since each is a two-quadrant converter, either may be used for regenerative braking of the motor. For this mode of operation, the braking converter, which operates in inversion mode, sinks the motor current aided by the back emf of the motor. The energy of the overhauling motor now returns to the ac source. It may be noted that the braking converter may be used to maintain the braking current at the maximum allowable level right down to zero speed. A complete acceleration–deceleration cycle of such a drive is indicated in Fig. During braking, the firing angle is maintained at an appropriate value at all times so that controlled and predictable deceleration takes place at all times. The innermost control loop indicated in Figure is for torque, which translates to an armature current loop for a dc drive. Speed- and position-control loops are usually designed as hierarchical control loops. Operation of each loop is sufficiently decoupled from the other so that each stage can be designed in isolation and operated with its special limiting features.

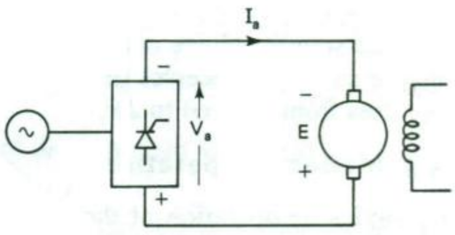
Braking operation (separately excited dc motor)

A fully-controlled rectifier-fed de separately excited motor is shown in figure. The polarities of output voltage, back emf, and armature current shown are for the motoring operation in the forward direction. The rectifier output voltage is positive and the firing angle is $0^0 \le \alpha \le 90^0$.The motor can be made to work under regenerative braking if the armature current can reverse. This is not possible because the rectifier can carry current only in one direction. The only alternative available for the reversal of the flow of power is to reverse both the rectifier output voltage V_a and the motor back emt E with respect to the rectifier terminals and make $E > V$



$0 \le \alpha \le 90°$ and $V_a > E$

(a) Motoring



$90° < \alpha < 180°$ and $|E| > |V_a|$

Regenerative braking

The reversal of the motor emf with respect to the rectifier terminals can be done by any of the following changes:

1. An active load coupled to the motor shaft may drive it in the reverse direction. This gives reverse regeneration (that is, operation in quadrant IV of the speed-torque plane). In this case no changes are required in the armature connection with respect to the rectifier terminals.
2. The field current may be reversed, with the motor running in the forward direction. This gives forward regeneration. In this case also no changes are required in the armature connection.
3. The motor armature connections may be reversed with respect to the rectifier output terminals, with the motor still running in the forward direction. This will give forward regeneration.

Steady-State Motor Performance Equations (continuous conduction)

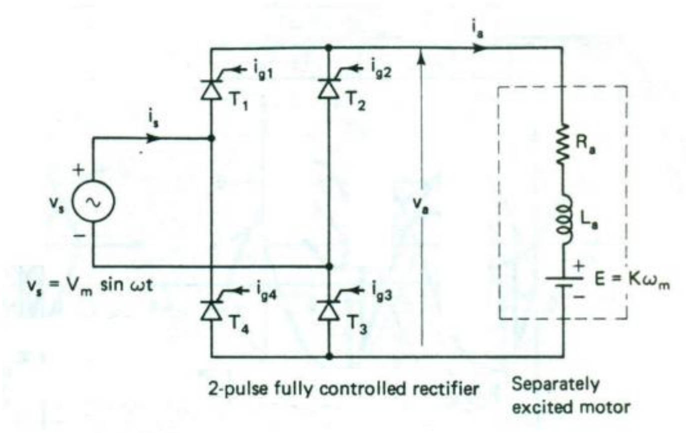
For the purpose of analysis, the following assumptions are made:

1. Thyristors are ideal switches-that is, they have no voltage drop when con-ducting and no leakage current when blocking. The main implication of this assumption is that the rectifier voltage drop and losses are neglected. This assumption should not be used with low-voltage motors.

2. The armature resistance and inductance are constant. The skin effect, which is present due to a ripple in the motor current, does alter the value of the resistance.
3. During a given steady-state operation, the motor speed is constant. The motor torque does fluctuate due to the ripple in the motor current. Because the mechanical time constant is very large compared to the period of current ripple, the fluctuation in speed is in fact negligible. At constant speed, one can assume the back emf E is an ideal direct voltage for a given steady-state operation.
4. Source inductance is negligible.

The rectifier output voltage consists of one or two of the following intervals:

1. Duty Interval
2. During this interval



Duty Interval. When T1 and T3 conduct

$$v_a = L_a \frac{di_a}{dt} + R_a i_a + Kw_m = V_m \sin wt \tag{2.36}$$

When T2 and T4 conduct

$$v_a = L_a \frac{di_a}{dt} + R_a i_a + Kw_m = -V_m \sin wt \tag{2.37}$$

Zero current Interval:

$$i_a = 0 \quad \text{and} \quad v_a = Kw_m \tag{2.38}$$

The average voltage can be found considering the any one pair of switches during the duty interval
Average motor terminal voltage Va = average voltage drop across R_a + average voltage drop across L_a + back emf

$$\text{Now } V_a = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin wt\, d(wt) = \frac{2V_m}{\pi}\cos\alpha = V_{a0}\cos\alpha \tag{2.39}$$

$$\text{Where } V_{a0} = \frac{2V_m}{\pi}$$

From the motor equation

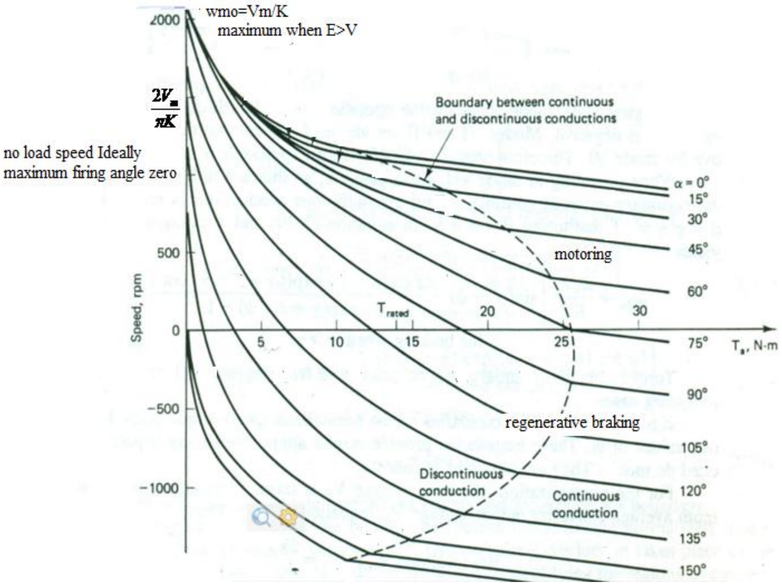
$$V_a = I_a R_a + Kw_m \ \text{(average voltage across La is zero)} \tag{2.40}$$

$$I_a = \frac{V_a - Kw_m}{R_a} = \frac{\left(\frac{2V_m}{\pi}\cos\alpha\right) - Kw_m}{R_a} \tag{2.41}$$

Comparing eq. (2.39), (2.40) and (2.41)

$$w_m = \frac{2V_m}{\pi K}\cos\alpha - \frac{R_a}{K_2}T_a \tag{2.42}$$

Speed-Torque Characteristics



For torques less than the rated value, a low-power drive operates predominantly in the discontinuous conduction. In continuous conduction, the speed-torque characteristics are parallel straight lines, whose slope, according to equation (2.42), depends on the armature circuit resistance Ra. The effect of discontinuous conduction is to make the speed regulation poor. In continuous conduction, for a given a, any increase in load causes E and Wm to drop so that la and Ta can increase. The average terminal voltage Va remains constant. On the other hand, in discontinuous conduction, any increase in load, and the accompanied increase in la causes β to increase. Consequently Va reduces, and the speed drops by a larger amount than in the case of continuous conduction. Other disadvantages of discontinuous conduction are the nonlinear transfer characteristics of the converter and the slower transient response of the drive.

Stability:

Stability of the closed-loop system can be checked by examining the frequency response of the open-loop system, the gain being adjusted to ensure (by means of design criteria known as gain and phase margins) that, when the loop is closed for the first time, there is no danger of instability. This is that if the d.c. loop gain is too high, some closed-loop systems exhibit self-sustaining oscillations. When a system behaves in this way it is said to be unstable, and clearly the consequences can be extremely serious, particularly if large mechanical elements are involved. Also unstable behavior is characteristic of linear systems of third or higher order. Whenever the closed-loop system has an inherently oscillatory transient response, increasing the proportional gain and/or introducing integral control generally makes matters worse that is the output response may become larger and settling time is more. So, the dc gain should be within its limit.

MODULE-3

ELRCTIC TRACTION

Traction system can be classified into non-electric and electric traction. Non-electric traction does not use electric at any stage. For ex. (steam engine drive and internal combustion drive).However, the electric drive system has certain advantages over other systems. So, electric drive system is widely used in rail traction.

Advantages:

- Due to cleanliness and pollution free, it can be used in underground railways.
- Starting torque is high, speed control is simple, braking is simple and efficient. By regenerative braking can be pumped back into the supply and saving the electric energy
- Less maintenance than steam locomotive.
- Put into service immediately
- The coefficient of adhesion is high
- Center of gravity is lower than steam locomotive. Hence it runs faster at curved routes.
- Saving high grade of coal and diesel

Disadvantages:

- High capital cost in erecting overhead supply
- Power failure for few minutes can cause dislocation of traffic for hours
- Communication lines gets interference

System of electric traction:

- DC system
- Single phase low frequency ac system
- Single phase high frequency ac system
- Three phase system
- Composite system

At present composite system is used for power supply. Composite systems are two types i) Kando system (single phase to three phase system) ii) Single phase to dc system

Kando System: This is the single phase,25 KV,50 hz supply that is converted to three phase at low frequency (½ to 10 hz) by the converter inside the locomotive. The three phase induction motor is used at low frequency due to the following advantages.

- Draws less current
- Improves the efficiency
- Speed control is easy

Advantages and disadvantages of 25KV, 50Hz supply

Advantages:

- Due to high voltage supply, the line current is less which will reduce the cross section of the conductor and makes the supporting structure light.
- Saving of substations
- Track is the return conductor
- Coefficient of adhesion is high

Disadvantages:

- Single phase ac system traction produces both voltage and current unbalance. So special care are to be taken
- Current unbalance produces heating in the alternator
- Voltage unbalance produces heating on induction motor
- Gives interference to the communication line
- Produces harmonics

Types of traction:

Three types of services for the passenger are available in rail traction.

- Urban service (Local train, the distance between stations <10 km)
- Suburban service (2.5 to 3.5 km)
- Main line service (intercity train and goods train, >10 km)

Goods train service also are three types:

- Main line freight service
- Local freight service
- Shunting service

In India, locomotives are classified according to their track gauge, motive power, the work they are suited for and their power or model number. The class name includes this information about the locomotive. It comprises 4 or 5 letters. The first letter denotes the track gauge. The second letter denotes their motive power (Diesel or Electric) and the third letter denotes the kind of traffic for which they are suited (goods, passenger, mixed or shunting). The fourth letter used to denote locomotives chronological model number.

The classification syntaxes

The first letter (gauge)

- W – Indian broad gauge (the "W" Stands for Wide Gauge - 5 ft 6 in)
- Y – meter gauge (the "Y" stands for Yard Gauge - 3 ft or 1000mm)
- Z – narrow gauge (2 ft 6 in)

The second letter (motive power)

- D – diesel
- C – DC electric (can run under DC overhead line only)
- A – AC electric (can run under AC overhead line only)
- CA – both DC and AC (can run under both AC and DC overhead line); 'CA' is considered a single letter
- B – Battery electric locomotive (rare)

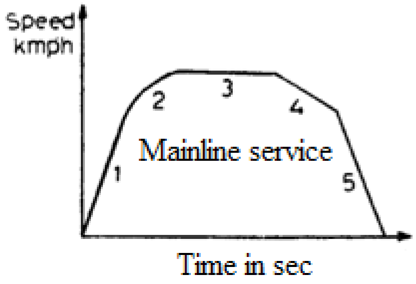
The third letter (job)

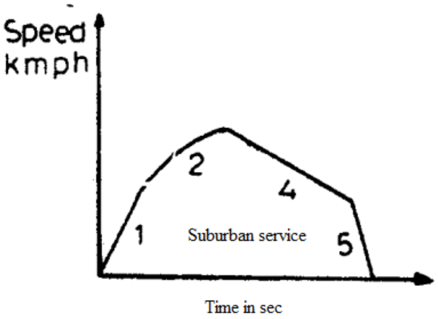
- G – goods
- P – passenger
- M – mixed; both goods and passenger
- S –shunting (also known as switching engines or switchers in the USA and some other countries)
- U – electric multiple unit (used to carry commuters in city suburbs)
- R – Railcars

For example, in "**WDM 3A**":

"W"- broad gauge, "D"- diesel, "M"- suitable for both passenger and goods train, "3A"- locomotive power is 3,100hp (3-stands for 3000hp, A-stands for 100hp more)

Speed –Time curve for train movement:



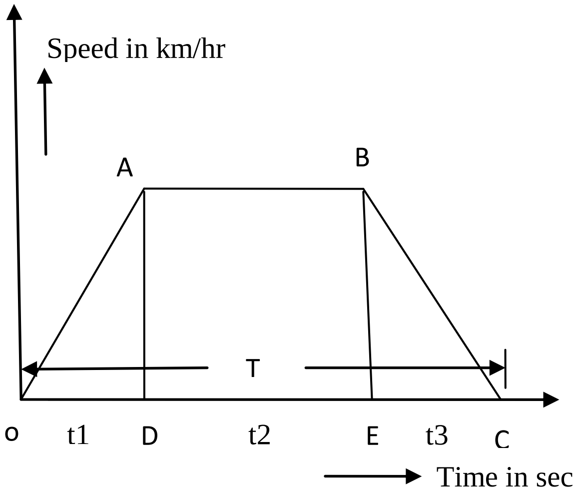


1. Acceleration with constant current (notching period called constant torque region)
2. Acceleration with constant voltage (constant power region)
3. Free running period (speed constant)
4. Coasting period (running with power cut off)
5. Braking period (retardation)

Distance and speed calculation fro curve:

The simplified version of curve is drawn for main line service for calculation of speed and time

Trapezoidal speed-time curve



D =Distance in km between two stations

T = running time between two stations in sec

α = acceleration in kmphpsec

β =retardation in kmphpsec

N_m =maximum speed in kmph

N_a = average speed in kmph

$t_1 = \frac{N_m}{\alpha} =$ time of acceleration

$t_3 = \frac{N_m}{\beta}$ =time of retardation

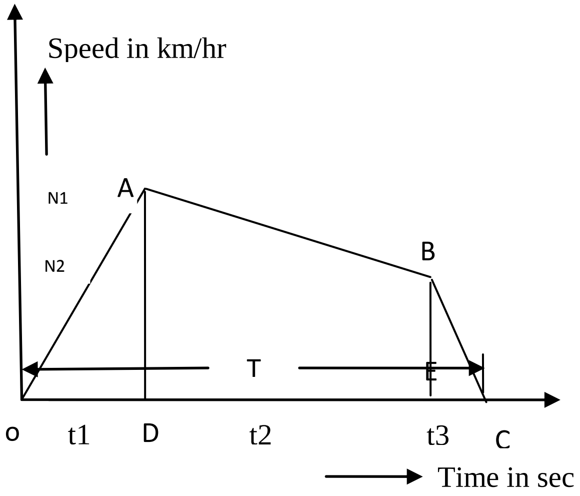
The total distance can be formulated from the above curve that

$$D = \frac{N_m}{3600}(T - N_m k) \quad \text{in km} \tag{3.1}$$

Where $k = \frac{\alpha + \beta}{2\alpha\beta}$

For urban and sub urban duties the quadrilateral speed-time curve can be done.

quadrilateral speed-time curve



β_c = coasting retardation in kmphpsec

β = retardation in kmphpsec

$t_2 = \frac{N_1 - N_2}{\beta_c}$ coasting time in sec

$t_2 = \frac{N_1}{\alpha}$ acceleration time in sec

$$D = \frac{1}{7200}[T(N_1 + N_2) - N_1 N_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)] \quad \text{in km} \tag{3.2}$$

Average speed and scheduled speed:

The average speed of a train is defined as the ratio of the distance between the consecutive stations to the time taken by the train to travel the distance.

Scheduled speed: The scheduled speed is defined as the distance between two consecutive stations to the actual time of run and time for stop. So, scheduled speed is always less than the average speed.

MODULE-4

HEATING AND COOLING

The size and power rating of the motor depends upon

- the heating effect
- loading condition and classes of duty
- Environmental condition

Insufficient power rating, either fails to drive or damages and shut down due to overloading of the motor and power modulator. However, induction and synchronous motor operates at a low power factor when operating below rated power.

When the motor operates heat is produced due to losses (copper, iron and friction)inside the machine and its temperature rises. The heat produced during no load is from iron parts to winding and when loaded heat is flows from winding to surrounding as more heat is seen on winding than winding. The temperature reaches at a steady state when the heat generated becomes equal to heat dissipated into the surrounding medium. This steady temperature depends on power loss, which in turn depends on the output power of the machine. Since the temperature has direct relation with output power, it is termed as thermal loading of the machine.

An electric machine is designed for a given temperature rise is decided by its insulation. The various insulations are Y,A,E,B,F,H,C and the corresponding temperature sustainable are $90^0, 105^0, 120^0\, 130^0$, $155^0, 180^0$ and above 180^0.

Thermal Modeling

It is very difficult to have the thermal modeling of the machine due to its heterogeneous material. During the no load heat is more in iron parts and it flows to the winding and during overload heat flows from winding to the surrounding. So, there is uniform of temperature gradient. For accurate thermal modeling the following assumptions are made.

- The machine is considered to be homogeneous
- Heat dissipation is proportional to the different of temperature between the body and the surrounding
- The rate of heat dissipation is constant.

Heating Curve:

Heat balance equation shows

Heat generated=heat dissipated to the surrounding + heat stored in the body

That is $$Wdt = A\lambda\theta dt + Gsd\theta \qquad (4.1)$$

Where W = power loss on the motor responsible for heat in time (dt) in watt

G = weight of the acyive parts of the motor in kg

s = specific heat of the material of the body in J/degree/kg

A =cooling surface in mt^2

λ = sp. Heat dissipation or emissivity in J/s/mt^3/degree

θ = temp. rise of the body (degree)

$d\theta$ = temp. rise due (dt)

When the temp. reaches a constant value the body is said to be reached a maximum value (θ_m). The change in temperature $d\theta = 0$

So there is no store of heat in the body.

The heat generated ($Wdt =$ heat dissipated $A\lambda\theta dt$)

Now

$$Wdt = A\lambda\theta_m dt \tag{4.2}$$

$$\Rightarrow W = A\lambda\theta_m$$

$$\theta_m = \frac{W}{A\lambda} \quad \text{(maximum temp. rise)} \tag{4.3}$$

Eq. (4.1) can be re arranged as $Wdt - A\lambda\theta dt = Gsd\theta$

$$(W - A\lambda\theta)dt = Gsd\theta$$

$$\frac{d\theta}{dt} = \frac{W}{Gs} - \frac{A\lambda}{Gs}\theta \tag{4.4}$$

If cooling medium is absent, then no dissipation takes place. So, eq (4.1) is reduced to

$$Wdt = Gsd\theta \tag{4.5}$$

Which gives linear relationship between θ and t.

Therefore,

$$\frac{\theta}{t} = \frac{W}{Gs} \tag{4.6}$$

If τ_1 is the time taken to reach θ_m, then

$$\frac{\theta_m}{\tau_1} = \frac{W}{Gs} \tag{4.7}$$

Substituting $\theta_m = \frac{W}{A\lambda}$ in eq (4.7) we have, $\tau_1 = \frac{Gs}{A\lambda}$ (4.8)

Where τ_1 is the thermal (heating) time constant.

Further eq (4.1) can be arranged to find the time during temperature rise as

$$Wdt = A\lambda\theta dt + Gsd\theta$$

$$Wdt - A\lambda\theta dt = Gsd\theta$$

$$dt = \frac{Gsd\theta}{W - A\lambda\theta} \quad \text{Integrating both sides}$$

$$\int dt = \left(\int \frac{Gsd\theta}{W - A\lambda\theta} \right)$$

$$\Rightarrow t = -\frac{Gs}{A\lambda}(\log(W - A\lambda\theta) - \log K) \tag{4.9}$$

Log K is constant to be found out at initial conditions

Eq. (4.9) can be written as $\log\left(\frac{W - A\lambda\theta}{K}\right) = -t\left(\frac{A\lambda}{Gs}\right)$

$$\Rightarrow \frac{W - A\lambda\theta}{K} = e^{-t\left(\frac{A\lambda}{Gs}\right)} \tag{4.10}$$

At t=0, $\theta = 0$ so, $K = W$

From eq.(4.10) $\theta = \theta_m\left(1 - e^{-\frac{t}{\tau}}\right)$ (4.11)

Where, $\tau = \frac{Gs}{A\lambda}$ and $\theta_m = \frac{W}{A\lambda}$

But in many cases the initial temperature is not zero. And hence at t=0, $\theta = \theta_0$ So, eq. (4.10) reduces to

$\frac{W - A\lambda\theta}{W - A\pi\theta_0} = e^{-\frac{t}{\tau}}$ which reduces $\theta = \theta_m\left(1 - e^{-\frac{t}{\tau}}\right) + \theta_0 e^{-\frac{t}{\tau}}$ (4.12)

Cooling Curve:

When the machine is switched off from the supply or when the load is reduced, the machine cools. For first case, the machine cools to the ambient temperature.

Case-1

$0 = A\lambda\theta dt + Gsd\theta$

$\Rightarrow dt = -\frac{Gsd\theta}{A\lambda\theta}$ integrating both sides $\Rightarrow t = -\frac{Gs}{A\lambda}(\log(W - \log K_1)$

$\Rightarrow \theta = K_1 e^{-\frac{t}{\tau_2}}$ where $\tau_2 = \frac{Gs}{A\lambda}$ (cooling time constant)

When t=0, $\theta = \theta_m$, $K_1 = \theta_m$

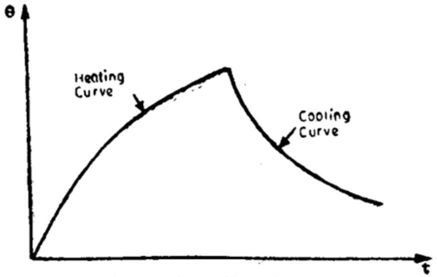
So, $\theta = \theta_m e^{-\frac{t}{\tau_2}}$ (4.13)

Case-2

When the load is reduced, let the temperature cools off from θ_f ,so, $\theta_f = \frac{W_1}{A\lambda}$ where W_1 is the total loss at reduced load, then

$$\theta = \theta_m\left(1 - e^{-\frac{t}{\tau_2}}\right) + \theta_f e^{-\frac{t}{\tau_2}} \tag{4.14}$$

For forced cooled $\tau_1 = \tau_2$



Heating and cooling time curves.

Drives for Specific Applications

Steel Mill:

The major function of rolling of steel mill is to reduce the cross section of the metal s while increasing the length proportionally. Steel mill usually produce blooms, slabs, rails, sheets, strips, beams, bar and angles.

Technologically steel mill is divided into four categories :

- Continuous cold rolling mills
- Reversing cold rolling mills
- Continuous hot rolling mills
- Reversing hot rolling mills

In reversing mill there is only one stand carrying the rolls that press the metal and metal is passed through this stand alternately forward and backward several times in order to reduce it to desired size. Each motion or travel is known as pass. A continuous mill consists of several stands, each one of them carrying pressing rolls. The metal passes through all the stands in only one direction and gets rolled.

Drives used in steel mills:

Dc motors is usually used in both reversible and continuous mills. Motor for reversing mills must have high starting torque, wide speed range, precise speed control, be able to withstand overload and pull out torque. Acceleration from zero to base speed and then to top speed and subsequent reversal from top speed backward to top speed forward must be achieved in few second. The moment of inertia of armature must be as small as possible and motors are enclosed and force ventilated. Ward-Leonard method for speed control is used. However, the speed control is replaced by thyristorised converter.

Paper Mills:

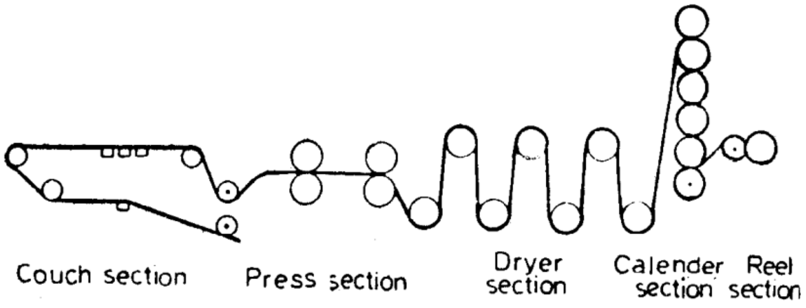
Pulp making and paper making are the two main important job for paper mills. The drive required each of them is quite different.

Pulp Making:

Pulp making requires grinding machines which almost run at constant speed that is acquired by synchronous motor. Motors run at speed of 200-300 rpm. However, pulp by mechanical means ,the motor runs at speed of 2000-3000 rpm for large grinder. Pulp is made by cutting the logs into several pieces and treated with alkalies and grass, rags etc. During the chemical treatment the material is continually beaten by the beater. Beater requires speed less than 200 rpm so, slip ring induction motor is used. The end product of the beater is passed to chipping and refining .so, synchronous motor is used.

Paper making:

The machine that makes the paper from pulp has to perform several jobs from five sections. i)Couch section(Wire section) ii) Press section iii) Dryer section iv) Calender section v) Reel section



Drive Requirement:

- Speed should be adjustable over a range of as large as 10:1
- In the wet end of paper machine the speed section should be independently adjustable.
- In the last two sections, speed control circuit must be good enough for tension control
- Control system employed should be flexible in nature.

Textile Mills:

From the raw material to finishing of cloths the mill has to perform several processes such as cotton to slivers, spinning, weaving and finishing.

Cotton to slivers: The process by which the seeds are separated from cotton is called ginning. The cottons are converted into slivers and then processed by drawing machine. The slivers are then made lap form.

Spinning: In this process, the slivers are made yarn is made of sufficient strength. This yarn is wound on bobbins by winding machine.

Weaving: The yarn is made in uniform layers. Weaving consists of two sets of threads, one which extends throughout the length of the fabric and other whose thread go across. This process is done in a loom.

Finishing: This consists a number of processes such as bleaching, dying, printing, calendaring, stamping and packing. The impurities like oil and grease are removed and the fabric is made white by bleaching.

Drives used in different sections:

In loom process loom motor is used where frequent start and stop is required. These results high temperature rise and sufficient ventilation is required. High torque three phase squirrel cage induction motor is used in loom motor. The motor is totally enclosed and must have capacity to absorbs the moisture content during the process.

Card motor: It is similar to loom motor but it runs continuously for card drum.

Spinning motor: For good quality of spinning the acceleration must be smooth. Three types of drives are required here, single speed motor (4 or 6 pole squirrel cage induction motor),two speed motor(4/6 or 8/6 pole motor) and two motor drive (two separate motor for driving single pulley)

Microprocessor Based drive:

Advantages:

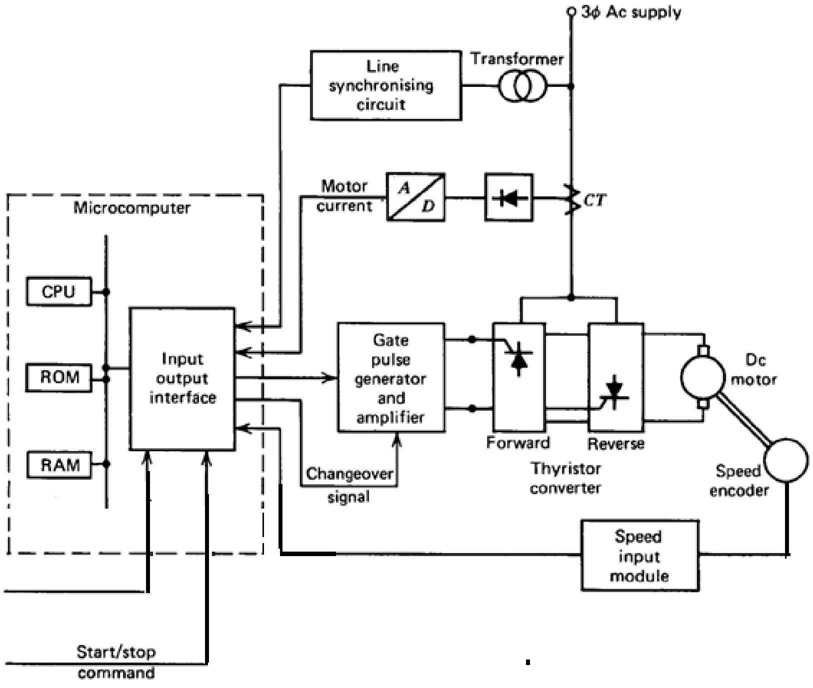
- The complexity of the system is reduced
- The software supported control using micro-processors performs the function of controllers, feedback, decision making of the drive system
- The hardware implementation in thyristerised controller unit four-quadrant operation using dual converter, vector control can be realized with software programs on micro-processor with least possible hardware
- Digital control has an inherent improved noise immunity

- The control is free from drift and parameter variation due to temperature

Limitations:

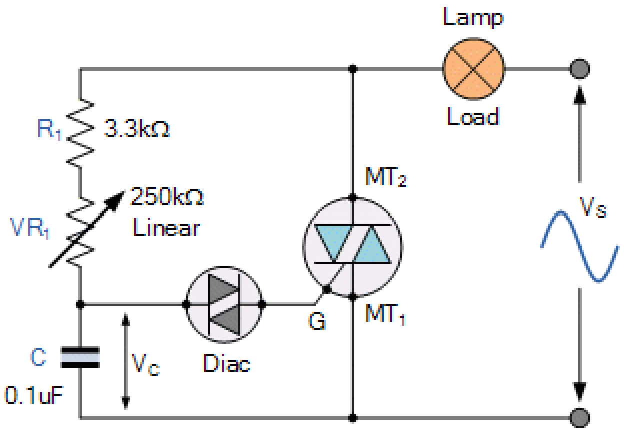
- Due to communication between the microprocessor and the analog circuitry done by A/D and D/A converter, there are sampling and quantizing error
- The response in micro-processor is slow in comparison with dedicated hardware
- The development of software may be costly and time consuming

Closed loop dc drive microprocessor based speed control



Block diagram of a reversing dc drive using microcomputer control system.

CONTENT BEYOND SYLLABUS



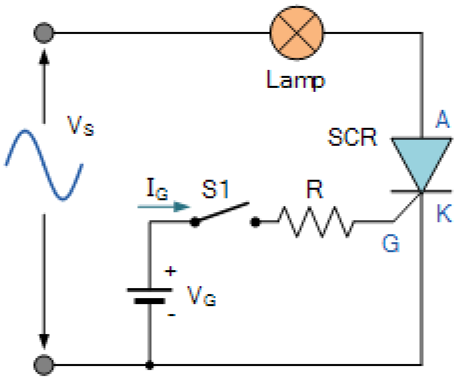
Diac Tutorial

The Diac is a two-junction bidirectional semiconductor device designed to break down when the AC voltage across it exceeds a certain level passing current in either direction

The **DI**ode **AC** switch, or **Diac** for short, is another solid state, three-layer, two-junction semiconductor device but unlike the transistor the *Diac* has no base connection making it a two terminal device, labelled A_1 and A_2.

Diac's are an electronic component which offer no control or amplification but act much like a bidirectional switching diode as they can conduct current from either polarity of a suitable AC voltage supply.

In our tutorial about *SCR's* and *Triacs*, we saw that in ON-OFF switching applications, these devices could be triggered by simple circuits producing steady state gate currents as shown.



When switch, S1 is open no gate current flows and the lamp is "OFF". When switch S1 is closed, gate current I_G flows and the SCR conducts on the positive half cycles only as it is operating in quadrant I.

We remember also that once gated "ON", the SCR will only switch "OFF" again when its supply voltage falls to a values such that its Anode current, I_A is less than the value of its holding current, I_H.

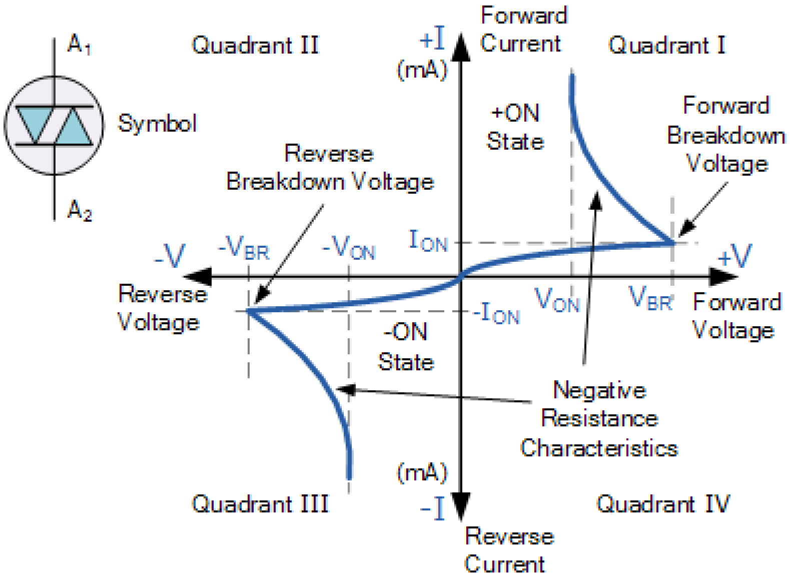
If we wish to control the mean value of the lamp current, rather than just switch it "ON" or "OFF", we could apply a short pulse of gate current at a pre-set trigger point to allow conduction of the SCR to occur over part of the half-cycle only. Then the mean value of the lamp current would be varied by changing the delay time, T between the start of the cycle and the trigger point. This method is known commonly as "phase control".

But to achieve phase control, two things are needed. One is a variable phase shift circuit (usually an RC passive circuit), and two, some form of trigger circuit or device that can produce the required gate pulse when the delayed waveform reaches a certain level. One such solid state semiconductor device that is designed to produce these gate pulses is the **Diac**.

The diac is constructed like a transistor but has no base connection allowing it to be connected into a circuit in either polarity. Diacs are primarily used as trigger devices in phase-triggering and variable power control applications because a diac helps provide a sharper and more instant trigger pulse (as opposed to a steadily rising ramp voltage) which is used to turn "ON" the main switching device.

The diac symbol and the voltage-current characteristics curves of the diac are given below.

Diac Symbol and I-V Characteristics



We can see from the above diac I-V characteristics curves that the diac blocks the flow of current in both directions until the applied voltage is greater than V_{BR}, at which point breakdown of the device occurs and the diac conducts heavily in a similar way to the zener diode passing a sudden pulse of voltage. This V_{BR} point is called the Diacs breakdown voltage or breakover voltage.

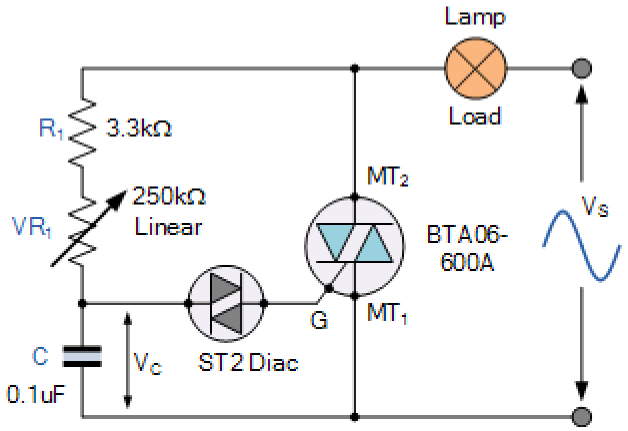
In an ordinary zener diode the voltage across it would remain constant as the current increased. However, in the diac the transistor action causes the voltage to reduce as the current increases. Once in the conducting state, the resistance of the diac falls to a very low value allowing a relatively large value of current to flow. For most commonly available diacs such as the ST2 or DB3, their breakdown voltage typically ranges from about ±25 to 35 volts. Higher breakover voltage ratings are available, for example 40 volts for the DB4 diac.

This action gives the diac the characteristic of a negative resistance as shown above. As the diac is a symmetrical device, it therefore has the same characteristic for both positive and negative voltages and it is this negative resistance action that makes the **Diac** suitable as a triggering device for SCR's or triacs.

Diac Applications

As stated above, the diac is commonly used as a solid state triggering device for other semiconductor switching devices, mainly SCR's and triacs. Triacs are widely used in applications such as lamp dimmers and motor speed controllers and as such the diac is used in conjunction with the triac to provide full-wave control of the AC supply as shown.

Diac AC Phase Control



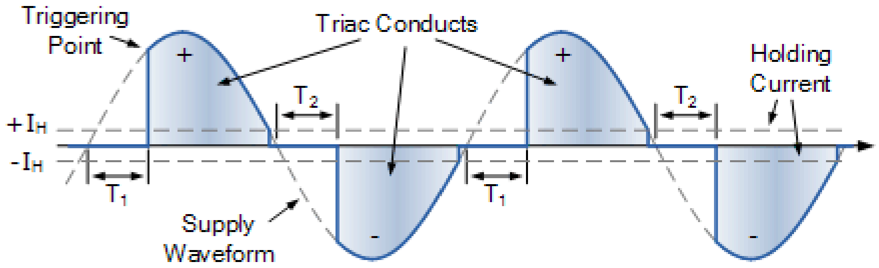
As the AC supply voltage increases at the beginning of the cycle, capacitor, C is charged through the series combination of the fixed resistor, $R1$ and the potentiometer, VR_1 and the voltage across its plates increases. When the charging voltage reaches the breakover voltage of the diac (about 30 V for the ST2), the diac breaks down and the capacitor discharges through the diac.

The discharge produces a sudden pulse of current, which fires the triac into conduction. The phase angle at which the triac is triggered can be varied using $VR1$, which controls the charging rate of the capacitor. Resistor, R1 limits the gate current to a safe value when VR1 is at its minimum.

Once the triac has been fired into conduction, it is maintained in its "ON" state by the load current flowing through it, while the voltage across the resistor–capacitor combination is limited by the "ON" voltage of the triac and is maintained until the end of the present half-cycle of the AC supply.

At the end of the half cycle the supply voltage falls to zero, reducing the current through the triac below its holding current, I_H turning it "OFF" and the diac stops conduction. The supply voltage then enters its next half-cycle, the capacitor voltage again begins to rise (this time in the opposite direction) and the cycle of firing the triac repeats over again.

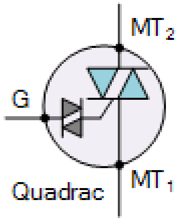
Triac Conduction Waveform



Then we have seen that the **Diac** is a very useful device which can be used to trigger triacs and because of its negative resistance characteristics this allows it to switch "ON" rapidly once a certain applied voltage level is reached. However, this means that whenever we want to use a triac for AC power control we will need a separate diac as well. Fortunately for us, some bright spark somewhere replaced the individual diac and triac with a single switching device called a **Quadrac**.

The Quadrac

The *Quadrac* is basically a *Diac* and *Triac* fabricated together within a single semiconductor package and as such are also known as "internally triggered triacs". This all in one bi-directional device is gate controlled using either polarity of the main terminal voltage which means it can be used in full-wave phase-control applications such as heater controls, lamp dimmers, and AC motor speed control, etc.



Like the triac, quadracs are a three-terminal semiconductor switching device labelled MT2 for main terminal one (usually the anode), MT1 for main terminal two (usually the cathode) and G for the gate terminal.

The quadrac is available in a variety of package types depending upon their voltage and current switching requirements with the TO-220 package being the most common as it is designed to be an exact replacement for most triac devices.

Diac Tutorial Summary

In this diac tutorial we have seen that the diac such as the ST2 or DB3 is a two-terminal voltage blocking device that can conduct in either direction. Diacs posses negative resistance characteristics which allows them to switch "ON" rapidly once a certain applied voltage level is reached.

Since the diac is a bidirectional device, when paired with the BTAxx-600A or IRT80 series of switching triacs it makes it useful as a triggering device in phase control and general AC circuits such as light dimmers and motor speed controls.

Quadracs are simply triacs with an internally connected diac. As with triacs, quadracs are bidirectional AC switches which are gate controlled for either polarity of main terminal voltage.

Industrial Applications of Power Electronics

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1. Introduction

In recent years, power electronics have been intensely contributing to the development and evolution of new structures for the processing of energy. It is becoming very common to generate electrical energy in different ways and convert it into another form in order to be able to use it—for instance, renewable sources, battery banks, and the transmission of electric power in direct current (DC), which makes the voltage of the network available in different levels, in detriment to the supplied voltage from the grid [1]. The main users of these signals are pieces of electronic equipment that use voltages at levels different from that which is available from the grid; the drives of electrical machines, which modify the voltage of the electrical network (amplitude and frequency) to control the machines and finally, in electrical systems, DC power transmission and frequency conversion [2].

Two leading trends are currently noticeable in the power systems field of study. The first trend is the increasingly and prevalent employment of renewable energy resources. The second trend is decentralized energy generation. This scenario raises many challenges. Therefore, the design, development, and optimization of power electronics and controller devices are required in order to face such challenges. New microprocessor control units (MCUs) could be utilized for power production control and for remote control operation, while power electronic converters are and could be utilized to control the power flow [3].

Nevertheless, power electronics can be used for a wide range of applications, from power systems and electrical machines to electric vehicles and robot arm drives [4]. In conjunction with the evolution of microprocessors and advanced control theories, power electronics are playing an increasingly essential role in our society [5].

Thus, in order cope with the obstacles lying ahead, original studies and modelling methods can be developed and proposed that could overcome the physical and technical boundary conditions and at the same time, consider technical, economic, and environmental aspects. The objective of this Special Issue was to present studies in the field of electrical energy conditioning and control using circuits and electronic devices, with an emphasis on power applications and industrial control. Therefore, researchers contributed their manuscripts to this Special Issue, and contribute models, proposals, reviews, and studies. In this Special Issue, 19 selected and peer-reviewed papers contribute to a wide range of topics, by addressing a wide variety of themes, such as motor drives, AC-DC and DC-DC Converters, multilevel converters and electromagnetic compatibility, among others.

A significant portion of the currently produced electricity worldwide is mostly generated by centralized systems, based on conventional fossil fuel plants or nuclear power [6,7]. The barriers that policy makers, researchers, and engineers have to overcome when it comes to the operation and control of conventional power plants, and the development of low voltage power generation systems, have paved the way for diverse opportunities of energy generation, closer to the load, by the customers themselves, also known as distributed generation (DG) [8]. Thus, concerning this topic, several papers are published in this Issue.

In [9], an efficient H7 single-phase photovoltaic grid-connected inverter for common mode current conceptualization and mitigation is proposed. Specifically, an extended H6 transformerless inverter that operates with an additional power switch (H7) is utilised for improving the common mode leakage current mitigation in a single-phase utility grid. A new control for a modular multilevel converter (MMC) based static synchronous compensator (STATCOM) is proposed in [10] as an effective interface between energy sources and the power grid. This study showed that the proposed control method led to an effective reduction in capacitor voltage fluctuation and losses. The protection of sensitive loads against voltage drop is a challenge for the power system, especially in face of the rising use of DG. Thus, in order to address this obstacle, a compound current limiter and circuit breaker is proposed in [11] and validated through experimental and simulation results. The authors argue that in this study that the proposed compound current limiter is able to limit the fault current and fast break in order to adjust voltage sags at the protected buses. A data-driven based voltage control strategy for DC-DC converters, which are increasingly used to integrate renewable energy resources, with the aim of applying them to DC microgrids is given in [12]. Because these converters can be used for so many applications, suitable modelling and control methods are necessary for their voltage regulation. Simulations performed in this study show a satisfying performance of the data-driven control strategy. Since DG will most likely cause a higher occurrence of fault current levels, in [13], a multi-inductor H bridge fault current limiter is proposed in order to reduce the frequency of occurrence of such types of problems. Positive results are obtained through experimental and simulated tests.

As for research studies revolving around converters, a full-bridge converter (FBC) for bidirectional power transfer is presented in [14]. The proposed FBC is an isolated DC-DC bidirectional converter, linked to a double voltage source—a voltage bus on one side, and a stack of super-capacitors (SOSC) on the other side and real prototype, compliant with automotive applications. In [15], a single DC source multilevel inverter with changeable gains and levels for low-power loads is proposed. The validation of this inverter was conducted through simulation and experimental tests using nine different modulations. A p-type module with virtual DC links to increase levels in multilevel inverters is proposed in [16], which are able to produce higher voltage levels with a lower number of components, making them appropriate for a wide range of applications. In another study [17], Janina Rząsa proposes an alternative carrier-based implementation of space vector modulation to eliminate common mode voltage in a multilevel matrix converter and evidences that part of the high-frequency output voltage distortion component is eliminated. The proposed modulation method is validated though simulation and experimental results. A comprehensive comparative analysis of impedance-source based DC-DC and DC-AC converters regarding passive component count and size, range of input voltage variation, and semiconductor stress is proposed in [18]. The authors analyzed the main impedance-source converters with or without inductor coupling and with or without a transformer; useing simulations and experiments to validate this.

Converters and inverters are used for several applications, as gathered from the above-mentioned studies, and as so, they are also used for permanent-magnet synchronous machines (PMSM). An improved model predictive torque control combined with discrete space vector modulation for a two-level inverter fed interior PMSM is proposed in [19] and the authors establish a cost function involving the excitation torque and reluctance torque. Simulation and experimental results are used to validate this study, in which the torque ripple and current ripple is reduced. The nonlinear effects, such as voltage and command voltage deviation, of a three-level neutral-point clamped inverter on speed sensorless control of an induction motor are studied in [20]. In this study a new voltage deviation compensation measure based on the volt-second balance principle is proposed and validated through experimental results.

Extensive research is also made in the area of motor drives. A new effective use and operation of fuzzy-logic controller-based two-quadrant operation of a permanent magnet brushless DC motor drive system for multipass hot-steel rolling processes is proposed in [21], with validation through simulation and experimental tests provided. Another study [22], proposed a field weakening control method that employs interpolation error compensation of the look-up table based PMSM method. As in the majority of these studies, the improvement reached by using the proposed method is validated through experimental and simulation tests.

Non-linear ceramic resistors such as metal oxide ZnO-based varistors are mainly utilised to protect electronic and electric circuits from overvoltage. The ZnO varistor, also known as metal oxide varistor, is the most well-known type of varistor, and, as such, it is a topic that attracts attention from the research community. This Special Issue is no exception. An experimental study on the effect of multiple lightning waveform parameters on the aging characteristics of ZnO varistors is proposed in [23], in which the aging rate and surface temperature rise of ZnO varistor under the impact of multi-pulse current was examined. Another experimental study was made in [24] by focusing on the failure mode of ZnO varistors under multiple lightning strokes, in which alterations were observed in the performance of these ZnO varistors after multiple lightning impulses. These changes were analyzed from micro and macro perspectives.

In the semiconductor area of research, a robust electrostatic discharge reliability design of an ultra-high voltage 300-V power n-channel lateral diffused MOSFETs, with elliptical cylinder super-junctions in the drain side, is given in [25]. The authors have concluded that this is a decent strategy and the human-body model capability of these ultra-high voltage n-channel lateral diffused MOSFETs could be successfully improved without altering the basic electrical properties or adding any extra cell area.

In this area of electromagnetism, a novel AC magnetic transmitter current source circuit is proposed in [26] for the application of frequency domain electromagnetic method prospecting. The proposed current source circuit is able to generate high frequency and high constant amplitude currents, which are the main technical problems for the frequency domain electromagnetic method. The results obtained through simulation and experimental tests show that the proposed circuit achieves the linearity of the rising/falling edge, a short reversal time, a low power loss, and a constant amplitude. Since the penetration of electric vehicles is constantly growing in the market, in another study [27], a new topology-based approach to improve vehicle-level electromagnetic radiation is proposed due to the fact that electric vehicles suffer from various electromagnetic interferences. The efficacy of this method was demonstrated through experimental tests that compared the predicted vehicle-radiated emissions at low frequency with the obtained experimental results.

Improved control methods, better energy efficiency and problem mitigation can be achieved at any level and in almost any system, as can be seen in the contributions to this Special Issue. Even though numerous challenges still remain, research and technology are vital tools for overcoming the challenges that arise in power electronics, specially by heading towards responsible and careful use of the environment. Power electronics plays a key role in the development of renewable energy systems and, therefore, in reducing greenhouse gases. Therefore, through small incremental steps, the objective is to strengthen the role of innovation, with the aim of facing the challenges that lay ahead [28] with efficient responses that, additionally, can ensure an economical, reliable and sustainable electrical supply, on which we have grown to become so dependent.